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Nonlinear Simulations of Energetic Particle Effects in Fusion Plasmas

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Outline:

- Need for self-consistent and conserving MHD + energetic particles (EP) models
- Commonly used MHD + EP coupling schemes and their limitations
- Variational approach to derivation of MHD-EP models
- NSTX example: effects of beam ion driven modes on electron transport.

Motivations for self-consistent MHD + fast particles models

- Energetic particles (EP) produced in fusion plasmas by heating and current drive mechanisms, or fusion reactions (alphas)

  - Energetic particle driven modes have effect on transport, and on EP redistribution and losses
  - Require non-perturbative approaches (EPM)

- Modeling particle acceleration:

  - Particle acceleration during reconnection [Drake et al., Phys. Plasmas 2019]
  - Self-consistent runaway-electron simulations [Hirvijoki et al., Phys. Plasma 2018]

- In many cases full kinetic simulations are too expensive, bulk plasma is described by MHD.

Commonly used MHD + fast particles coupling models

Current coupling scheme (CCS)

$$\frac{\partial(\rho\mathbf{V})}{\partial t} = -\nabla\cdot(\rho\mathbf{V}\mathbf{V}) - \nabla p + (\mathbf{J}-\mathbf{J}_{EP})\times\mathbf{B} - en_{EP}\mathbf{E}$$

Bulk plasma momentum equation (exact)

Pressure coupling scheme (PCS)

$$\frac{\partial(\rho\mathbf{V})}{\partial t} + \cancel{\frac{\partial(m_i n_{EP}\mathbf{V}_{EP})}{\partial t}} = -\nabla\cdot(\rho\mathbf{V}\mathbf{V}) - \nabla p - \nabla\cdot\mathbf{P}_{\mathbf{EP}} + \mathbf{J}\times\mathbf{B}$$

Whole plasma momentum equation; assumes that $n_{EP}V_{EP} << nV$

$$\mathbf{E} = -\mathbf{V}\times\mathbf{B} \quad + \cancel{\frac{\mathbf{J}-\mathbf{J}_{\mathbf{EP}}}{en_e}\times\mathbf{B}}$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \qquad \mathbf{J} = \nabla\times\mathbf{B}$$

$$\partial p^{1/\gamma}/\partial t = -\nabla\cdot(\mathbf{V}p^{1/\gamma}), \quad \partial\rho/\partial t = -\nabla\cdot(\mathbf{V}\rho)$$

ρ, p, $\mathbf{V}$ are bulk plasma density, pressure and velocity;
n_{EP}, $\boldsymbol{P}_{EP}$ and $\boldsymbol{J}_{EP}$ are energetic particle density, pressure and current,
$n_e = n_i + n_{EP}$, and $n_e >> n_{EP}$ is assumed.

CCS - could be energy conserving depending on EP model;
PCS – does not conserve energy, unless particle dynamic is modified.

Energetic particle models: Vlasov ions

Full-orbit ions: $$\frac{\partial F}{\partial t} + \mathbf{v}\cdot\frac{\partial F}{\partial \mathbf{x}} + (\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\frac{\partial F}{\partial \mathbf{v}} = 0$$

Current coupling scheme (CCS) – conserves energy, momentum, cross-helicity

- Hamiltonian [Tronci, J.Phys.A'10].
- Numerically efficient: $\mathbf{J}_{EP} = \int \mathbf{v}F\, d^3\mathbf{v},\ \mathrm{n}_{EP} = \int F\, d^3\mathbf{v}$
- No Hall term → ω/ω_{ci} <1 assumed.
- Implemented in HYM code [Belova,PoP'17].

Pressure coupling scheme (PCS) – does not conserve energy, unless Vlasov equation is modified; numerically more expensive:

$$\mathbf{P}_{EP} = \int \mathbf{v}\mathbf{v}F\, d^3\mathbf{v}, \quad \text{or} \quad \mathbf{P}_{EP} = \int (\mathbf{v}-\mathbf{u})(\mathbf{v}-\mathbf{u})F\, d^3\mathbf{v}$$

– Leads to unphysical instability for $k_{||}v_A \gtrsim \omega_{ci}$ [Tronci,PPCF'14]

NSTX-U

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Energetic particle models: Drift-kinetic particles

Drift-kinetic ions: $\frac{\partial F}{\partial t} + \nabla\cdot\left[\mathbf{v}_{gc}F\right] + \frac{\partial}{\partial v_\parallel}\left[\frac{1}{B_\parallel^*}\left(\mathbf{B}^*\cdot\mathbf{E}^*\right)F\right] = 0, \quad \mathbf{v}_{gc} = \frac{1}{B_\parallel^*}\left(v_\parallel \mathbf{B}^* - \mathbf{b}\times\mathbf{E}^*\right), \quad F = F(\mathbf{X}, v_\parallel, \mu)$

Current coupling (CCS)

$$\boxed{\mathbf{J}_{EP} = \int \mathbf{v}_{gc} F\, dv_\parallel d\mu + \nabla\times\mathbf{M}}$$

– could be made conservative,

– MEGA code [Todo, PoP'06], no energy conservation because $B_{||}^* \rightarrow B$, and $\mathbf{M} = -\int \mu\mathbf{b}\, F dv_{||} d\mu$

– Hamiltonian if "moving-dipole correction" included in $\mathbf{M} = -\int [\mu\mathbf{b} - v_{||}\mathbf{v}_{gc\perp}/\mathrm{B}]\, F dv_{||} d\mu$ [Burby, PPCF'17, Kaufman,PF'86]

Pressure coupling (PCS)

– does not conserve energy

– NIMROD [Hou,PoP'18], M3D-K [Fu,PRL'95], reduced MHD-DK code [Briguglio,PoP'95]

$$\boxed{\mathbf{P}_{EP} \rightarrow \mathbf{P}_{CGL} = \int \left[v_\parallel^2 \mathbf{bb} + \mu B(\mathbf{I} - \mathbf{bb})\right] F\, dv_\parallel d\mu}$$

– Could be made conservative [Close,JPP'18]

DK ordering: $\omega/\omega_{ci} \sim \rho_i/L \sim k_{||}/k_\perp \ll 1$

Energetic particle models: Gyrokinetic particles

Gyro-kinetic ions: $\frac{\partial F}{\partial t}+\nabla\cdot\left[\mathbf{v}_{gy}F\right]+\frac{\partial}{\partial v_{\parallel}}\left[\frac{1}{B_{\parallel}^{**}}\left(\mathbf{B}^{**}\cdot\mathbf{E}^{**}\right)F\right]=0,\ \ \mathbf{v}_{gy}=\frac{1}{B_{\parallel}^{**}}\left(v_{\parallel}\mathbf{B}^{**}-\mathbf{b}\times\mathbf{E}^{**}\right),\ F=F(\mathbf{X},v_{\parallel},\mu)$

Current coupling (CCS) – HYM code [Belova,JCP'97] for straight B_0; conserves energy, but not momentum or particle number in curved B_0.

– includes CAW, no contradiction with GK ordering if $n_{EP} << n_e$

– generalized using variational methods [Burby, PPCF'17]

Pressure coupling (PCS) – does not conserve energy

– M3D-K [Fu,PoP'06], MHD-GK code [Briguglio,PoP'98]

$$\boxed{\mathbf{P}_{EP}\rightarrow\mathbf{P}_{CGL}=\int\left[v_{\parallel}^{2}\mathbf{bb}+\mu B(\mathbf{I}-\mathbf{bb})\right]\delta(\mathbf{X}+\boldsymbol{\rho}-\mathbf{x})F\,dv_{\parallel}d\mu}$$

– Neglects difference between GC and GY coordinates

– No variational formulation yet

GK ordering: $\omega/\omega_{ci}\sim\rho_i/L\sim\delta B/B=O(\varepsilon)$, but $k_{\perp}\rho_i=O(1)$

Full kinetic simulation approach to EP effects

All-kinetic-ions option (bulk and energetic) avoids coupling issues.

- More expensive to run: resolution, number of particles
- Hybrid scheme (full-orbit ions + fluid e) includes Hall term

  HYM code (FRC simulations)

- GK/DK codes use (φ, $\delta A_{||}$) fields; can not do CAW, conservation laws?

Include kinetic electron physics; allows EP / turbulence interaction, TAE interaction with continuum, KAW.

Variational approach to MHD-EP coupling models

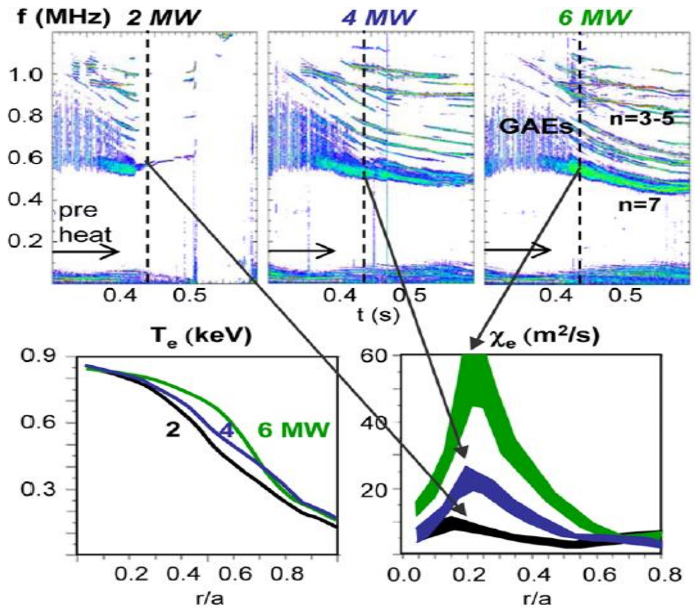
- Dropping EP terms in MHD equations destroys conservation properties.
- Hamiltonian PCS is derived by neglecting terms ~$n_{EP}v_{EP}$ in MHD-Vlasov (MHD-DK) Hamiltonian [Tronci,J.Phys.A'10; Close, JPP'18].
- Variational methods are very successful in deriving improved CCS.
- Variational PCS conserve energy, momentum, cross-helicity etc. MHD equations are the same with p $\rightarrow$ p + $\mathbf{P}_{EP}$, but Vlasov equation includes extra terms, corresponding to a shift to fluid frame [Close,JPP'18]:

$$\frac{\partial F}{\partial t} + (\mathbf{v}+\mathbf{u})\cdot\frac{\partial F}{\partial \mathbf{x}} + [\mathbf{E} + (\mathbf{v}+\mathbf{u})\times\mathbf{B} - \nabla\mathbf{u}\cdot\mathbf{v}]\cdot\frac{\partial F}{\partial \mathbf{v}} = 0$$

- Same could be achieved by keeping EP momentum in fluid equations, making PCS and CCS conservation properties identical.

Correlation between Alfven eigemodes activity and flattening of electron temperature profile has been observed in NSTX [Stutman, PRL'09]

- Sub-cyclotron frequency Alfven eigenmodes are driven by cyclotron resonance with beam ions.
- Flattening of T_e profile with increased beam power [Stutman, PRL 2009]
- Proposed mechanisms:
  - enhanced electron transport due to orbit stochasticity in the presence of multiple modes [Gorelenkov, NF 2010]
  - energy channeling due to mode conversion and coupling to KAW [Belova, PRL 2015]
- Anomalously low T_e potentially can have significant implications for future fusion devices, especially low aspect ratio tokamaks.



Correlation between GAE activity, T_e flattening, and central electron heat diffusivity χ_e in NSTX H modes with 2, 4, and 6MW neutral beam.

Self-consistent MHD + fast ions coupling scheme (HYM)

Background plasma - fluid:

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_b) \times \mathbf{B} - n_b (\mathbf{E} - \eta \mathbf{j})$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{j}$$

$$\mathbf{B} = \mathbf{B}_0 + \nabla \times \mathbf{A}$$

$$\partial \mathbf{A} / \partial t = -\mathbf{E}$$

$$\mathbf{j} = \nabla \times \mathbf{B}$$

$$\partial p^{1/\gamma} / \partial t = -\nabla \cdot (\mathbf{V} p^{1/\gamma})$$

$$\partial \rho / \partial t = -\nabla \cdot (\mathbf{V} \rho)$$

Fast ions – delta-F scheme:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{E} - \eta \mathbf{j} + \mathbf{v} \times \mathbf{B}$$

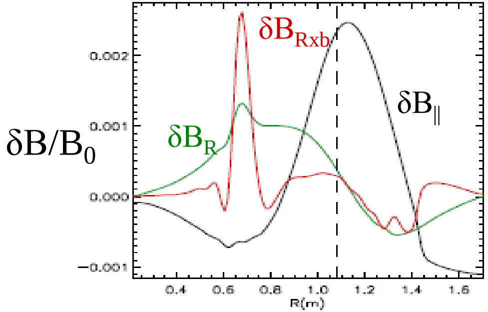
$w = \delta F / F$ - particle weight

$$\frac{dw}{dt} = -(1 - w) \frac{d(\ln F_0)}{dt}$$

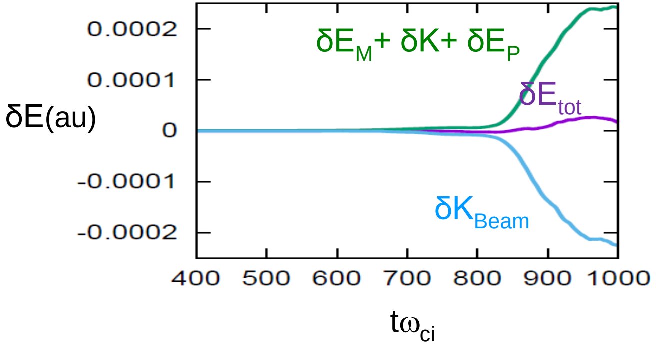
$$F_0 = F_0(\varepsilon, \mu, p_\phi)$$

ρ, **V** and p are thermal plasma density, velocity and pressure, n_b and $\mathbf{j}_b$ are beam ion density and current, and $n_b << n_e$ – is assumed.

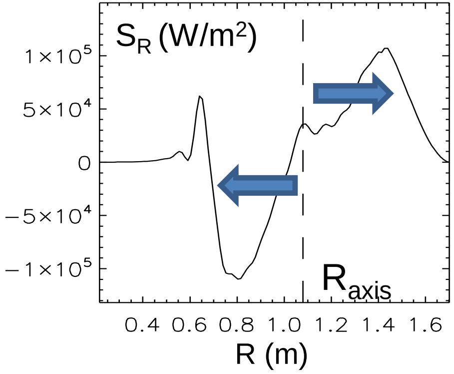
Relation between mode conversion and T_e flattening?



Radial profiles of magnetic field perturbation for the n=4 compressional Alfven eigemode (CAE).



Time evolution of the fluid energy (green), the beam ion energy, and the total energy of the system.



Radial component of Poynting vector **S**=<**E**x**B**>. Energy flux is directed away from magnetic axis to resonance location.

Change of energy flux across resonant layer at R~0.7m corresponds to power absorption at the high-field-side resonance of P~ 0.2 MW for $\delta B_{\|}/B_0$~ 3×10^{-3}. CAE/KAW coupling can provide an efficient energy channeling mechanism.

Conclusions

- Most of existing nonlinear MHD-EP codes use coupling schemes which do not conserve energy.
- Energy conserving current coupling schemes (CCS) have been developed for Vlasov, drift- and gyro- kinetic energetic particle models.
- CCS is more accurate and has better conservation properties compared to PCS, yet most codes use PCS.
- Pressure coupling schemes generally do not conserve energy, but can be fixed by changing particle dynamics.