Realistic 2D quasilinear modeling of fast ion relaxation

Vinícius Duarte, Nikolai Gorelenkov, Mario Podestà and Roscoe White

Princeton Plasma Physics Laboratory, Princeton, NJ, USA

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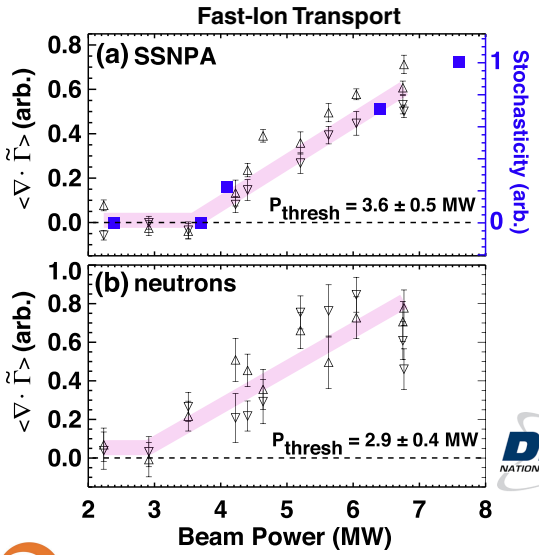
Fast ion transport predictions for whole device modeling are needed for fusion plasmas

DIII-D critical gradient experiments

Stiff, stochastic fast ion transport gives credence in using a quasilinear approach

C. Collins et al, PRL 2016



- Fully nonlinear modeling of fast ion interaction with Alfvénic modes in a realistic tokamak is numerically expensive
- Reduced (but still realistic) modeling can be exploited if linear mode properties do not change faster than the equilibrium, e. g.,
  - eigenstructure
  - resonance condition
- Simulations need to cope with the simultaneous excitation of multiple unstable Alfvénic instabilities
- Reduced models need to be strongly verified

Outline

- The Resonance Broadened Quasilinear (RBQ) model
- RBQ1D verification exercises
- RBQ interfacing with TRANSP
- Ongoing extension to 2D

The relevant diffusion path for particles resonating with a low-frequency Alfvénic mode is nearly 1D

Equilibrium – 3 invariants

$$\mathcal{E}=v^2/2,$$

$$\mu=\mathcal{E}_\perp/B,$$

$$P_\varphi=z\psi(R,Z)/Mc-$$

$$-\sigma_\parallel\sqrt{2\mathcal{E}}\sqrt{1-\mu B/\mathcal{E}}(RB_\varphi/B),$$

Particle trajectory is one point in $(\mathcal{E},P_\varphi,\mu)$ space

In the presence of low-frequency perturbations:

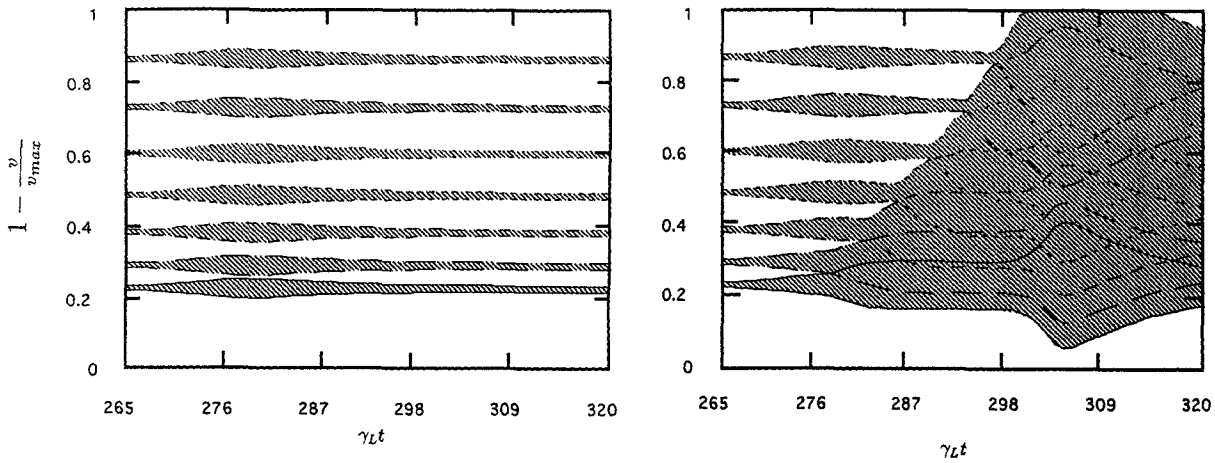
- the invariance of μ is roughly maintained;
- the invariances of $\mathcal{E}$ and P_φ are broken
- if the ansatz $e^{-i(n\varphi+\omega t)}$ holds, a new invariant emerges:

$$\boxed{\mathcal{E}'=\mathcal{E}+\omega P_\varphi/n}$$

Particle motion is 1D upon single mode interaction

Resonance broadened quasilinear (RBQ) model: motivation for the 2D generalization

- Designed to address both regimes of isolated and overlapping resonances
  - the fast ion distribution function relaxes while self-consistently evolving the amplitude of modes
  - interfaced with linear ideal/kinetic codes, NOVA/NOVA-K, which provide eigenstructures, damping rates and wave-particle interaction matrices for resonances in the constant of motion space



The ultimate goal of the SciDAC RBQ work is to evolve particle distribution in 2D in whole-device modeling

H. Berk, B. Breizman, J. Fitzpatrick, and H. Wong, Nucl. Fusion 35, 1661 (1995).

Resonance-broadened quasilinear (RBQ) diffusion model

Formulation in action and angle variables[2,3]

- Diffusion equation:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial I}\left(\sum_{n_k,p,m,m'} D(I;t)\right)\frac{\partial f}{\partial I} + \left(\left|\frac{\partial \Omega_{\mathbf{l}}}{\partial I}\right|_{I_r}\right)^{-2} \nu^3_{scatt,\mathbf{l}} \frac{\partial^2 (f - f_0)}{\partial I^2}$$

$$D(I;t) = \pi C_k^2(t)\,\mathcal{E}^2 \frac{\mathcal{F}_{\mathbf{l}}(I - I_r)}{\left|\frac{\partial \Omega_{\mathbf{l}}}{\partial I}\right|} G^*_{m'p} G_{mp}$$

Broadened delta, a function of $\Delta\Omega$

eigenstucture information

- Mode amplitude evolution:

$$\frac{dC_n^2(t)}{dt} = 2\left(\gamma_{L,n} - \gamma_{d,n}\right) C_n^2(t)$$

$$\frac{\partial}{\partial I} = \omega \frac{\partial}{\partial \mathcal{E}} - n \frac{\partial}{\partial P_\varphi}$$

Physics-based determination of the window function is pending

Broadening is the platform that allows for momentum and energy exchange between particles and waves:

$$\Delta\Omega = a\omega_b + b\nu_{eff}$$

[1]Berk, Breizman, Fitzpatrick and Wong, NF 1995.
[2]Kaufman PoF, JPP 1972 (no broadening due to growth rate!).
[3]Gorelenkov, Duarte, Podestà and Berk, NF 2018.

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Verification: determining the parametric dependencies of the broadening from single mode saturation levels – collisional case

We use analytic results for determining a and b: $\Delta\Omega = a\omega_b + b\nu_{eff}$

Limit near marginal stability[3]
→ $b = 3.1$

$$\omega_b = 1.18\nu_{eff}\left(\frac{\gamma_{L0}-\gamma_d}{\gamma_{L0}}\right)^{1/4}$$

Limit far from marginal stability[4]
→ $a = 2.7$

$$\omega_b = 1.2\nu_{eff}\left(\frac{\gamma_{L0}-\gamma_d}{\gamma_d}\right)^{1/3}$$

Resonance-broadened quasilinear formalism can cope with both situations of isolated and overlapping modes

[3]H. L. Berk et al. Plasma Phys. Rep, 23(9), 1997
[4]H. L. Berk and B. N. Breizman. Phys. Fluids B, 2(9), 1990

Verification: RBQ replicates analytical predictions for mode saturation amplitude

Definitions: initial linear growth rate γ_L, mode damping rate γ_d and trapping (bounce) frequency ω_b (proportional to square root of mode amplitude)

Collisionless case

- Undamped case

$$\omega_b \cong 3.2\gamma_L$$

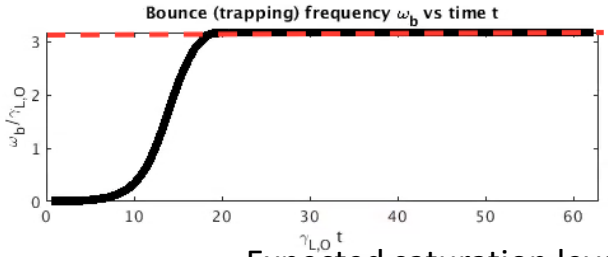
Collisional cases

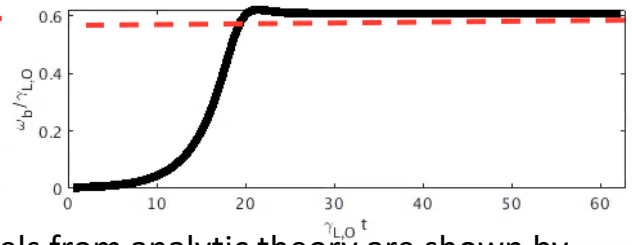
- Close to marginal stability: $\nu_{\text{eff}} \gg \omega_b$

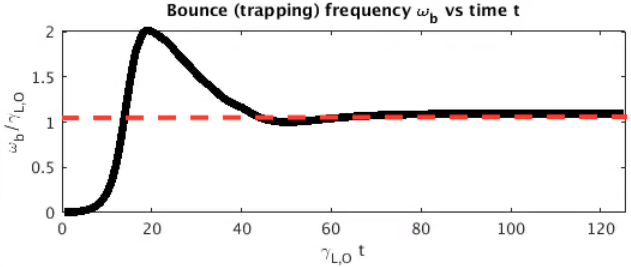
$$\omega_b = 1.18\nu_{\text{eff}}\left(\frac{\gamma_L - \gamma_d}{\gamma_d}\right)^{1/4}$$

- Far from marginal stability: $\omega_b \gg \nu_{\text{eff}}$

$$\omega_b = 1.2\nu_{\text{eff}}\left(\frac{\gamma_L - \gamma_d}{\gamma_d}\right)^{1/3}$$







Expected saturation levels from analytic theory are shown by - - -

Verification: analytical collisional mode evolution near threshold

- Near marginal stability, the wave amplitude evolution is governed by [Berk, Breizman and Pekker, PRL 1996]

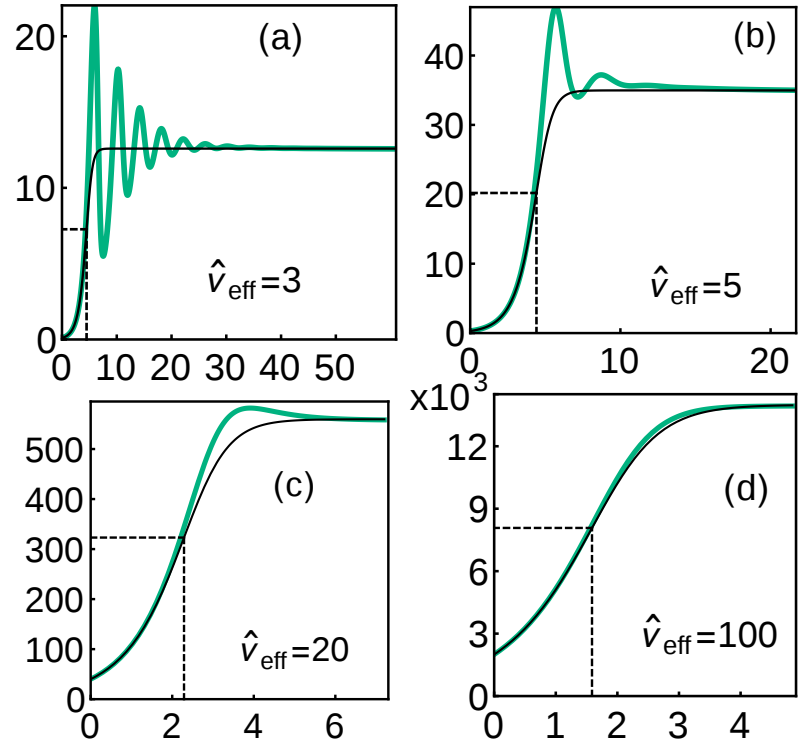
$$\frac{dA(t)}{dt} = A(t) - \frac{1}{2}\int d\Gamma \mathcal{H}\left\{\int_0^{t/2} dz z^2 A(t-z)\times \right.$$
$$\left.\times \int_0^{t-2z} dy e^{-\hat{\nu}_{eff}^3 z^2 (2z/3+y)} A(t-z-y)A^*(t-2z-y)\right\}$$

- An approximate analytical solution is found when $\hat{\nu}_{eff} \gg 1$: [Duarte & Gorelenkov, NF 2019]

$$A(t) = \frac{A(0)e^t}{\sqrt{1 - gA^2(0)\left(1-e^{2t}\right)}}$$

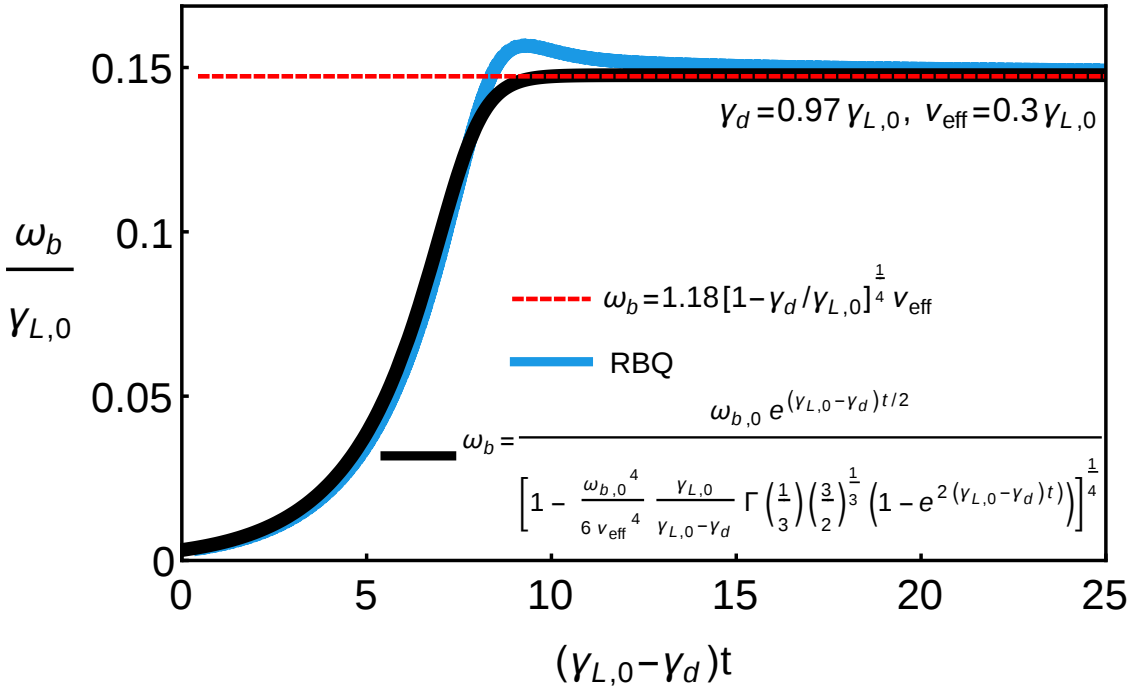
$g \equiv \int d\Gamma \mathcal{H} \frac{\Gamma(1/3)}{6\hat{\nu}_{eff}^4}\left(\frac{3}{2}\right)^{1/3}$ is a resonance-averaged collisional contribution evaluated by NOVA-K

Amplitude A vs time t for the full cubic equation (green) and the analytical solution (black)



[Duarte & Gorelenkov, NF 2019]

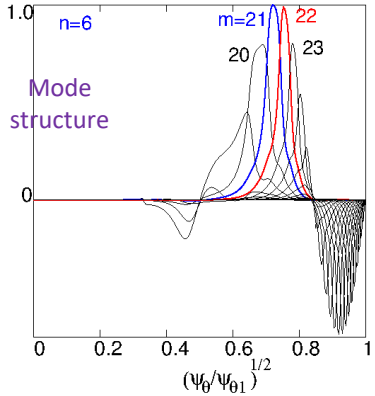
Verification of RBQ vs analytical solution for mode evolution

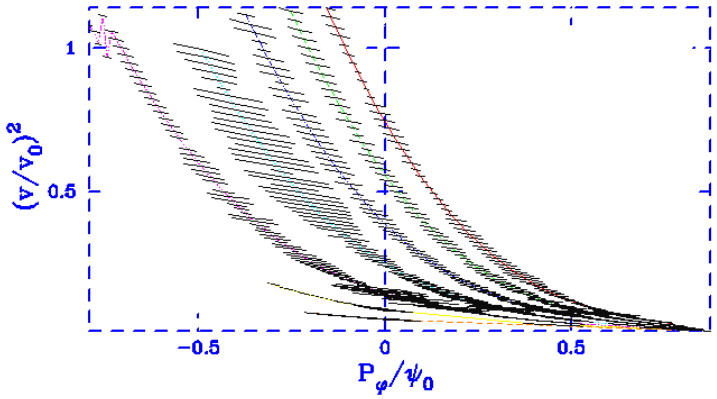


(Berk, Breizman, Pekker, PPR 1997)

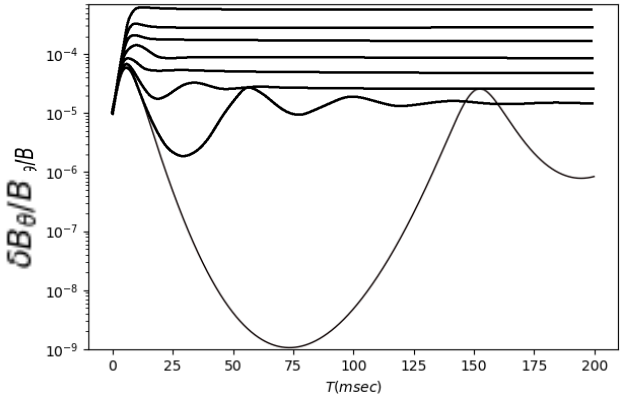
(Duarte & Gorelenkov, NF 2019)

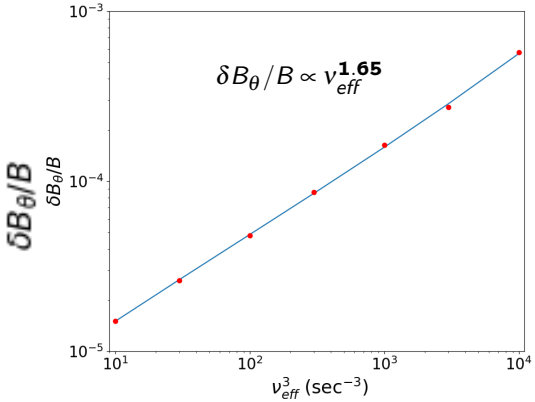
Verification of RBQ runs for the collisional evolution of an Alfvénic wave





DIII-D discharge 159243





Idealized bump-on-tail scaling:

$$\delta B_\theta/B = \nu_{eff}^2$$

Gorelenkov *et al*, PoP (submitted)

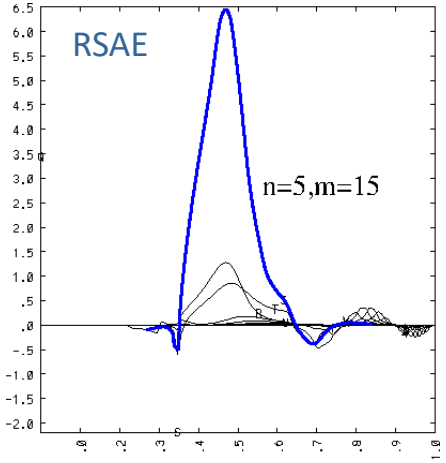
RBQ results for single mode shows

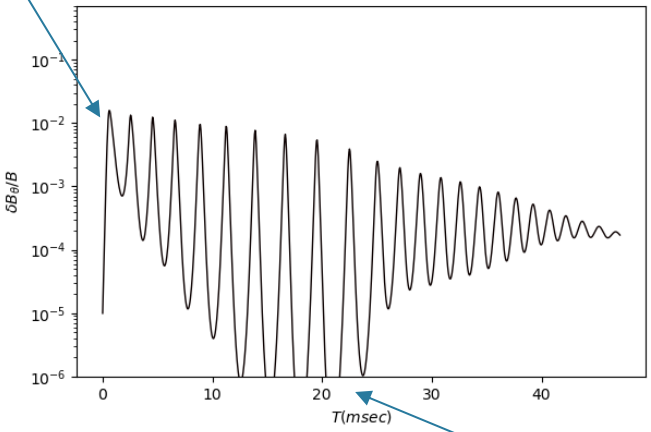
Interplay between mode drive, background damping and collisions

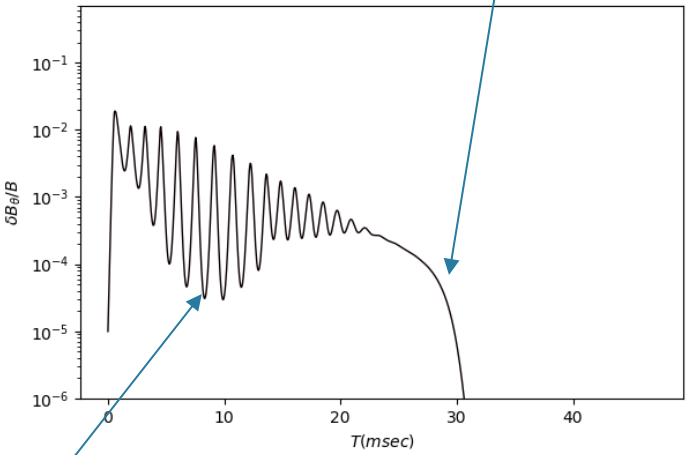
Eigenstructure limits the resonance width
(White et al, PoP 2018)

doubling the collisionality

Absence of sources leads to mode decay (currently being addressed in the 2D version)







At low amplitude, scattering collisions build up the gradient inside the resonance→peaks are sharper than the troughs

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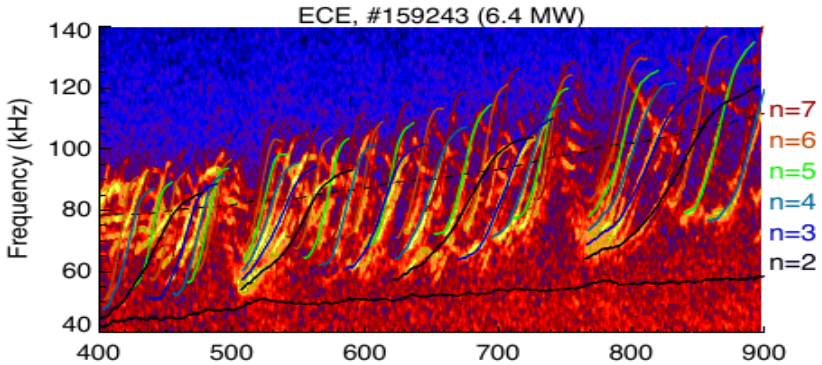
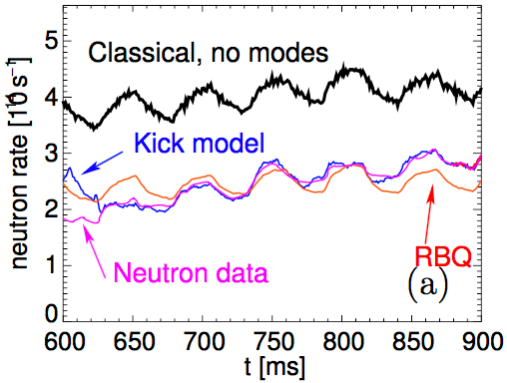
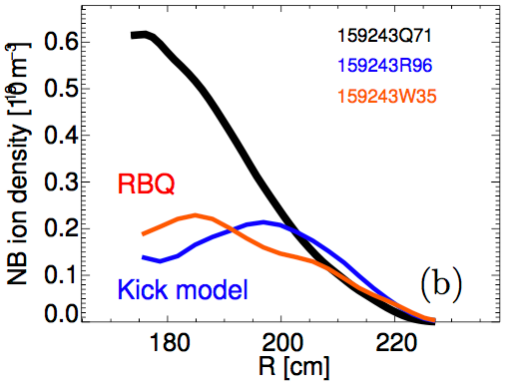
RBQ is interfaced with TRANSP: multi-mode case

11 unstable Alfvénic modes at 805ms







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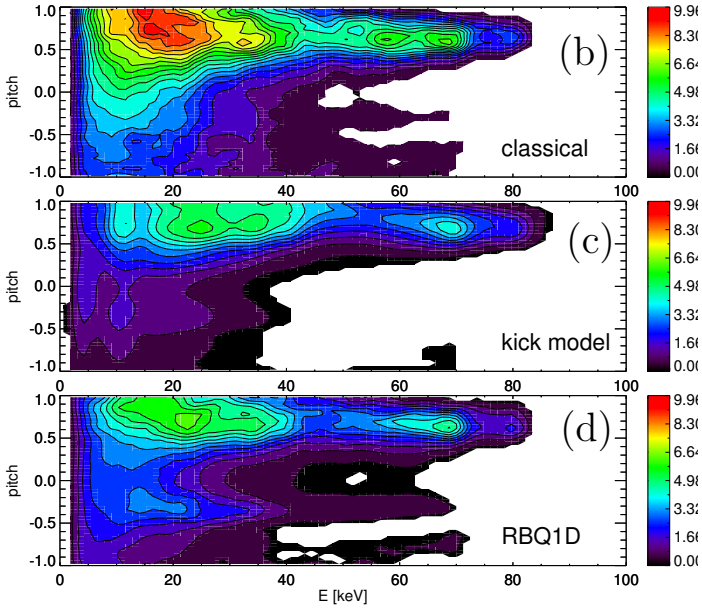
Distribution at 805ms



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Current work is aligned with milestones for ISEP SCiDAC

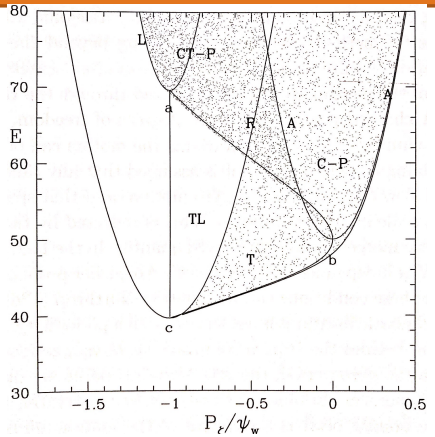
"Formulate RBQ2D approach for full resolution in velocity. Use NOVA-K interface for single modes. Implement slanted phase space diffusion. Apply RBQ1D for VV with velocity space resolution."

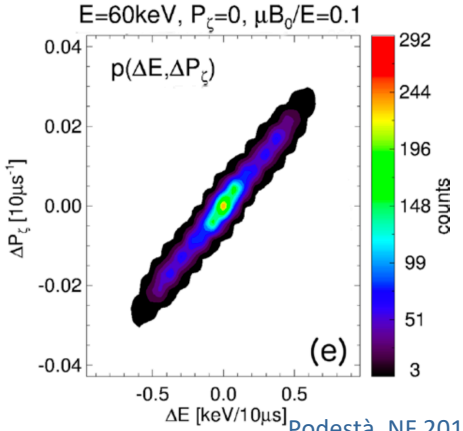
- Alternating direction implicit (ADI) slanted scheme being implemented: reduction of the computation time (McKee *et al*, J. Comp. Phys. 1996)

$$\left[1-\frac{\delta t}{\delta P_\varphi^2}\frac{1}{2}n_k^2 D_{i,j}^n \delta_{P_\varphi}^2+\frac{\delta t}{\delta P_\varphi}\frac{1}{2}n_k\left(n_k\left(\frac{\partial D}{\partial P_\varphi}\right)_{i,j}^n-\omega_k\left(\frac{\partial D}{\partial \mathcal{E}}\right)_{i,j}^n\right)\nabla_{P_\varphi}\right]f_{i,j}^{n+1*}=$$
$$=\left[1+\frac{\delta t}{\delta \mathcal{E}^2}\omega_k^2 D_{i,j}^n\delta_{\mathcal{E}}^2-\frac{\delta t}{\delta \mathcal{E}}\omega_k\left(\omega_k\left(\frac{\partial D}{\partial \mathcal{E}}\right)_{i,j}^n-n_k\left(\frac{\partial D}{\partial P_\varphi}\right)_{i,j}^n\right)\nabla_{\mathcal{E}}+\frac{\delta t}{\delta P_\varphi^2}\frac{1}{2}n_k^2 D_{i,j}^n\delta_{P_\varphi}^2-\right.$$
$$\left.-\frac{\delta t}{\delta P_\varphi}\frac{1}{2}n_k\left(n_k\left(\frac{\partial D}{\partial P_\varphi}\right)_{i,j}^n-\omega_k\left(\frac{\partial D}{\partial \mathcal{E}}\right)_{i,j}^n\right)\nabla_{P_\varphi}+\frac{\delta t}{\delta P_\varphi\delta\mathcal{E}}\frac{1}{4}(-2)\,n_k\omega_k D_{i,j}^n\delta_{P_\varphi\mathcal{E}}\right]f_{i,j}^n$$

$$\left[1-\frac{\delta t}{\delta \mathcal{E}^2}\frac{1}{2}\omega_k^2 D_{i,j}^n\delta_{\mathcal{E}}^2+\frac{\delta t}{\delta \mathcal{E}}\frac{1}{2}\omega_k\left(\omega_k\left(\frac{\partial D}{\partial \mathcal{E}}\right)_{i,j}^n-n_k\left(\frac{\partial D}{\partial P_\varphi}\right)_{i,j}^n\right)\nabla_{\mathcal{E}}\right]f_{i,j}^{n+1}=f_{i,j}^{n+1*}-$$
$$-\left[\frac{\delta t}{\delta \mathcal{E}^2}\frac{1}{2}\omega_k^2 D_{i,j}^n\delta_{\mathcal{E}}^2-\frac{\delta t}{\delta \mathcal{E}}\frac{1}{2}\omega_k\left(\omega_k\left(\frac{\partial D}{\partial \mathcal{E}}\right)_{i,j}^n-n_k\left(\frac{\partial D}{\partial P_\varphi}\right)_{i,j}^n\right)\nabla_{\mathcal{E}}\right]f_{i,j}^n$$

- Challenge: non-rectangular domain due to loss boundary
- Burning plasma modeling will be computationally expensive

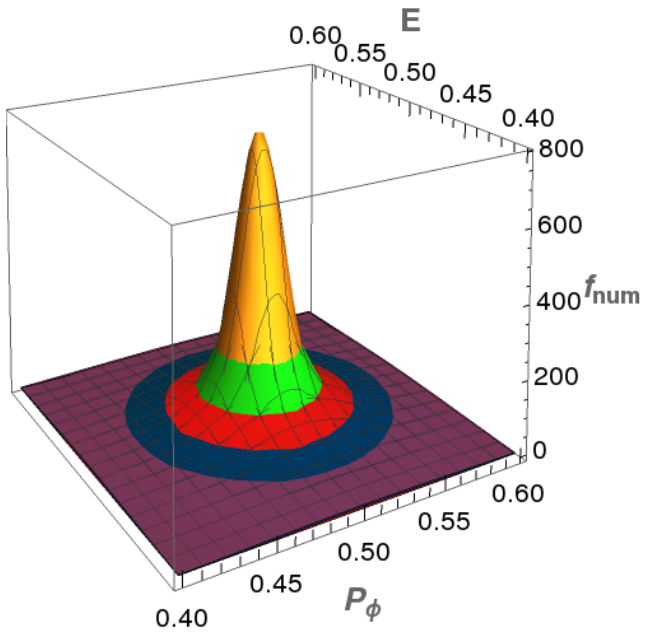




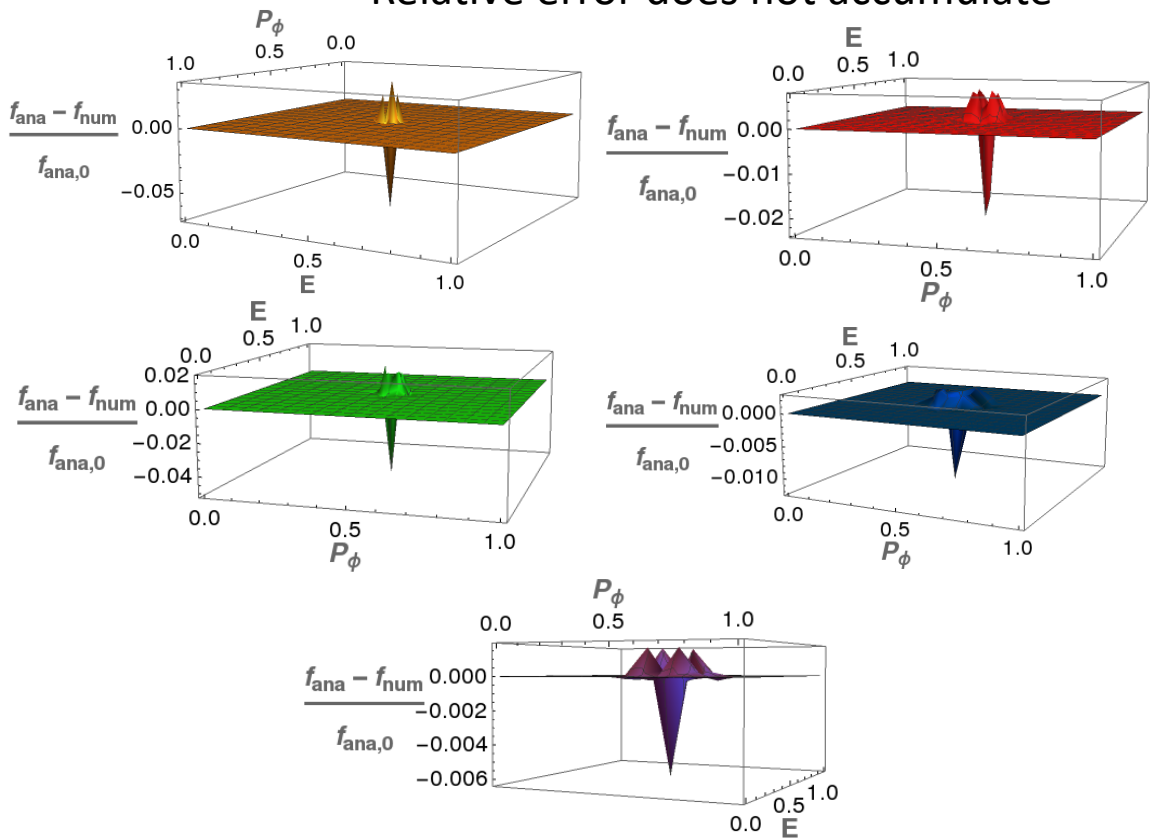
Podestà, NF 2016

Verification of the 2D diffusion scheme against analytic prediction

Numerical distribution function: unconditionally stable

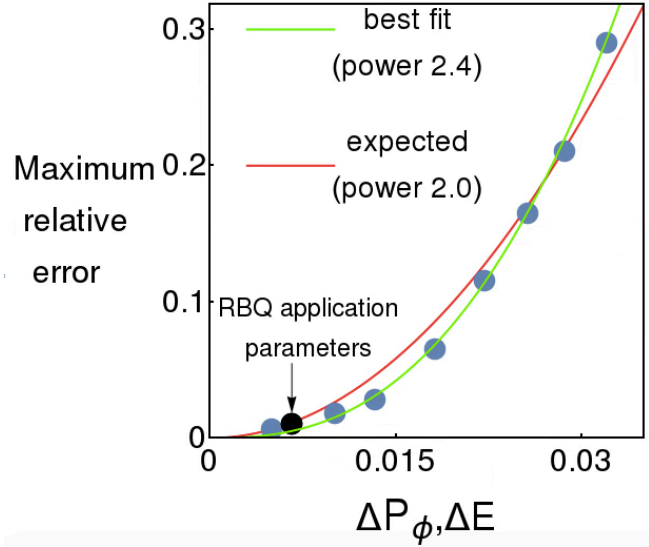


Relative error does not accumulate

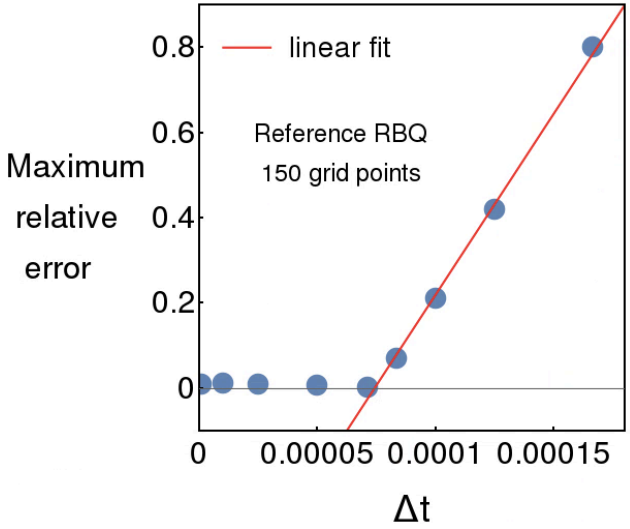


Scaling laws for numerical error

Grid spacing dependence



Time stepping dependence



Future verifications:

- change diffusion directions to verify against known analytic solutions;
- include multiple modes;
- explore diffusion paths in the presence of overlap;

Summary

- RBQ1D addresses both isolated and overlapping resonances
- oscillating and quasi-steady evolutions are recovered
- RBQ1D is verified against known analytical solution in limiting cases
- integration with TRANSP allows whole-device reduced modeling, which captures hollow profiles
- **Ongoing work: full 2D multi-mode resonant dynamics (with strong V&V exercises)**