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HYM simulations of GAEs in DIII-D

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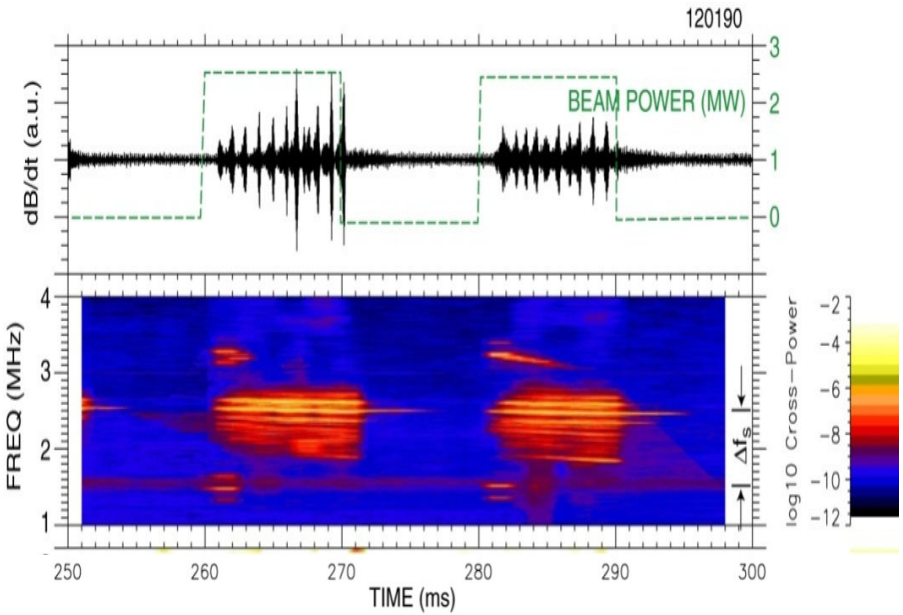
Sub-cyclotron frequency Alfven Eigenmodes were observed in low toroidal field experiments in DIII-D



Time evolution of the magnetic signal and spectra in a discharge with periodic injection of 80 keV left beams; B_T = 0.6 T; f_{ci}=4.5Mhz. [Heidbrink, NF2006]

- High-frequency AEs were observed in DIII-D in low toroidal field discharges (NSTX similarity experiments) mostly when $v_b \gtrsim v_A$.
- These modes are counter-propagating, and driven unstable by Doppler shifted cyclotron resonance with beam ions.
- Mode polarization was not measured directly, but large dB_φ was observed near the edge.
- They were identified as compressional Alfven eigenmodes (CAE) based on high frequency ($f \sim 0.6 f_{ci}$) and comparison with previous theoretical instability conditions.
- For comparison:
  $f_{CAE} \sim 0.3\text{-}0.5\ f_{ci}$, $f_{GAE} \sim 0.1\text{-}0.3\ f_{ci}$ (NSTX);
  $f_{GAE} \sim 0.4\ f_{ci}$ (NSTX-U)

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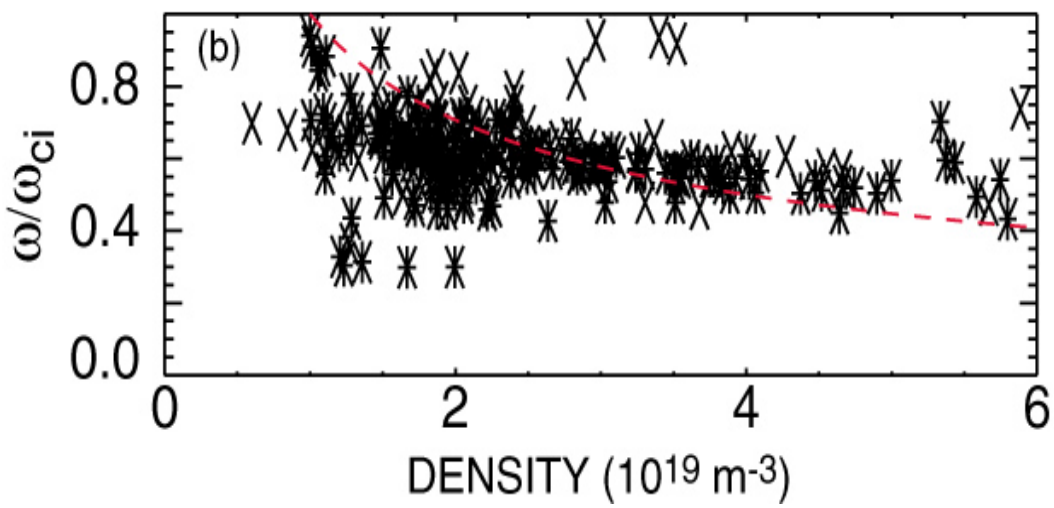
Some observed features could not be explained by CAE theory



Normalized frequency of the strongest mode vs n_e did not show $\omega \sim n_e^{-1/2}$ scaling (red line) [Heidbrink, NF2006]

- Frequency correlate strongly with B_{tor}, but weakly with n_e.
- f/f_{ci} for the most unstable modes remains ≈ const as B_{tor} is varied
- Three levels of frequency splitting were observed, not consistent with CAE dispersion:
  $\Delta f/f \sim 0.8\text{MHz}/2.5\text{MHz}=0.32$,
  $\Delta f/f \sim 110\text{kHz}/2.5\text{MHz}=0.04$,
  $\Delta f/f \sim 20\text{kHz}/2.5\text{MHz}=0.008$,

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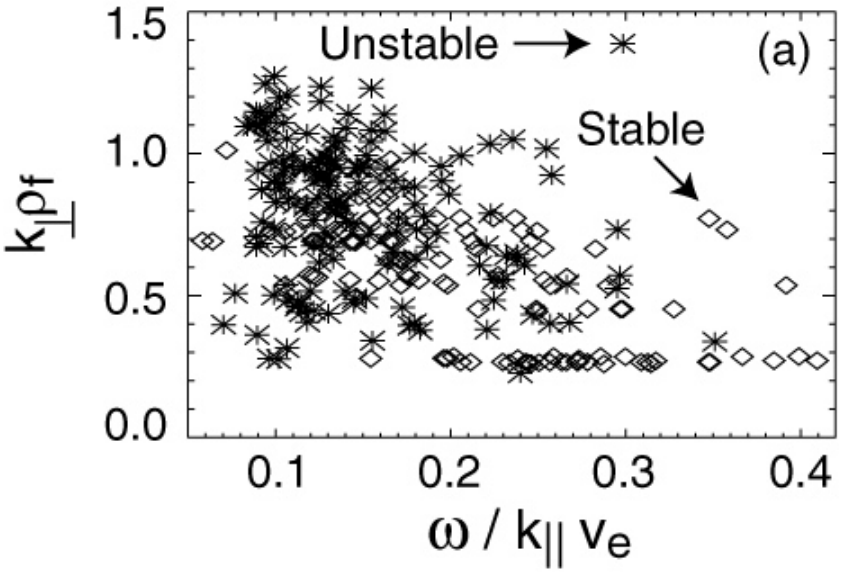
Early CAE/GAE theory predicted instability for larger values of $k_\perp \rho_b$



[Heidbrink, NF2006]

- Experimental esimates got $k_\perp \rho_b \lesssim 1$ (based on CAE dispersion ie from observed ω/v_A) and local B value.
- Re-scaling for B on axis gives even lower values: $k_\perp \rho_b \lesssim 0.6\, k_\perp v_A/\omega$.
- Theory predicted the CAE instability for:

  $1 < k_\perp \rho_b < 2$
- For Global Alfven eigenmode (GAE) instability:

  $2 < k_\perp \rho_b < 4$
- New GAE/CAE theory [Belova,2019, Lestz, 2020] predicts stronger instability for small $k_\perp/k_{||}$ ($k_\perp \rho_b \ll 1$), and GAEs more unstable than CAEs.

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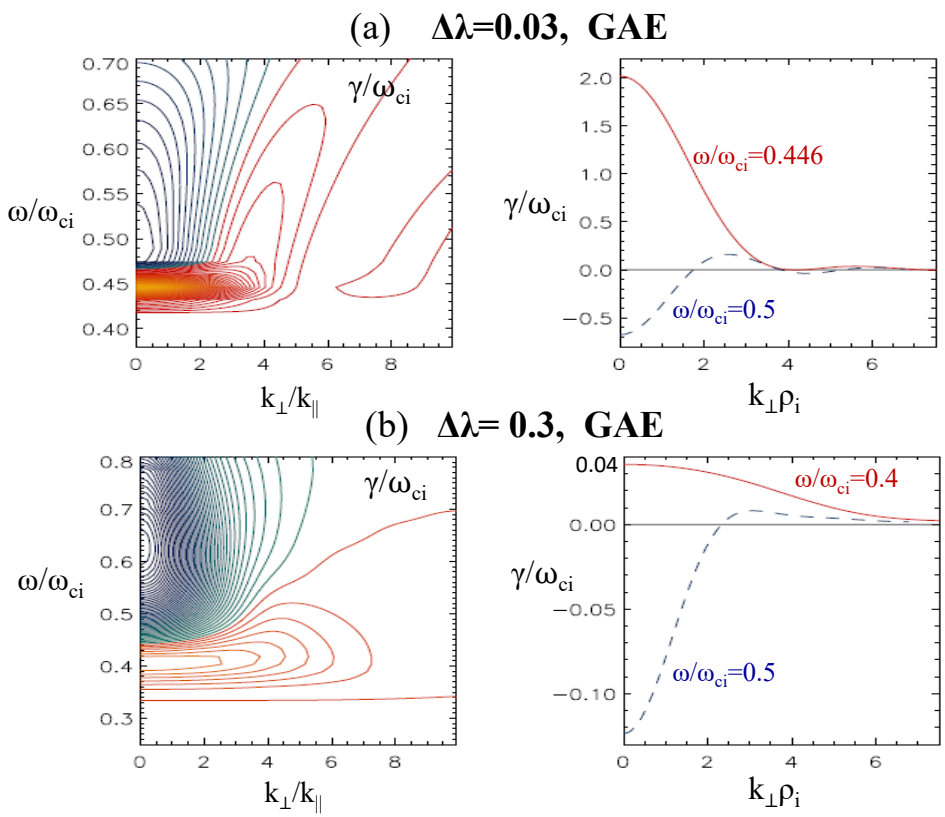
New analytic theory of counter-GAE instability has been developed



Contour plots of growth rate and plots of γ vs $k_\perp\rho_i$ for very narrow and for wide pitch parameter distributions; blue-green contours correspond to negative values, and orange-red to positive;

(a) $\Delta\lambda$=0.03, γ/ω_{ci} values are between -0.7÷2.0;

(b) $\Delta\lambda$= 0.3, γ/ω_{ci} values are between -0.22÷0.036.

$$\gamma \approx \pi \frac{n_b}{n_i}\frac{\omega_{ci}^2}{2\omega} v_{||res}^3 A \int_0^{\lambda_m} \frac{d\lambda}{(1-\lambda)^2}\lambda \frac{\partial f}{\partial \lambda}\frac{J_1^2}{\xi^2}\Bigg|_{v_{||}=v_{res}},$$

$v_{||res} = (\omega_{ci} - \omega)/|k_{||}|$, $\lambda_m = 1 - v_{||res}^2/v_0^2$

v_0 – injection velocity,

$f = A \exp[-(\lambda-\lambda_0)^2/\Delta\lambda^2] / (v^3+v_*^3)$.

- Resonant beam ions drive the instability provided: $\partial f/\partial\lambda>0$, i.e. when $\lambda<\lambda_0$, and stabilizing otherwise.
- Most unstable modes have $k_\perp\rho_b<1$, and are in the range [1,2]:

$$(1 + v_0/v_A)^{-1} < \omega/\omega_{ci} \le (1 + v_0/v_A\sqrt{[1-\lambda_0]})^{-1}$$

  - follows from cyclotron resonance condition and instability condition: $\lambda_m<\lambda_0$
- Higher frequency modes with $\omega/\omega_{ci} > (1 + v_0/v_A\sqrt{[1-\lambda_0]})^{-1}$ have smaller growth rates and unstable if: $2< k_\perp\rho_b<4$ (Bessel regime [3]).

[1] Belova, Phys. Plasmas 2019
[2] Lestz, Phys. Plasmas 2020
[3] Gorelenkov, NF03.

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Dependence on frequency and $k_\perp \rho_i$ for counter-GAEs

$$\gamma \approx \pi \frac{n_b}{n_i} \frac{\omega_{ci}^2}{2\omega} v_{||res}^3 A \int_0^{\lambda_m} \frac{d\lambda}{(1-\lambda)^2} \lambda \frac{\partial f}{\partial \lambda} \frac{J_1^2}{\xi^2} \Big|_{v_{||}=v_{res}},$$

$\lambda = v_\perp^2/v^2$, $\lambda_m = 1 - v_{||res}^2/v_0^2$

$v_{||res} = (\omega_{ci} - \omega)/|k_{||}|$

$\xi^2 = (k_\perp v_\perp/\omega_{ci})^2 = \lambda/(1-\lambda)\,(k_\perp v_{||res}/\omega_{ci})^2$

$f = A \exp[-(\lambda-\lambda_0)^2/\Delta\lambda^2] / (v^3+v_*^3)$

The sign of the integrand is determined by sign of $\partial f/\partial\lambda = 2(\lambda_0 - \lambda)/\Delta\lambda^2 f$

→ particles with small $v_\perp$ $(\lambda<\lambda_0)$ <u>are always destabilizing</u>,

→ particles with large $v_\perp$ $(\lambda>\lambda_0)$ <u>are always stabilizing</u>.

1. For $\omega < \omega_{ci}$, $v_{||res} \sim v_0$ and $\lambda_m<1$
   - γ is always positive if $\lambda_m \leq \lambda_0$ – sufficient condition for instability gives an approximate range of unstable frequencies: $\omega/\omega_{ci} \leq (1 + v_0/v_A\sqrt{[1-\lambda_0]})^{-1}$
   - most unstable modes have $k_\perp\rho_i$ <<1 when $(J_1/\xi)^2=1/4$
2. High-frequency limit $\omega \approx \omega_{ci}$, $v_{||res} << v_0$ and $\lambda_m \approx 1$
   - for small $k_\perp\rho_i \rightarrow (J_1/\xi)^2=1/4$ and γ is always negative
   - for $k_\perp\rho_i \gtrsim 2$, Bessel factor <u>reduces stabilizing effect</u> of large $v_\perp$ $(\lambda>\lambda_0)$ particles.

<u>This clarifies the role of fast ion FLR effects:</u> small $k_\perp\rho_i$ are destabilizing $(\lambda<\lambda_0)$, while large $k_\perp\rho_i$ reduces the stabilizing effect of particles with $\lambda>\lambda_0$.

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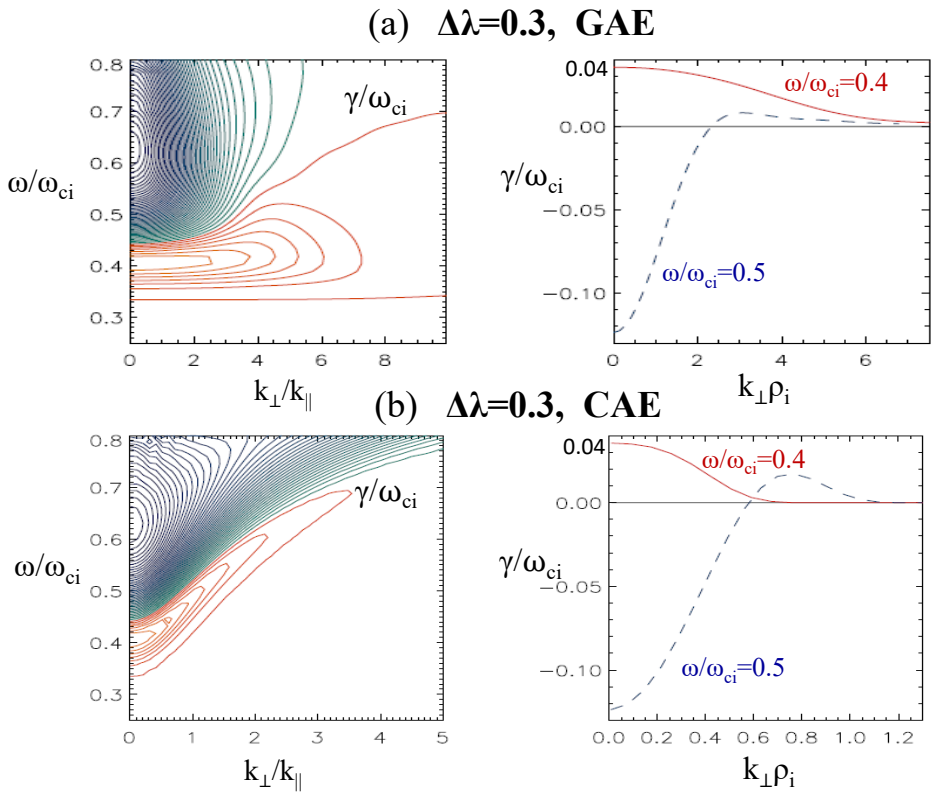
Counter-CAE are predicted to be less unstable than GAEs



Contour plots of growth rate and plots of γ vs $k_\perp\rho_i$ for wide pitch parameter distributions; blue-green contours correspond to negative values, and orange-red to positive.

(a) GAE: $\Delta\lambda_0$=0.3, γ/ω_{ci} values are between -0.22÷0.036;

(b) CAE: $\Delta\lambda_0$=0.3, same range of γ/ω_{ci} values as in (a).

$$\gamma \approx \pi \frac{n_b}{n_i}\frac{\omega_{ci}^2}{2\omega} v_{||res}^3 A \int_0^{\lambda_m} \frac{d\lambda}{(1-\lambda)^2}\lambda \frac{\partial f}{\partial \lambda}\frac{(J_0^2 - J_2^2)}{4}\Bigg|_{v_{||}=v_{res}},$$

$v_{||res}=(\omega_{ci}-\omega)/|k_{||}|$, $\lambda_m = 1 - v_{||res}^2/v_0^2$.

v_0 – injection velocity,

$f=A\exp[-(\lambda-\lambda_0)^2/\Delta\lambda^2]/(v^3+v_*^3)$ [1,2].

- Same (sufficient) instability condition: $\lambda_m<\lambda_0$ as GAEs. Same γ in the limit $k_\perp$=0.
- Most unstable modes have $k_\perp\rho_b<1$, and are in the range:

$$(1+\alpha v_0/v_A)^{-1} < \omega/\omega_{ci} < (1+\alpha v_0/v_A\sqrt{[1-\lambda_0]})^{-1}$$

where $\alpha=|k_{||}/k|$; follows from cyclotron resonance condition and $\lambda_m<\lambda_0$.

- Counter-CAEs have much smaller growth rates than GAEs for $k_\perp/k_{||} \gtrsim 1$.
- Two-fluid effects / coupling to shear AW reduce growth rate of CAEs [2].

[1] Belova et al, Phys. Plasmas 2019
[2] Lestz et al., Phys. Plasmas 2020.

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Theory predict scaling of most unstable GAE with B_{tor},n_e,λ_0

Predicted range of mostunstable counter-GAEs:

$$\frac{1}{(1+v_0/v_A)} < \frac{\omega}{\omega_{ci}} \leq \frac{1}{(1+v_0/v_A\sqrt{1-\lambda_0})} \quad ,$$

- $\omega \sim \omega_{ci} \rightarrow$ Nearly linear scaling with B_{tor}.
- Weaker than $1/\sqrt{n_e}$scaling with density.
- Larger v_0/v_A results in smaller values of ω/ω_{ci}:

  $\omega/\omega_{ci} \approx 0.6$ for $v_0/v_A \sim 1$ and $\lambda_0 \sim 0.6$(DIII-D?),

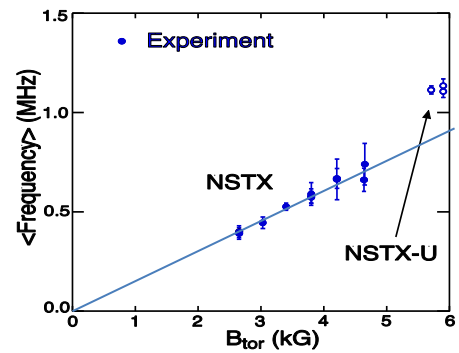
  $\omega/\omega_{ci} \approx 0.4$ for $v_0/v_A \sim 2$ (NSTX-U),

  $\omega/\omega_{ci} \lesssim 0.2$ for $v_0/v_A \gtrsim 4$ (NSTX),

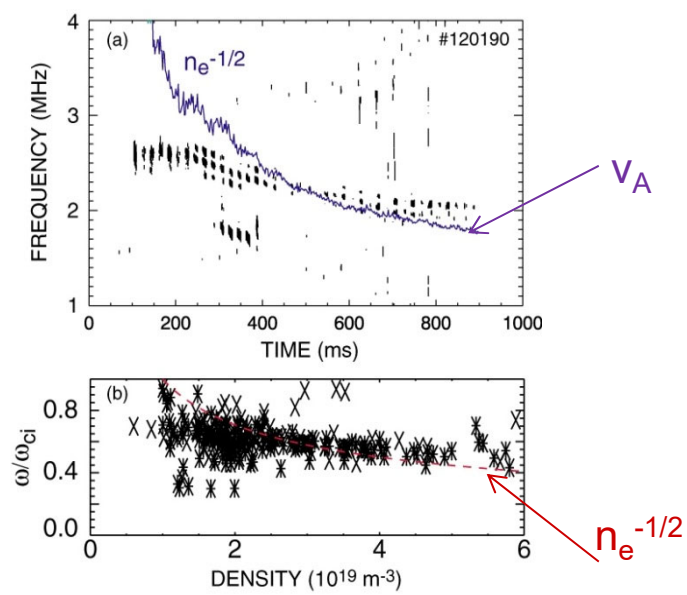
- Scaling with λ_0: larger $\lambda_0 \rightarrow$ larger ω/ω_{ci}

$\omega/\omega_{ci} \approx 0.6$ for λ_0=0.5,
$\omega/\omega_{ci} \approx 0.7$ for λ_0=0.8 and $v_0/v_A \sim 1$

– consistent with DIII-D 'left' and 'right' (ω/ω_{ci}=0.56 and ω/ω_{ci}=0.69 [Heidbrink,NF06]) beam sources.



A roughly linear scaling with B_{tor} is seen for GAE frequency in **NSTX-U** [Fredrickson,NF18]



DIII-D: (a) Spectra for B_{tor}=0.6 T with 80 keV left beams; (*b*) Frequency of the strongest mode vs the line-average n_e for all the discharges in the database [Heidbrink,NF06].

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HYM – HYbrid and MHD code

Applications

- NSTX(-U)

  - Sub-cyclotron frequency Alfven eigenmodes (GAE and CAE)

- ICC Theory and Modeling

  - Hybrid simulations of spheromak merging

  - FRC: Effects of beam ions on stability

    - Rotation control

    - n=2 rotational and n=1 wobble modes

Code description

- 3-D nonlinear.
- Physical models:
  - Resistive MHD & Hall-MHD
  - Hybrid (fluid electrons, particle ions)
  - **MHD/particle (one-fluid thermal plasma, + energetic particle ions)**
  - Drift-kinetic particle electrons
- Full-orbit kinetic ions.
- For particles: delta-f / full-f numerical scheme.
- Parallel (3D domain decomposition, MPI)

Self-consistent MHD + fast ions coupling scheme

Background plasma - fluid:

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_b) \times \mathbf{B} - n_b(\mathbf{E} - \eta \mathbf{j})$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{j}$$
$$\mathbf{B} = \mathbf{B}_0 + \nabla \times \mathbf{A}$$
$$\partial \mathbf{A} / \partial t = -\mathbf{E}$$
$$\mathbf{j} = \nabla \times \mathbf{B}$$
$$\partial p^{1/\gamma} / \partial t = -\nabla \cdot (\mathbf{V} p^{1/\gamma})$$
$$\partial \rho / \partial t = -\nabla \cdot (\mathbf{V} \rho)$$

Fast ions – delta-F scheme:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$
$$\frac{d\mathbf{v}}{dt} = \mathbf{E} - \eta \mathbf{j} + \mathbf{v} \times \mathbf{B}$$

$w = \delta F / F$ - particle weight

$$\frac{dw}{dt} = -(1 - w) \frac{d(\ln F_0)}{dt}$$

$$F_0 = F_0(\varepsilon, \mu, p_\phi)$$

ρ, **V** and p are thermal plasma density, velocity and pressure, n_b and $\boldsymbol{j}_b$ are beam ion density and current, and $n_b << n_e$ – is assumed.

NSTX-U

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Fast ions – delta-f scheme: $F_0=F_0(\varepsilon,\mu,p_\varphi)$

Equilibrium distribution function $F_0 = F_1(v)\, F_2(\lambda)\, F_3(p_\varphi, v)$

$$F_1(v) = \frac{1}{v^3 + v_*^3}, \text{ for } v < v_0$$

$$F_2(\lambda) = \exp(-(\lambda - \lambda_0)^2 / \Delta\lambda^2)$$

$$F_3(p_\phi, v) = \frac{(p_\phi - p_0)^\beta}{(R_0 v - \psi_0 - p_0)^\beta}, \text{ for } p_\phi > p_0$$

where $v_0 = 2\text{-}5v_A$, $v_* = v_0/2$, $\lambda = \mu B_0/\varepsilon$ – pitch angle parameter, $\lambda_0 = 0.5\text{-}0.7$,
and $\mu = \mu_0 + \mu_1$ includes first-order corrections [Littlejohn'81]:

$$\mu = \frac{(\mathbf{v}_\perp - \mathbf{v}_\mathbf{d})^2}{2B} - \frac{\mu_0 v_\parallel}{2\mathrm{B}}[\hat{b}\cdot\nabla\times\hat{b} - 2(\hat{a}\cdot\nabla\hat{b})\cdot\hat{c}]$$

$\mathbf{v}_d$ is magnetic gradient and curvature drift velocity, $\hat{c} = \mathbf{v}_\perp / v_\perp$, $\hat{a} = \hat{b}\times\hat{c}$.

Parameters are chosen to match TRANSP beam profiles.

HYM simulations for DIII-D

Two basic cases are considered:

1. NSTX-similarity experiments on DIII-D from [W. Heidbrink et al, NF 2006]

   B_{tor}= 0.6T, R_0= 1.63m, a= 0.56m, I= 0.6MA, q_0=1.2, q_{max}=4.5, β_{tot} ~9%
   Beam parameters: E= 80keV, V_0/V_A= 1.5, n_b/n_e ~4%, β_{beam} ~3%
   Observed mode parameters: f= 2.5MHz, f_{ci}= 4.5MHz

2. More recent dedicated GAE/CAE experiments [N. Crocker, IAEA 2020]

   B_{tor}= 1.24T, R_0= 1.72m, I= 0.62MA, β_{tot} ~2%
   Beam parameters: E= 78keV, V_0/V_A= 0.8, n_b/n_e ~3%, β_{beam}~0.5%
   Observed mode parameters: f= 5.5MHz, f_{ci}= 9.5MHz

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Case 1: NSTX-similarity experiments on DIII-D

B_{tor}= 0.6T, R_0= 1.63m, a= 0.56m, I= 0.6MA, q_0=1.2, q_{max}=4.5, β_{tot} ~9%
Beam parameters: E= 80keV, V_0/V_A= 1.5, n_b/n_e ~4%, β_{beam} ~3%
Observed mode parameters: f= 2.5MHz, f_{ci}= 4.5MHz

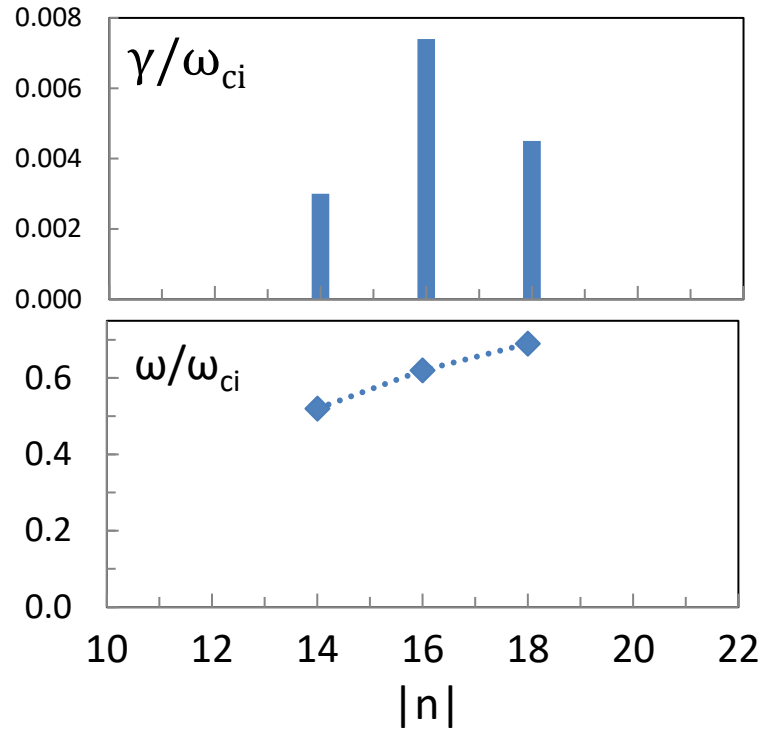
HYM parameters:

V_0/V_A= 1.5, n_b/n_e =4%, λ_0= 0.75, $\Delta\lambda$= 0.2

- Unstable modes have shear Alfven polarization with $\delta B_{||} << \delta B_{\perp}$ and are counter-rotating – GAEs
- |n|=14-18 with m~2-3,
- frequencies ω/ω_{ci}=0.52-0.69, and growth rates γ/ω_{ci}=0.003 – 0.007.
- Can estimate unstable |n| from $n \approx R_0 k_{||}$ and resonant condition to get

$n=R_0(\omega_{ci}-\omega)/v_{||}$,

and from $\lambda<\lambda_0 \rightarrow v_{||}>0.5v$, so that n= 13-20.



Toroidal mode numbers were not measured in experiments, but estimated as n=-O(10). For #120196 shot n=-16+/-5 was inferred from correlation with changes in Mirnov signal [W. Heidbrink et al, NF 2006]

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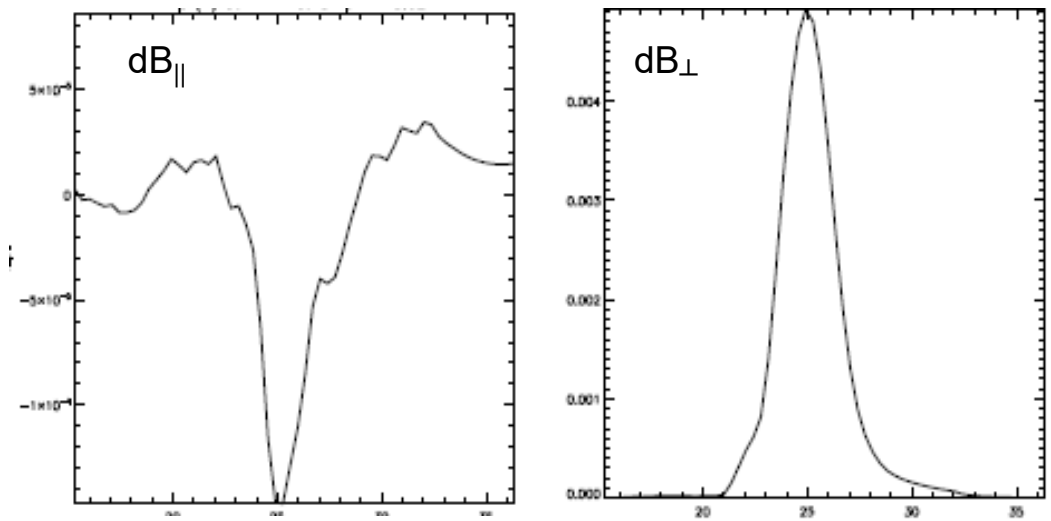
Modes have shear polarization (GAEs)



Radial profiles of **δB** for *n= -16* counter-GAE

- Simulations show unstable counter-propagating GAEs with $\delta B_{||} < 0.1 \delta B_{\perp}$
- Located near the magnetic axis

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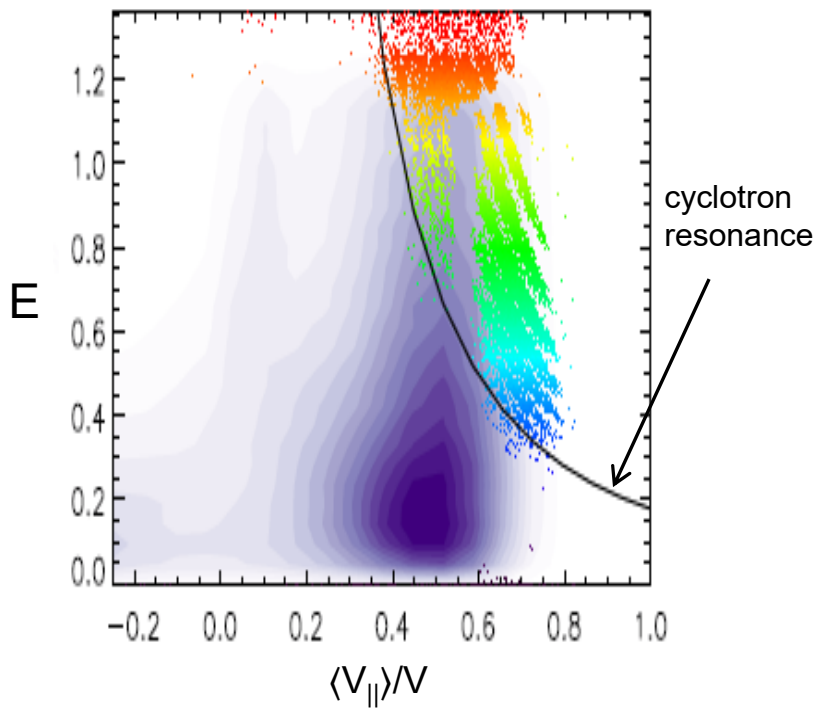
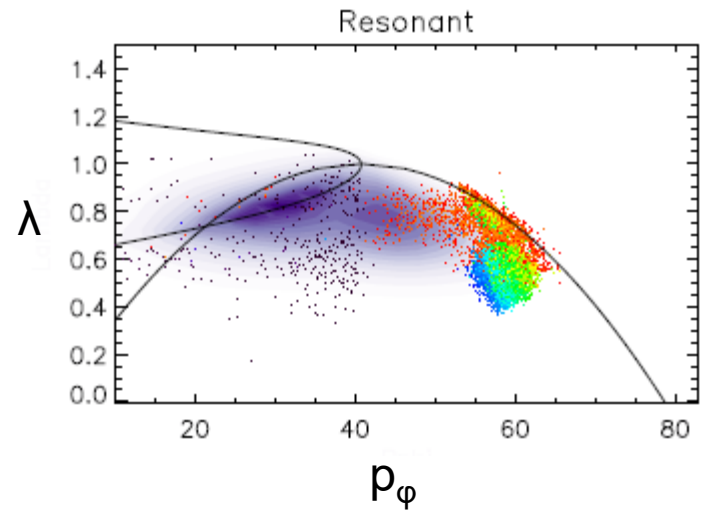
Large number of sideband resonances can be seen for each unstable GAE



HYM fast-ion energy vs pitch distribution from n=-16 GAE simulations; resonant line is shown for $v_{||}=0.8v_A$; colour dots show resonant particles.



Location of resonant particles in phase space: $\lambda=\mu B_0/\varepsilon$ vs p_φ. Particle color corresponds to different energies: from E=0 (purple) to E=80 keV (red).

NSTX-U

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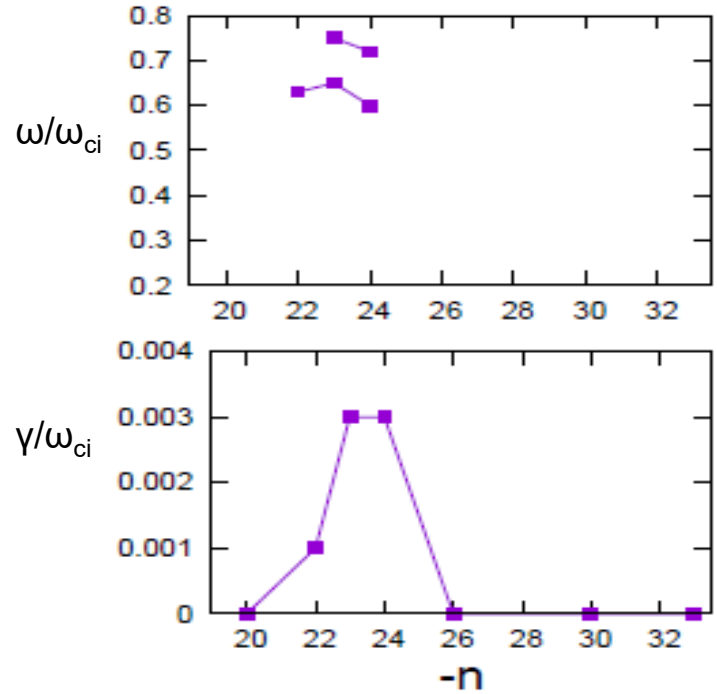
Case 2: Higher B_{tor} experiments on DIII-D

B_{tor}= 1.24T, R_0= 1.72m, I= 0.62MA, q_0=0.94, q_{max}=6, β_{tot} ~2%
Beam parameters: E= 75-80keV, V_0/V_A= 0.8, n_b/n_e =3%, β_{beam} ~0.5%
Observed mode parameters: f= 5.5 MHz, f_{ci}= 9.5 MHz

HYM parameters:

V_0/V_A= 0.8-0.9, n_b/n_e =3-6%,
λ_0=0.65-0.75, $\Delta\lambda$= 0.2

- Unstable modes have shear Alfven polarization with $\delta B_{||} << \delta B_{\perp}$ and are counter-rotating – GAEs
- $|n|$=22-24 with m~3-4,
- frequencies ω/ω_{ci}=0.6-0.75, and growth rates γ/ω_{ci}=0.001 – 0.003.
- Estimate for $\lambda<\lambda_0 \rightarrow v_{||}>0.5v$ gives:
  $|n|=R_0(\omega_{ci}-\omega)/v_{||}$ ~23-33,



Growth rates and frequencies of unstable counter-GAEs from HYM simulations for v_0/v_A=0.8, n_b=6%, λ_0=0.75.

Measured toroidal mode numbers were n≈ -7, possibly aliased by -20 [S.Tang, 2020].

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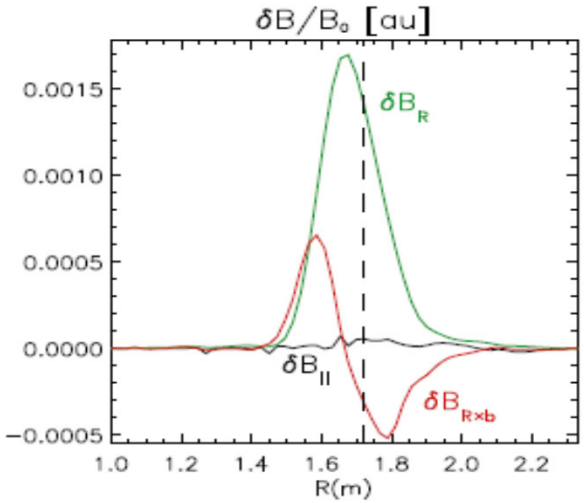
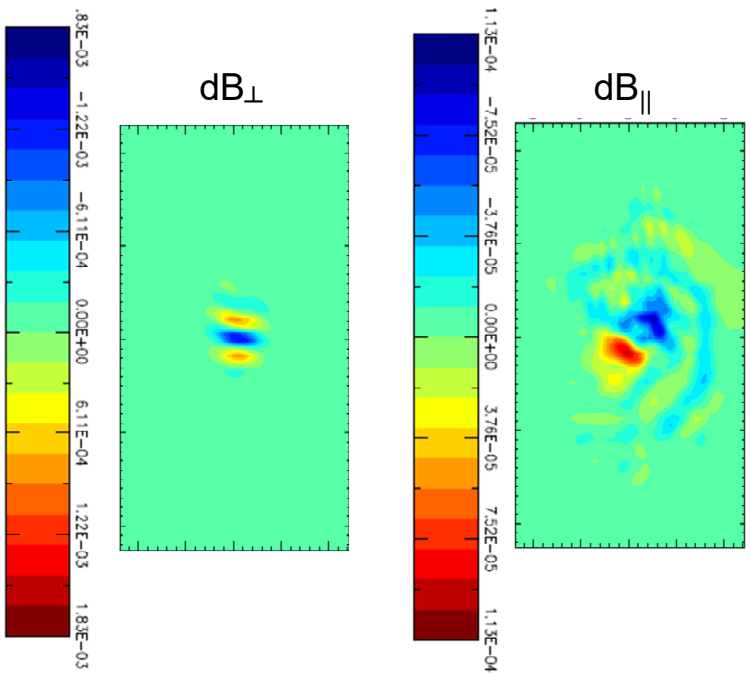
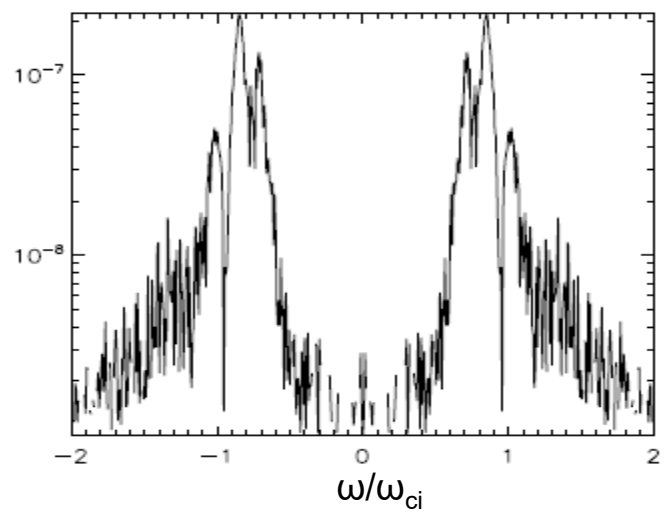
HYM simulations demonstrate that unstable modes in DIII-D have SAW polarization (GAEs)



Radial profile of **δB** for *n*= *-22* counter-GAE

- Simulations show unstable counter-propagating GAEs with $\delta B_{||}<0.1\delta B_{\perp}$,
- High toroidal mode numbers |n|>20
- Frequency ω/ω_{ci}~0.6-0.7,
- $k_{\perp}\rho_b$~0.5,
- Located near the magnetic axis





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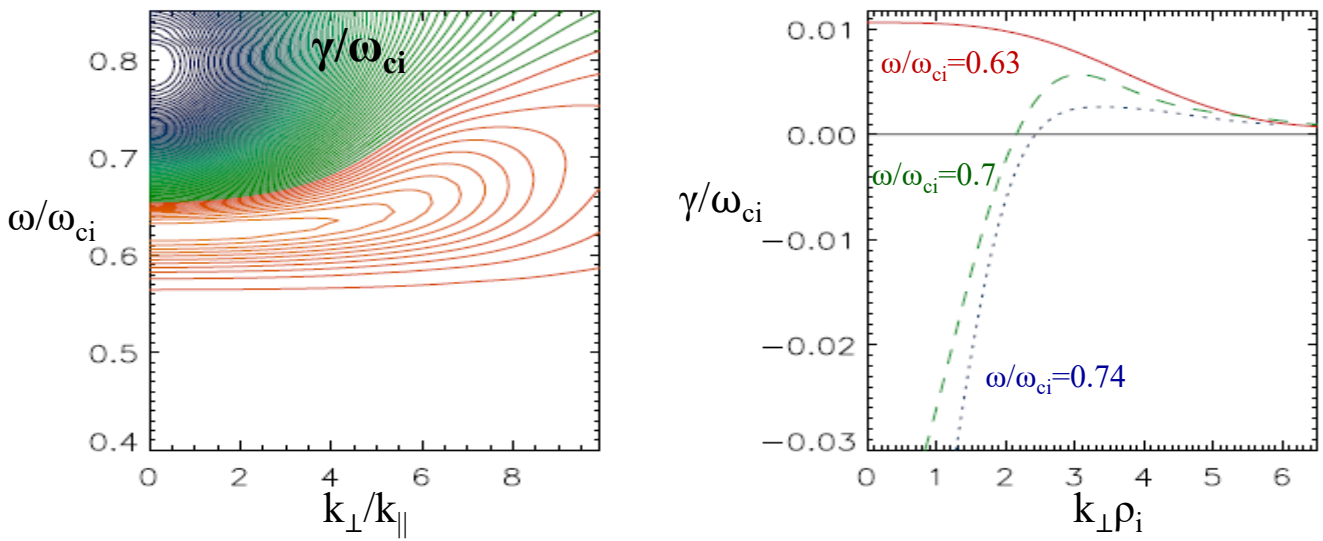
HYM simulations for DIII-D agree with analytic predictions



From local dispersion relation:(a) *contour plot of GAE growth rate and (b) plot of $\gamma(k_\perp\rho_i)$ for DIII-D parameters; blue-green contours correspond to negative values, and orange-red to positive; γ/ω_{ci} values are between -0.08:0.011.*

- Frequency of most unstable modes: ω/ω_{ci}~0.6-0.7,
- GAE linear growth rate is largest for small values of $k_\perp$ ($k_\perp\rho_i$<1)
- NBI injection velocity V_0/V_A=0.9 and density n_b/n_e=0.05.
- λ_0=0.7, $\Delta\lambda$=0.2

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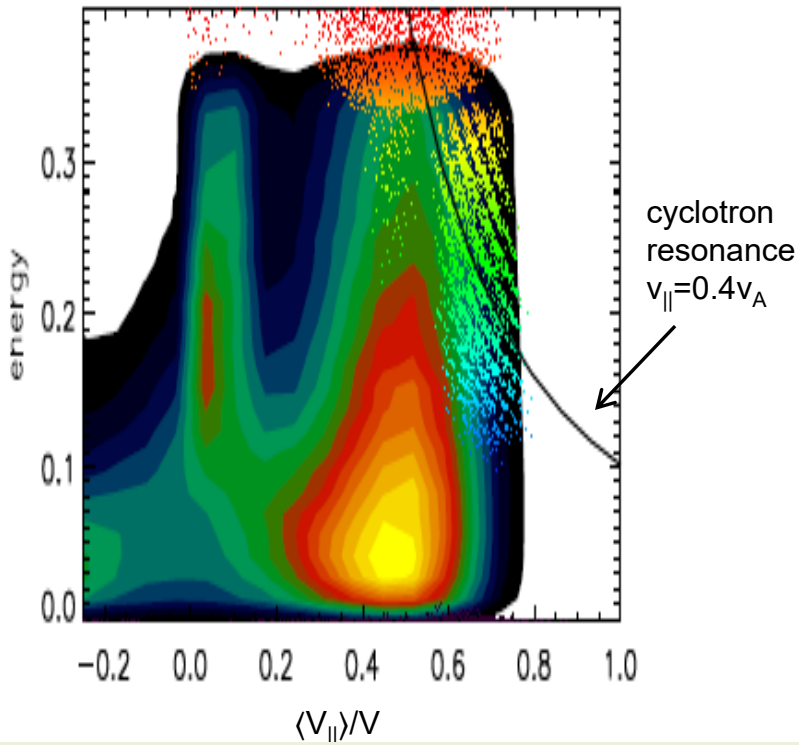
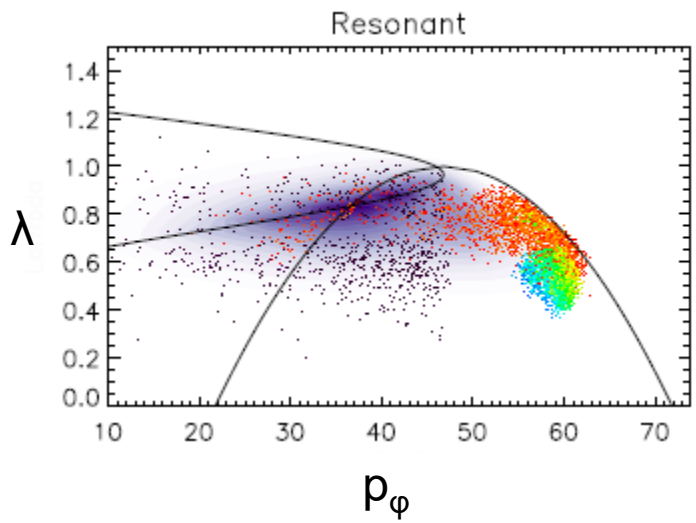
Large number of sideband resonances can be seen for each unstable GAE





HYM fast-ion energy vs pitch distribution from n=-22 GAE simulations; resonant line is shown for $v_{||}=0.4v_A$; colour dots show resonant particles.

Location of resonant particles in phase space: $\lambda=\mu B_0/\varepsilon$ vs p_φ. Particle color corresponds to different energies: from E=0 (purple) to E=80 keV (red).

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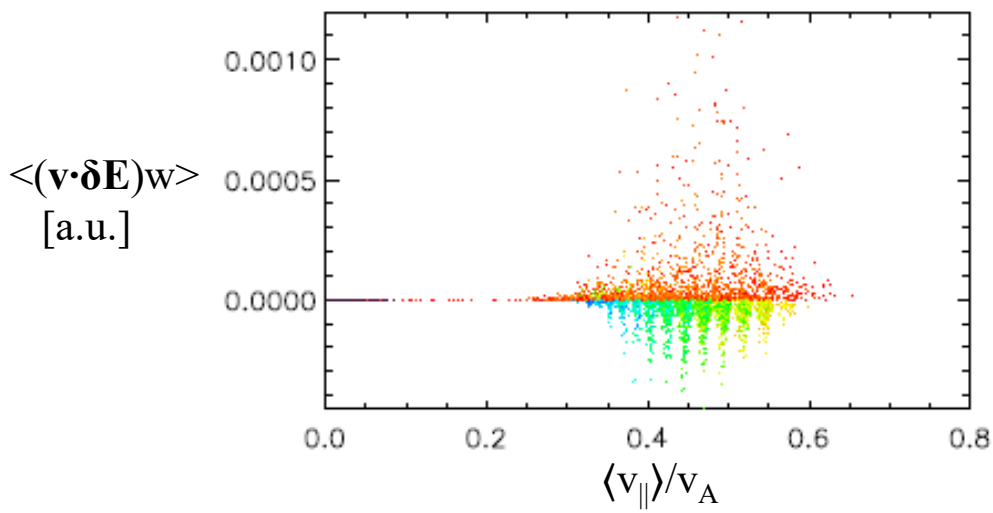
Two groups of resonant particles: driving ($\lambda<\lambda_0$) or damping ($\lambda>\lambda_0$)

Energy exchange rate between the beam ions and the mode $\Delta K = \int (\boldsymbol{\delta j_b} \cdot \boldsymbol{\delta E}) d^3x = \sum (\boldsymbol{v}_m \cdot \boldsymbol{\delta E}) w_m$.



Scatter plot of resonant particles from linear phase of n=-22 GAE simulations. Time-averaged values of $(\boldsymbol{v} \cdot \boldsymbol{\delta E})w$ [a.u.] of resonant particles. Particle color corresponds to different energies: from E=35keV (green) to E=80keV (red).

Summary and Future Work

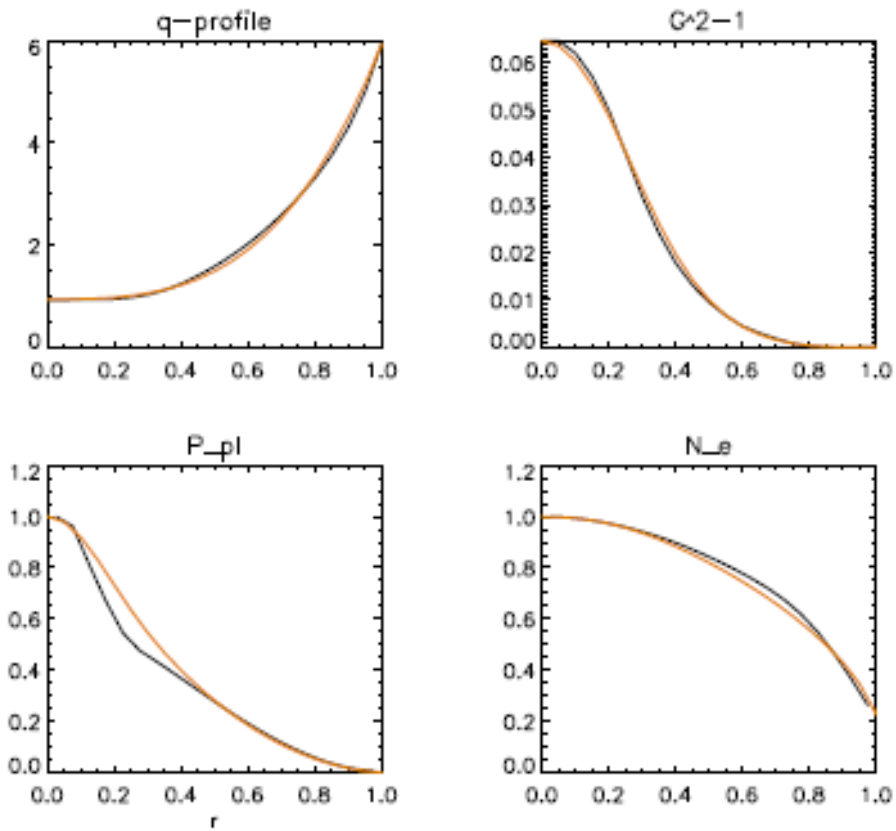
- HYM simulations show that sub-cyclotron frequency modes in DIII-D are counter-GAE, with very small compressional component.
- Simulations reproduce experimentally observed frequencies and toroidal mode numbers.
- A simple analytical theory based on local dispersion relation is very successful in predicting the counter-GAE instabilities.
- New analytic theory explains range of most unstable modes, and GAE frequency scaling across different devices.

Future work:

- Need to include 2-fluid (Hall) effects in thermal plasma description to account for finite frequency effects ~$O(\omega/\omega_{ci})$. At present, HYM overestimates unstable frequencies, and underestimates toroidal mode numbers.
- Need to implement more efficient coupling between field and particle grids for simulations of large |n| and large aspect ratio devices.

Case 2: Higher B_{tor} experiments on DIII-D

B_{tor}= 1.24T, R_0= 1.72m, I= 0.62MA, q_0=0.94, q_{max}=6, β_{tot} ~2%
Beam parameters: E= 75-80keV, V_0/V_A= 0.8, n_b/n_e =3%, β_{beam} ~0.5%
Observed mode parameters: f= 5.5 MHz, f_{ci}= 9.5 MHz



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