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Time-resolved biphase signatures of quadratic nonlinearity observed in coupled eigenmodes on the DIII-D tokamak

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Why are we here?

Understanding the **mechanism of saturation** in toroidicity induced Alfven eigenmodes (TAEs) is crucial for the success of next-generation fusion devices such as ITER

Previous work [e.g., Todo 2018, *Reviews of Modern Plasma Physics*] has suggested **nonlinear mode-mode coupling** as a viable explanation for overestimates of saturation amplitude in models

Transport of energetic ions facilitated by coupling with TAEs may detrimentally affect reactor output by reducing confinement time, damaging vessel walls, etc.

Interaction of counter-propagating cylindrical modes furnishes frequency gap[1,2]

Shear Alfven wave

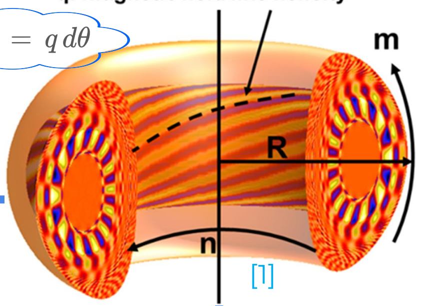
$$\omega_A(r) = k_{\parallel}(r) v_A(r)$$

$$k_{\parallel m}(r) = \frac{1}{R}\left(n - \frac{m}{q(r)}\right)$$

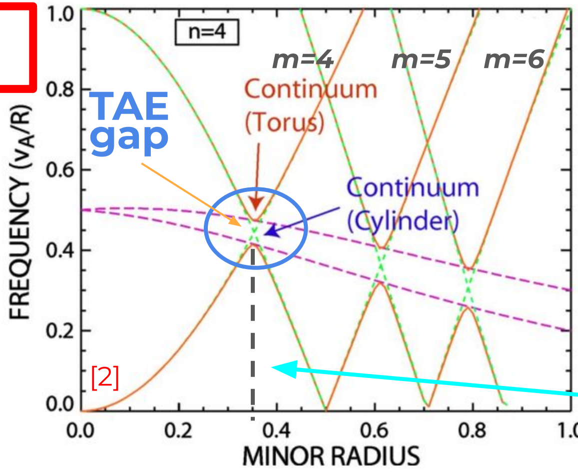
$$d\varphi = q\, d\theta$$

Coupling between adjacent modes

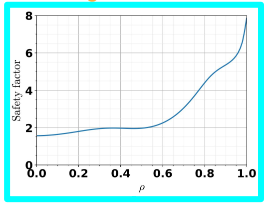
$$k_{\parallel m}(r_0) = -k_{\parallel (m+1)}(r_0)$$



Alfven continuum



Safety factor



Localization

$$q_{\mathrm{TAE}}(r_0) = \frac{m + 1/2}{n}$$

TAE frequency in plasma frame

$$f_{\mathrm{TAE}} = \frac{v_A}{4\pi q_{\mathrm{TAE}} R} \approx \frac{1}{(4\pi)^{3/2}} \frac{nB}{R(m+1/2)\sqrt{m_i n_i}}$$

[1] Garcia-Muñoz, M.et al.2019, PPCF 61, 054007

Resonance condition for fast ions

$$\omega + (m + l)\omega_\theta - n\omega_\varphi \simeq 0$$

[1] Pinches, S. 1996, University of Nottingham
[2] Heidbrink, W. 2008, PoP 15, 055501

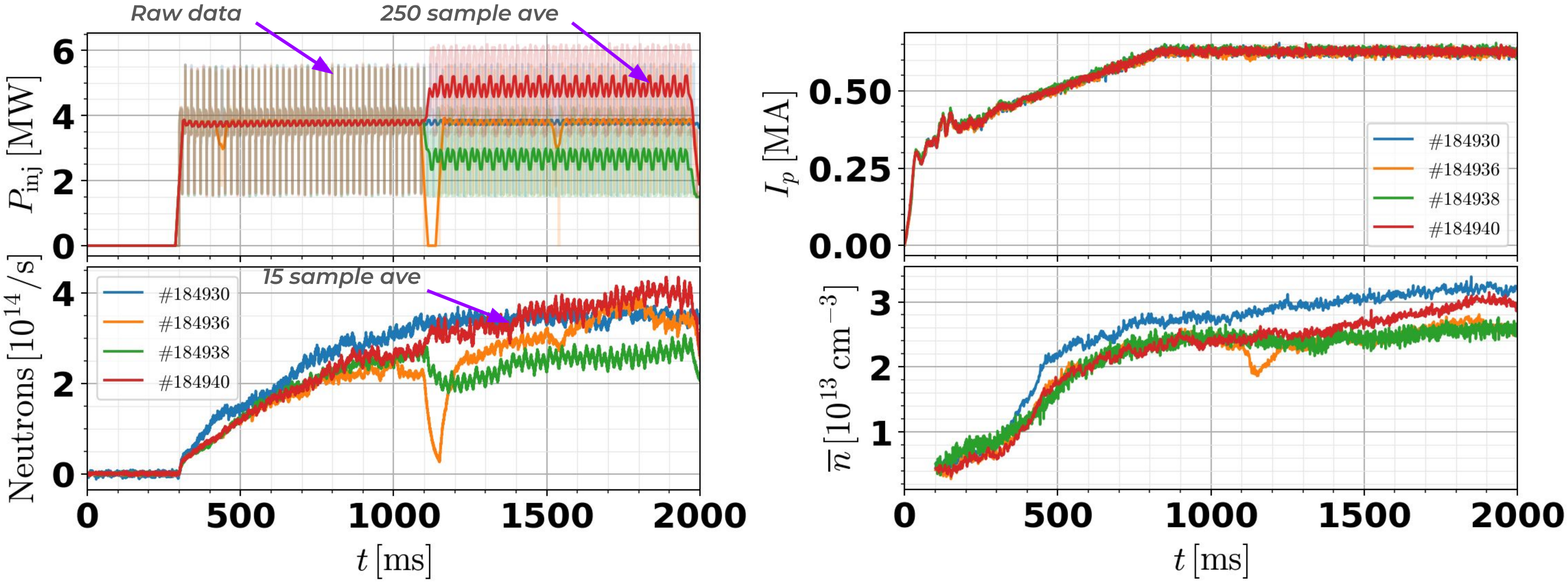
Growth/damping rate

$$\gamma \propto \omega \frac{\partial f}{\partial W} + n \frac{\partial f}{\partial p_\varphi}$$

stable *unstable!*

$$p_\varphi = mRv_\varphi - Ze\Psi$$

Experiment scanned energetic particle drive with NBI in low-field (1.25 T) configuration



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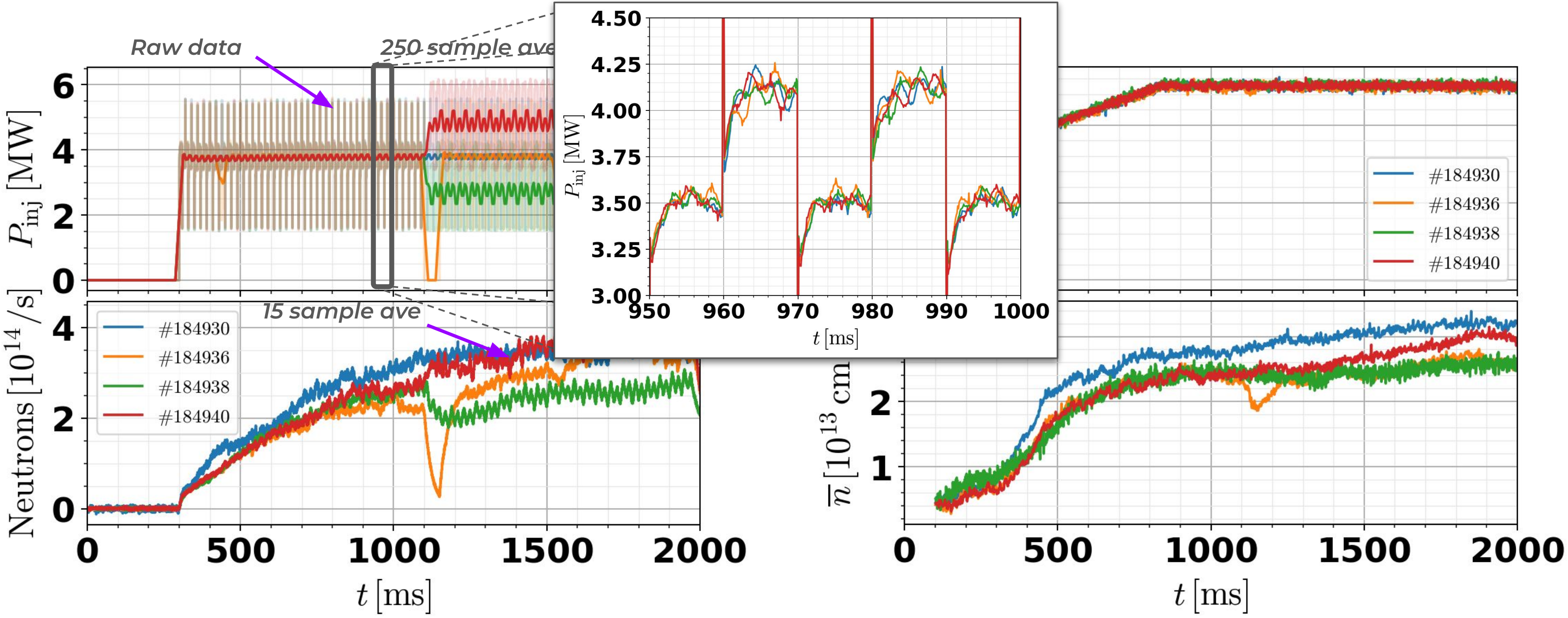

Bispectral analysis detects complementary phase relationships between frequency triples[3]

Average of **Fourier-transformed triple correlation** is auto-bispectrum

$$C_{xxx}(t_1, t_2) = \int_{-\infty}^{\infty} x(\tau + t_1)\, x(\tau + t_2)\, x(\tau)\, d\tau$$

Fourier transform

$$\widetilde{\mathcal{B}}_{xxx}(f_1, f_2) = \hat{x}(f_1)\, \hat{x}(f_2)\, \overline{\hat{x}(f_1 + f_2)}$$

Average in time

$$\mathcal{B}_{xxx}(f_1, f_2) = \left\langle \left| \widetilde{\mathcal{B}}_{xxx}(f_1, f_2) \right| e^{i\beta(f_1, f_2)} \right\rangle \in \mathbb{C}$$

Squared bicoherence given by Cauchy-Schwarz inequality:

$$b^2_{xxx}(f_1, f_2) = \frac{\left|\mathcal{B}_{xxx}(f_1, f_2)\right|^2}{\left\langle \left|\hat{x}(f_1)\hat{x}(f_2)\right|^2 \right\rangle \left\langle \left|\hat{x}(f_1 + f_2)\right|^2 \right\rangle} \in [0, 1]$$

[3] Kim, Y., Powers, E., 1978, Phys. Fluids 21, 1452–1453

Bispectral analysis detects complementary phase relationships between frequency triples[3]

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$$C_{xxx}(t_1, t_2) = \int_{-\infty}^{\infty} x(\tau + t_1)\, x(\tau + t_2)\, x(\tau)\, d\tau$$

Simple model for amplitude:

$$\frac{d\hat{x}_j}{dt} = \gamma_j \hat{x}_j + \sum_{j_1 + j_2 = j} \Lambda_{j_1 j_2} \hat{x}_{j_1} \hat{x}_{j_2}$$

x $\overline{\hat{x}_j}$ *and + c.c. leads to*

$$\frac{d}{dt}|\hat{x}_j|^2 = 2\gamma_j |\hat{x}_j|^2 + \sum_{j_1 + j_2 = j} \Lambda_{j_1 j_2} \hat{x}_{j_1} \hat{x}_{j_2} \overline{\hat{x}_{j_1 + j_2}} + \text{c. c.}$$

$$\widetilde{\mathcal{B}}_{xxx}(f_1, f_2) = \hat{x}(f_1)\, \hat{x}(f_2)\, \overline{\hat{x}(f_1 + f_2)}$$

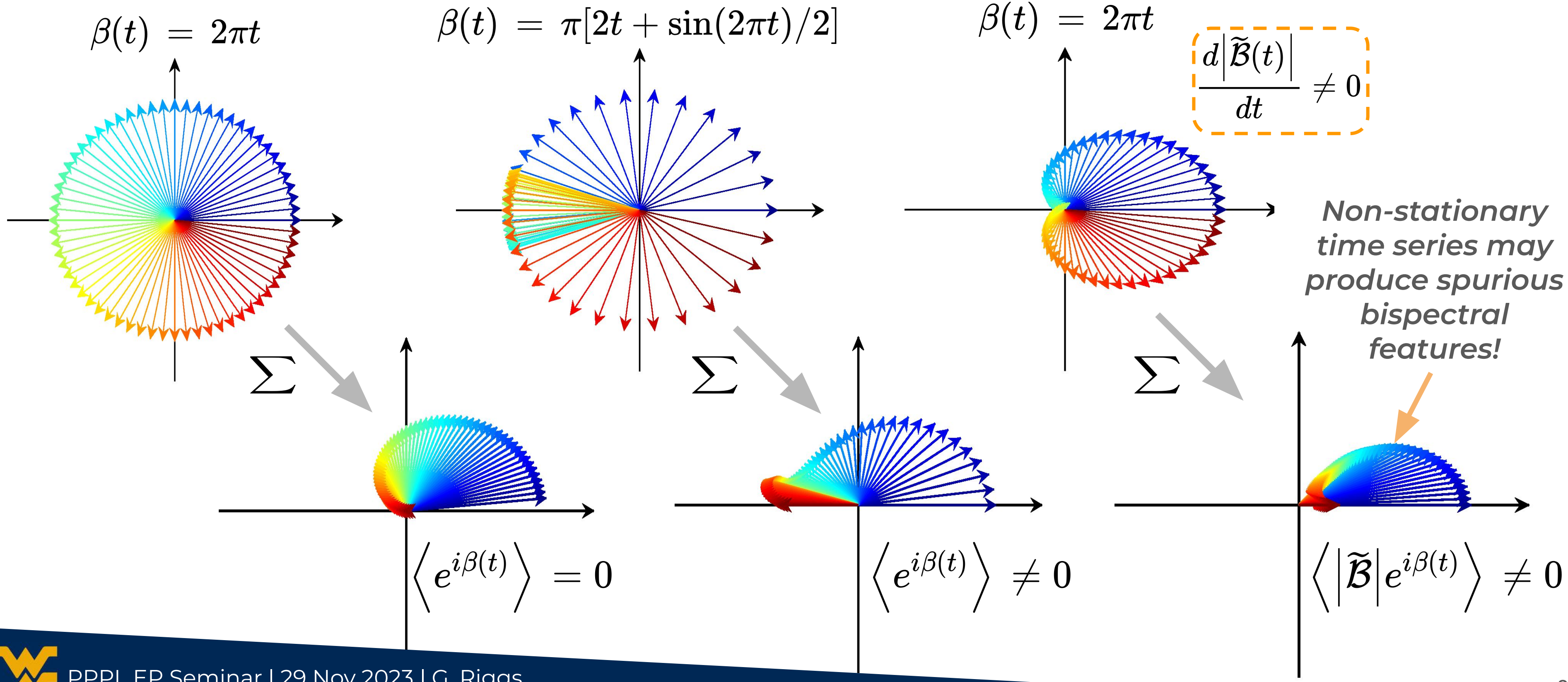
$$\mathcal{B}_{xxx}(f_1, f_2) = \left\langle \left| \widetilde{\mathcal{B}}_{xxx}(f_1, f_2) \right| e^{i\beta(f_1, f_2)} \right\rangle \in \mathbb{C}$$

given by Cauchy-Schwarz inequality:

$$b^2_{xxx}(f_1, f_2) = \frac{|\mathcal{B}_{xxx}(f_1, f_2)|^2}{\langle |\hat{x}(f_1)\hat{x}(f_2)|^2 \rangle \langle |\hat{x}(f_1 + f_2)|^2 \rangle} \in [0, 1]$$

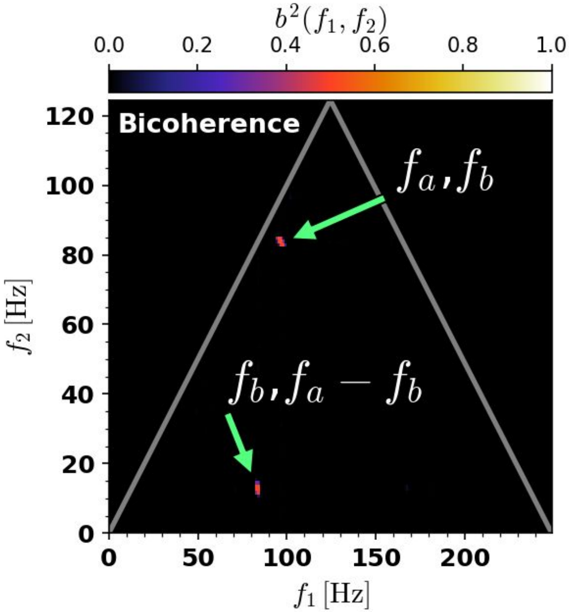
[3] Kim, Y., Powers, E., 1978, Phys. Fluids 21, 1452–1453

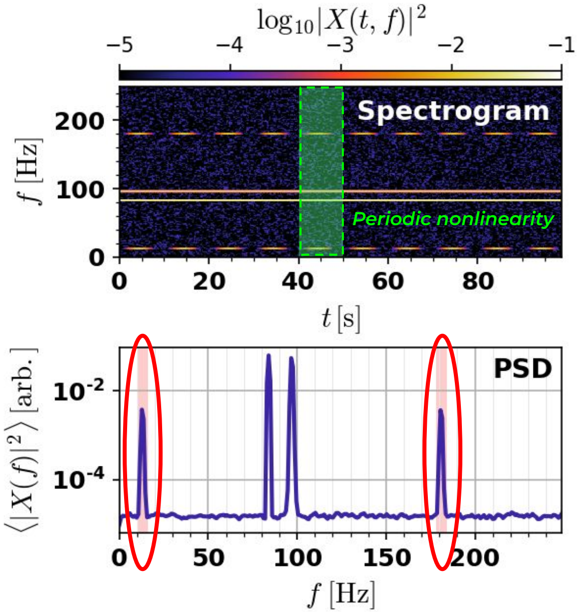
Phase or amplitude modulation require careful interpretation

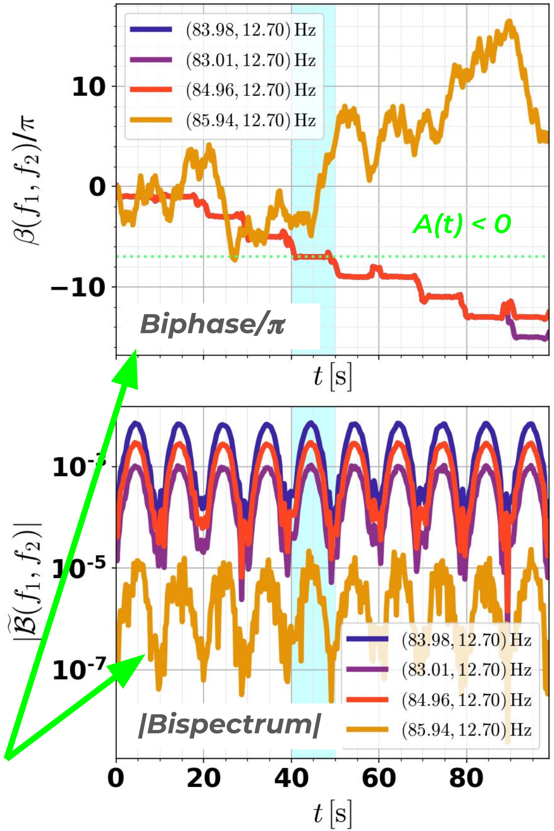


Non-stationary time series may produce spurious bispectral features!

Quadratic nonlinearity contributes coherent sum and difference frequencies





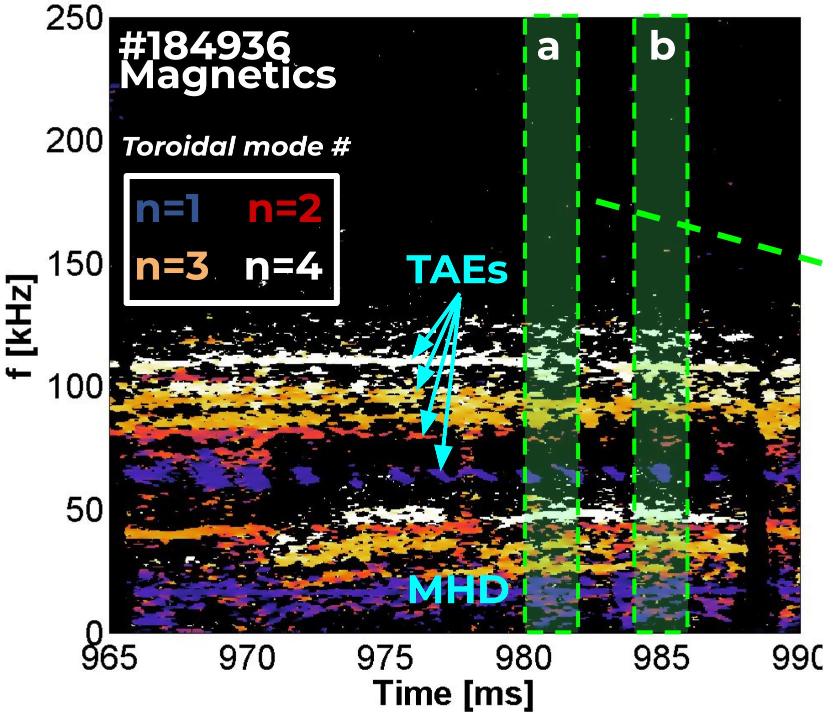


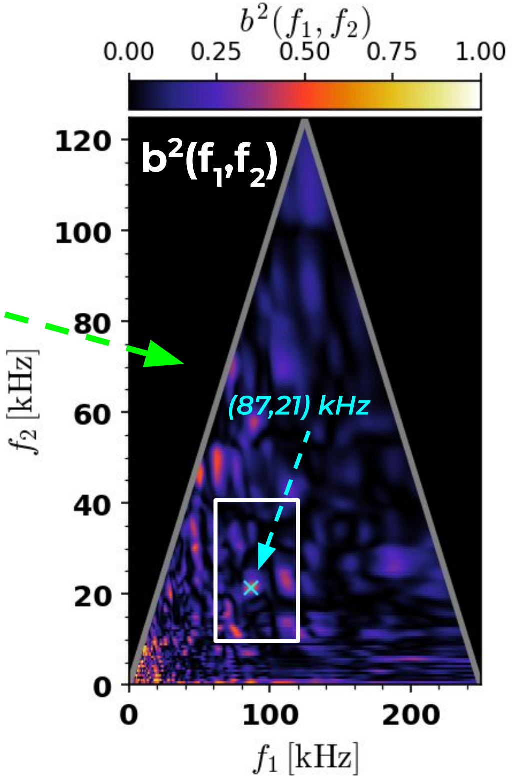
$$\cos(\omega_a t + \varphi_a) + \cos(\omega_b t + \varphi_b) + A(t)\cos(\omega_a t + \varphi_a)\cos(\omega_b t + \varphi_b)$$

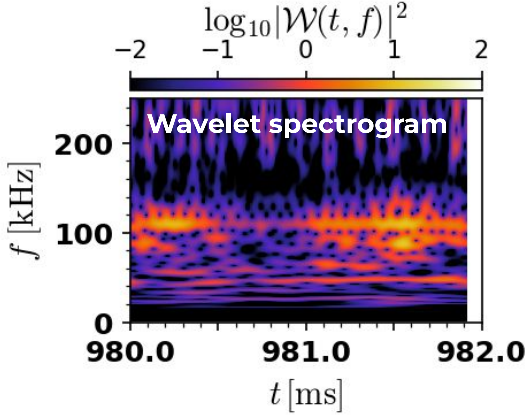
Nonlinearly generated modes

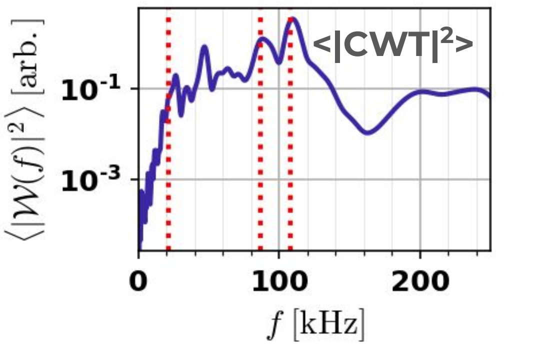
Line-outs @ (fb, fa-fb)

Quadratic coupling found to correlate with n=1 chirping in saturated density regime

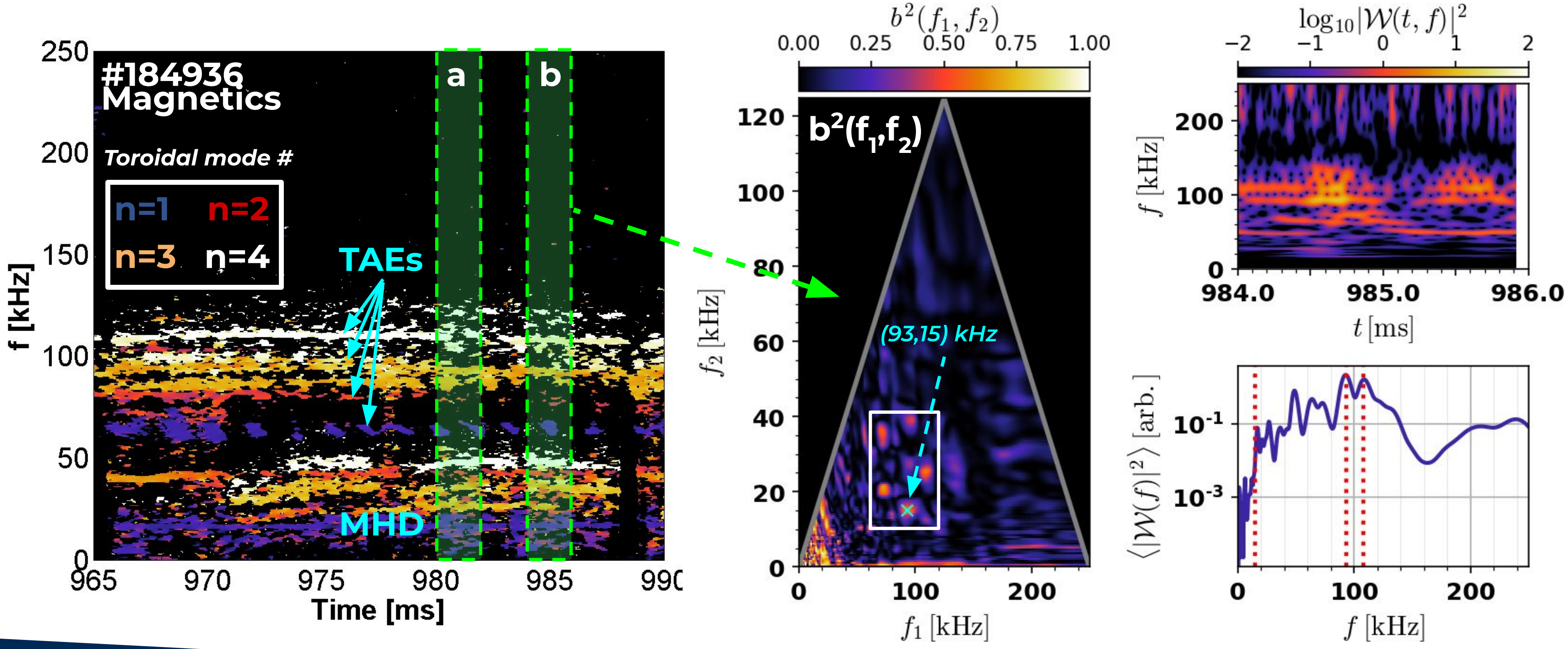




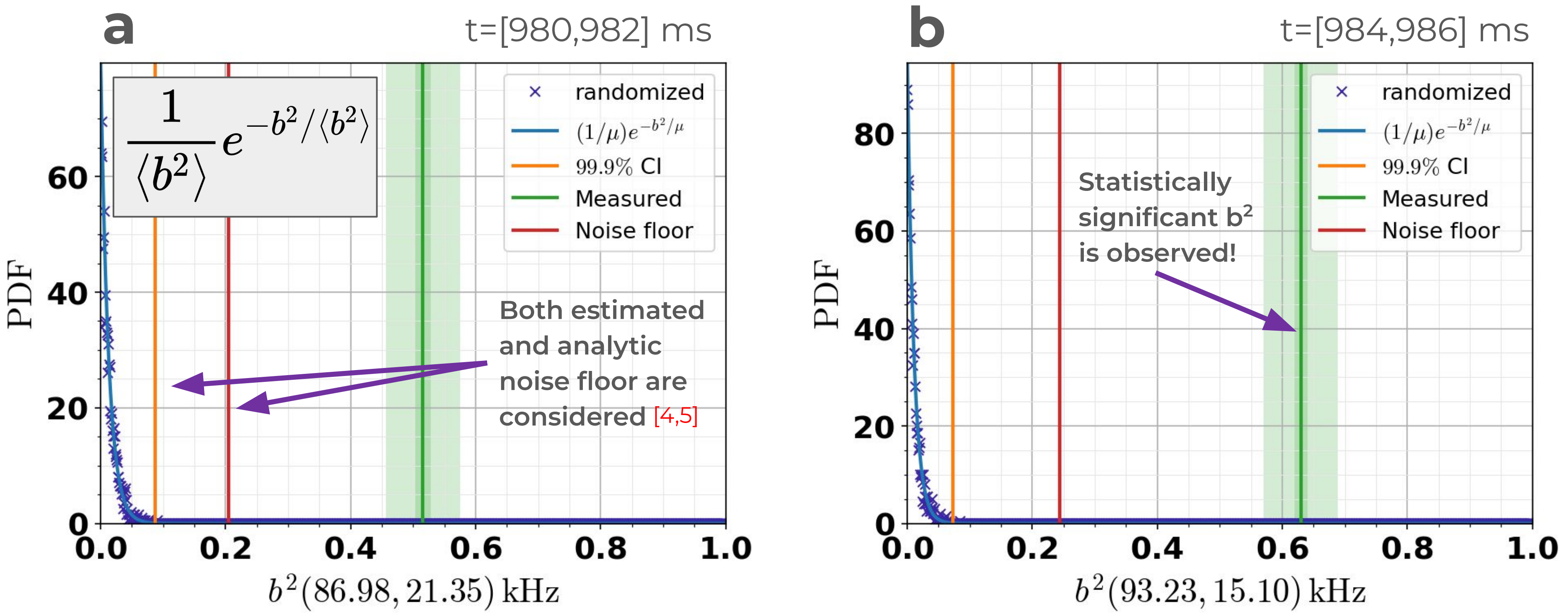




Quadratic coupling found to correlate with n=1 chirping in saturated density regime

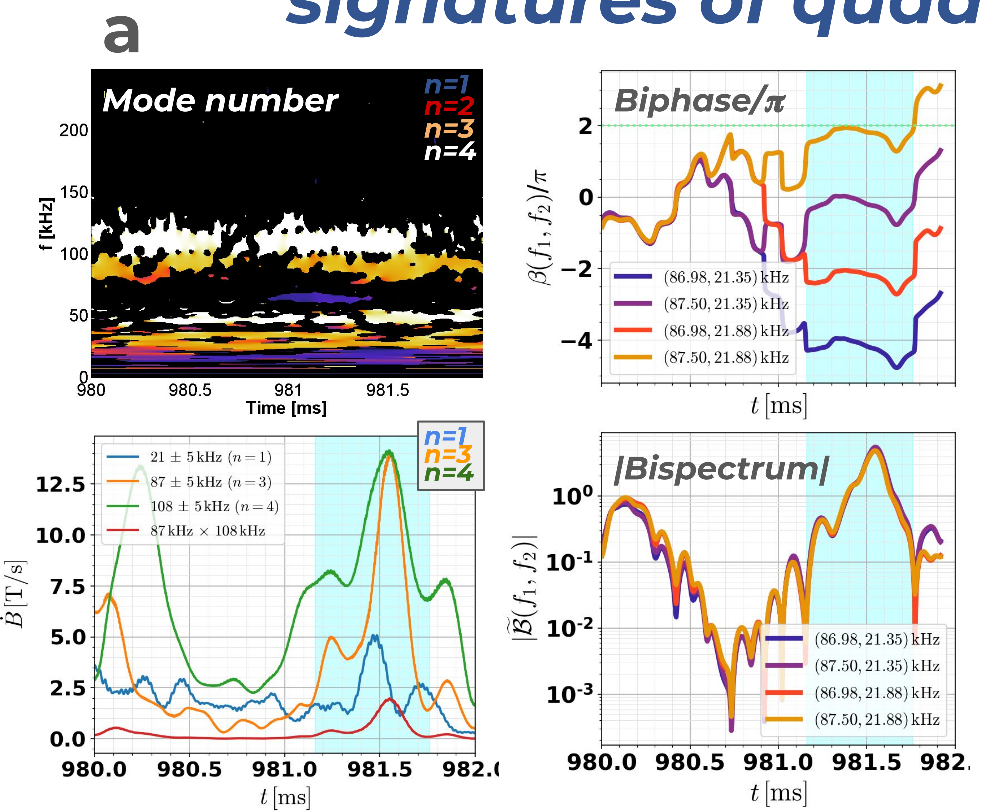


Uncertainty quantified with ensemble of random-phase realizations



[4] Poloskei, P., et al. 2018, arXiv:1811.02973
[5] Van Milligen, B., et al. 1995, PRL 74, 395

Changes in TAE amplitude coincident with signatures of quadratic nonlinearity

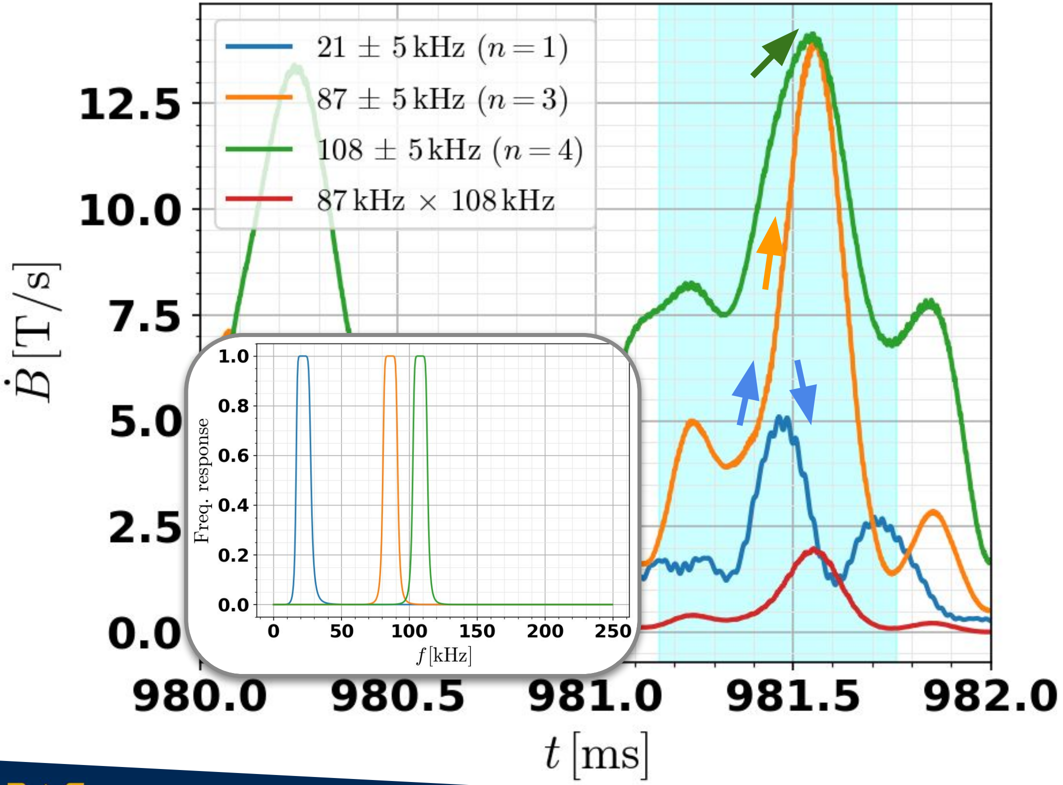


Mode number analysis consistent with $n_3 = |n_1 - n_2|$

Period of stationary biphase correlates with enhanced local bispectral modulus

Changes in TAE amplitude coincident with signatures of quadratic nonlinearity

a



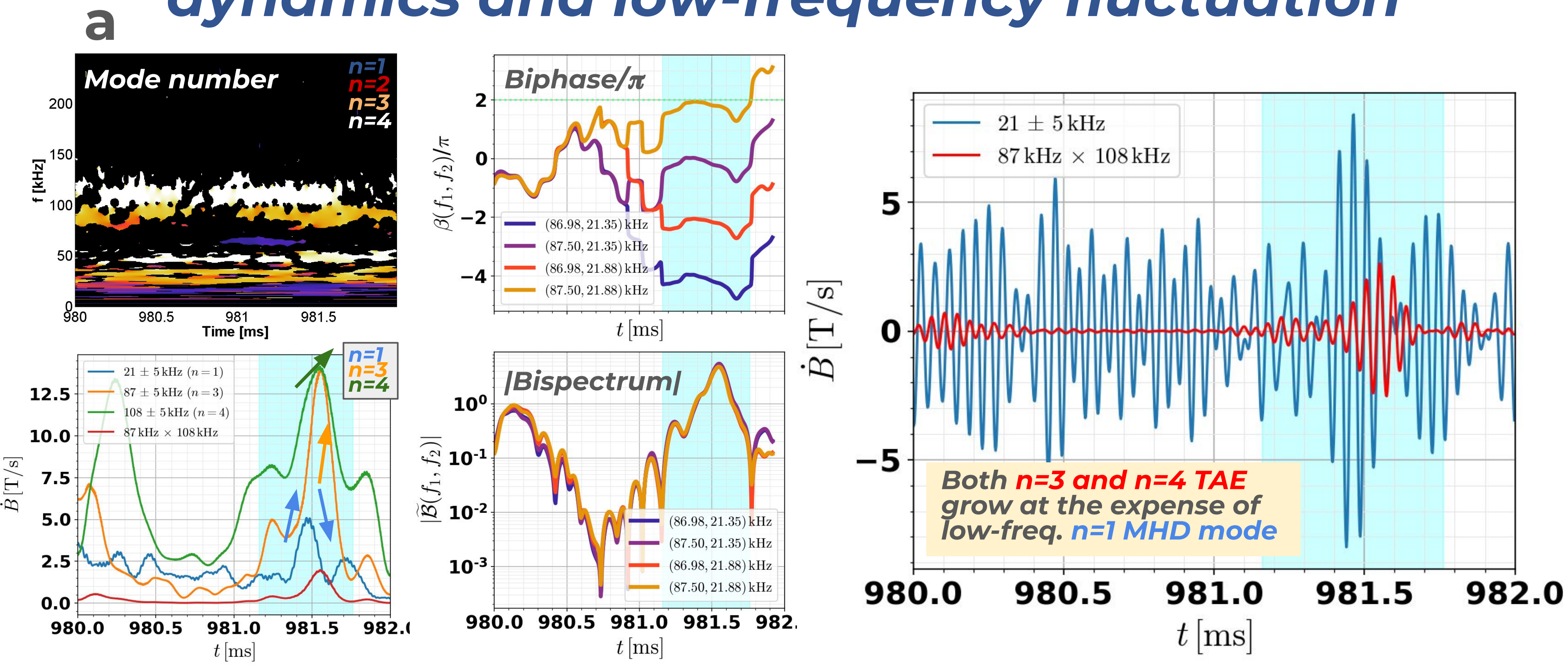
Mode number analysis consistent with $n_3 = |n_1 - n_2|$

Period of stationary biphase correlates with enhanced local bispectral modulus

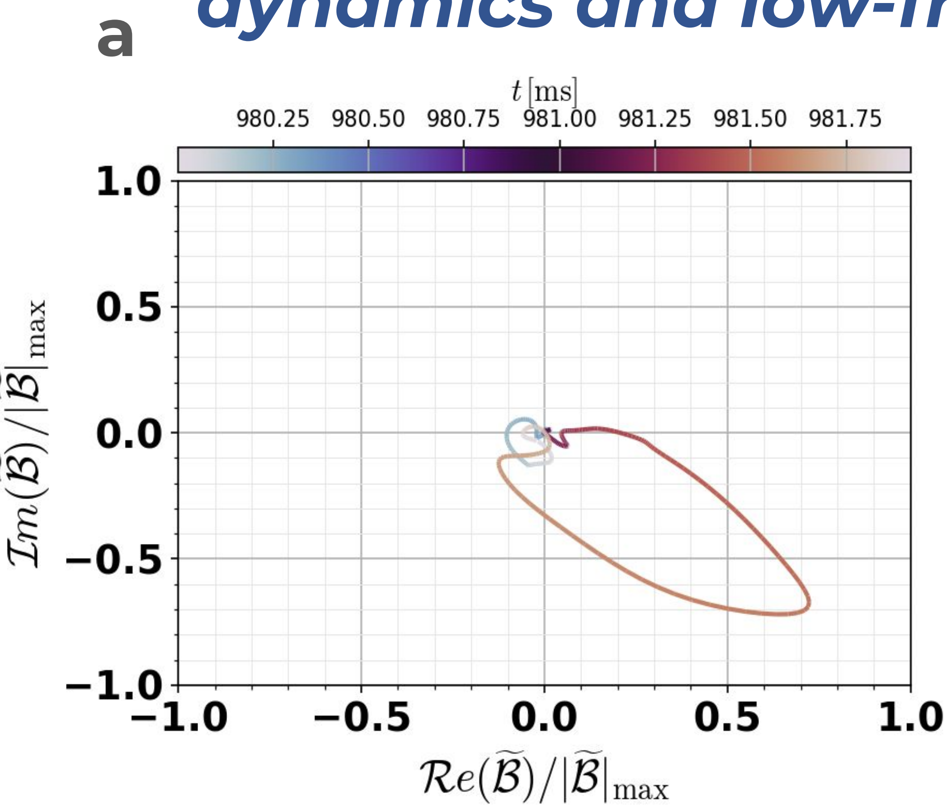
Low-frequency fluctuation is attenuated as TAEs continue to grow

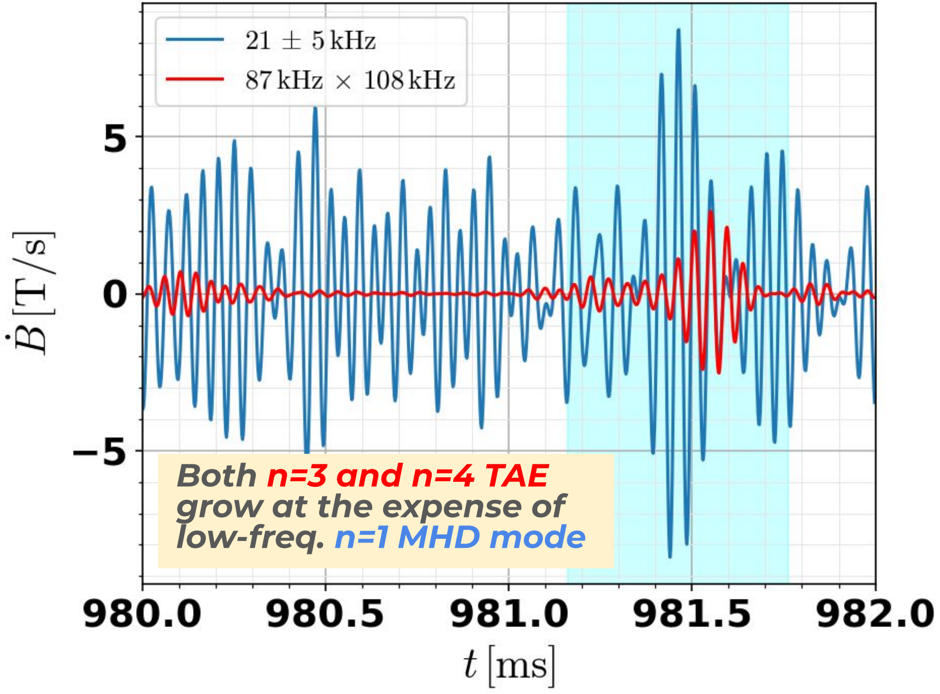
(5th order Butterworth filter)

Phase coherence observed between beat dynamics and low-frequency fluctuation

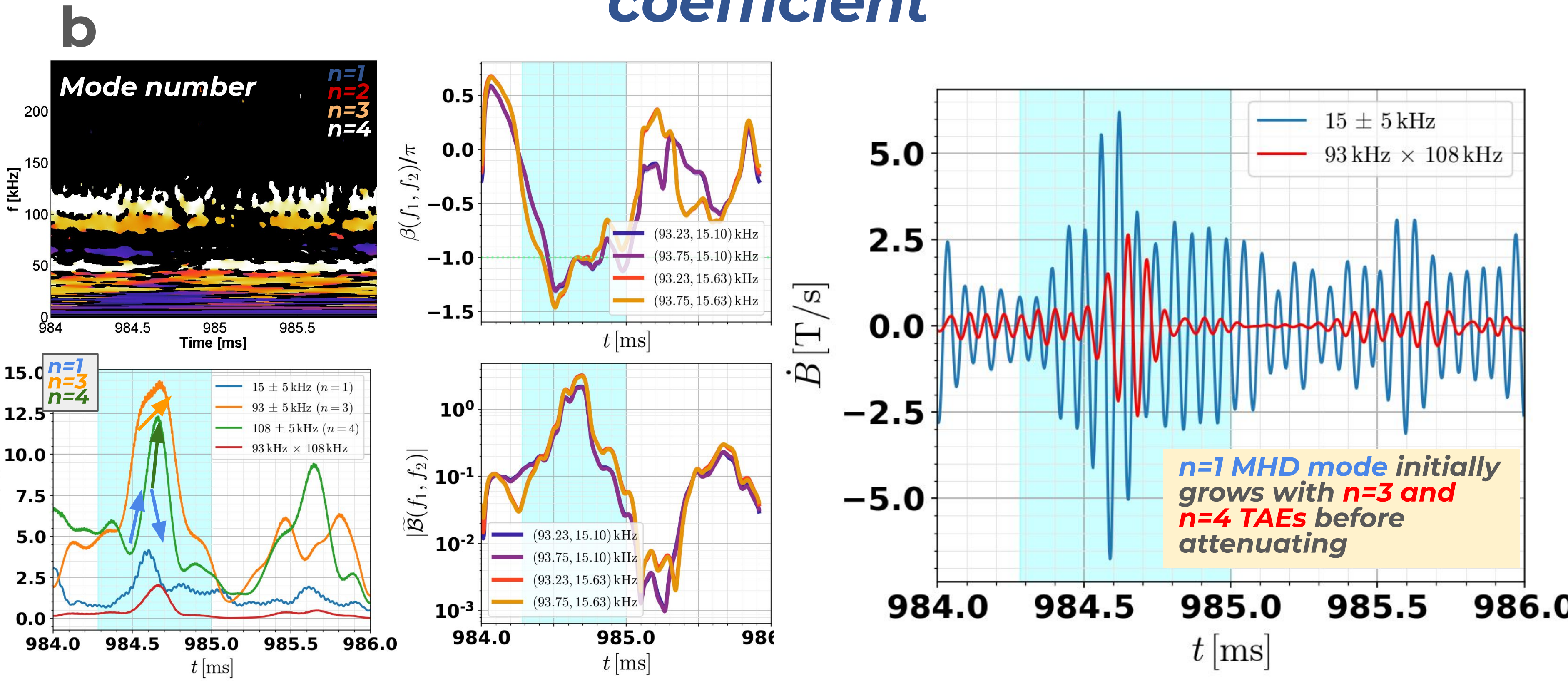


Phase coherence observed between beat dynamics and low-frequency fluctuation



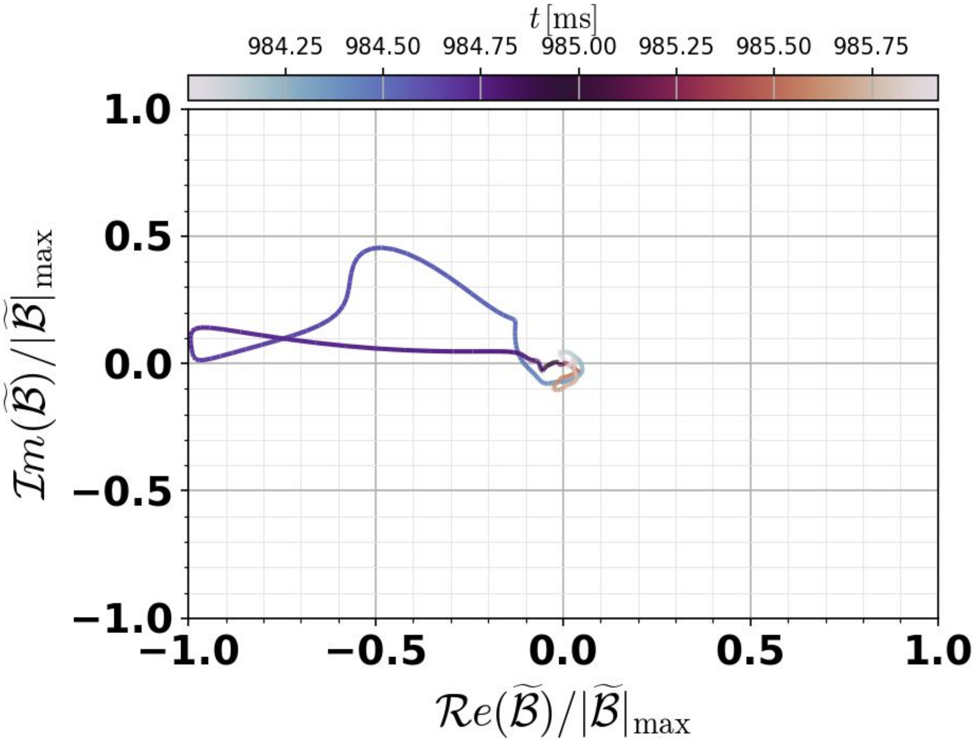


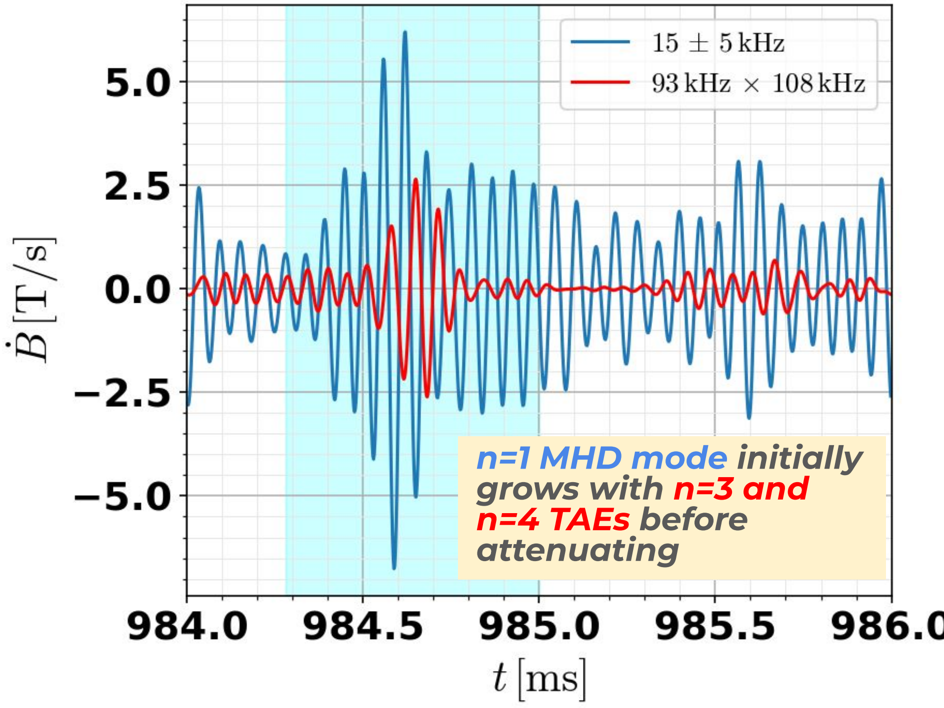
Parity of biphase/π corresponds to sign of coupling coefficient

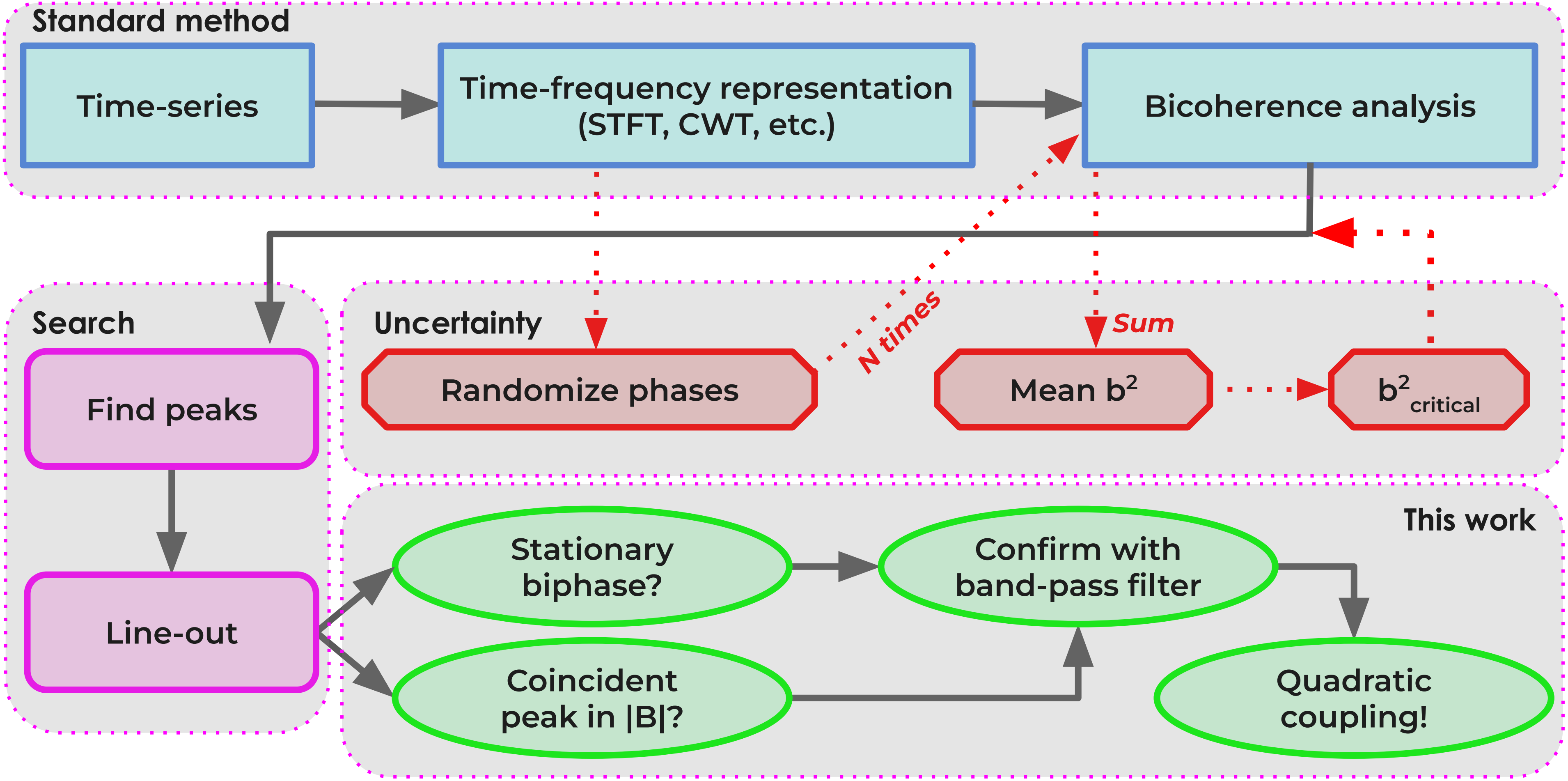


Parity of biphase/π corresponds to sign of coupling coefficient

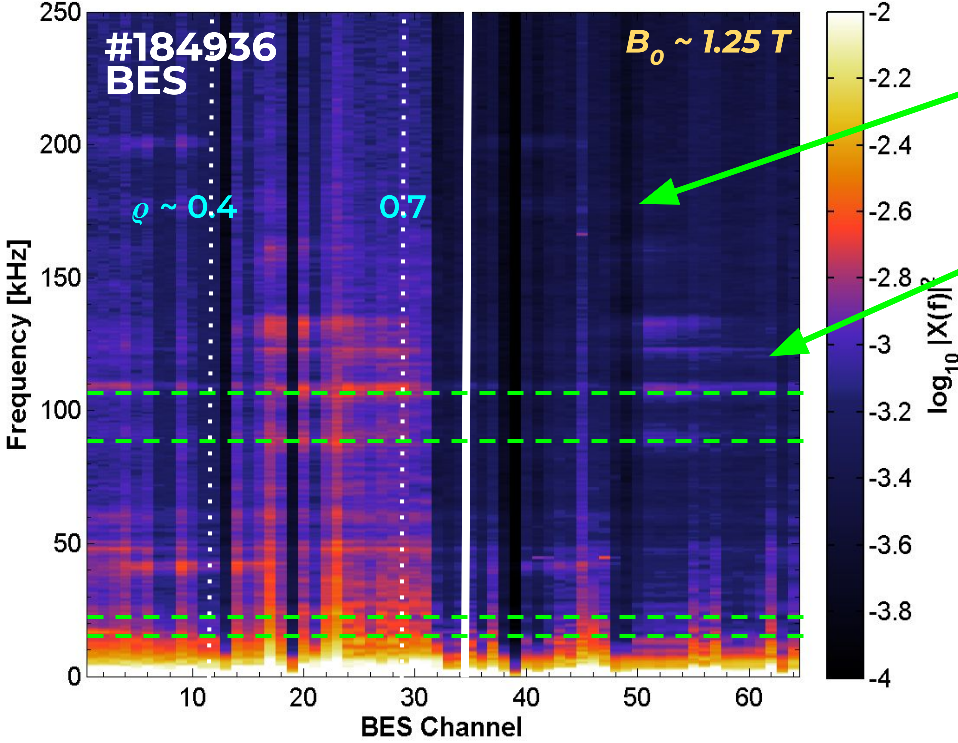
b





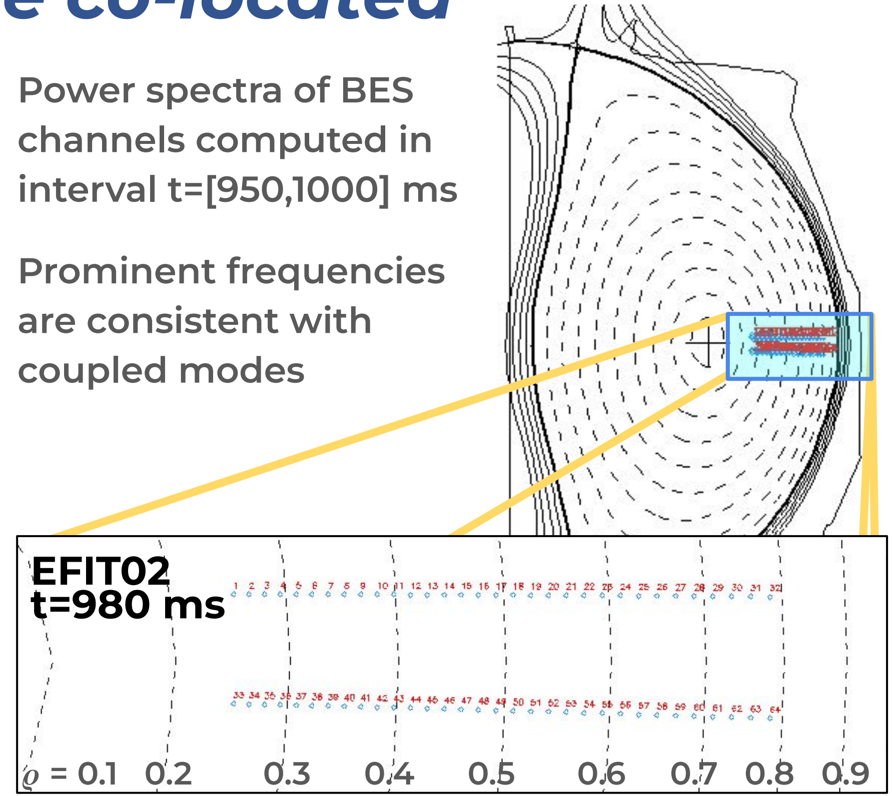

Standard method
Time-series
Time-frequency representation (STFT, CWT, etc.)
Bicoherence analysis
Search
Find peaks
Line-out
Uncertainty
Randomize phases
N times
Sum
Mean b²
b² critical
This work
Stationary biphase?
Coincident peak in |B|?
Confirm with band-pass filter
Quadratic coupling!

BES data indicate that TAEs and low-frequency fluctuations are co-located



Power spectra of BES channels computed in interval t=[950,1000] ms

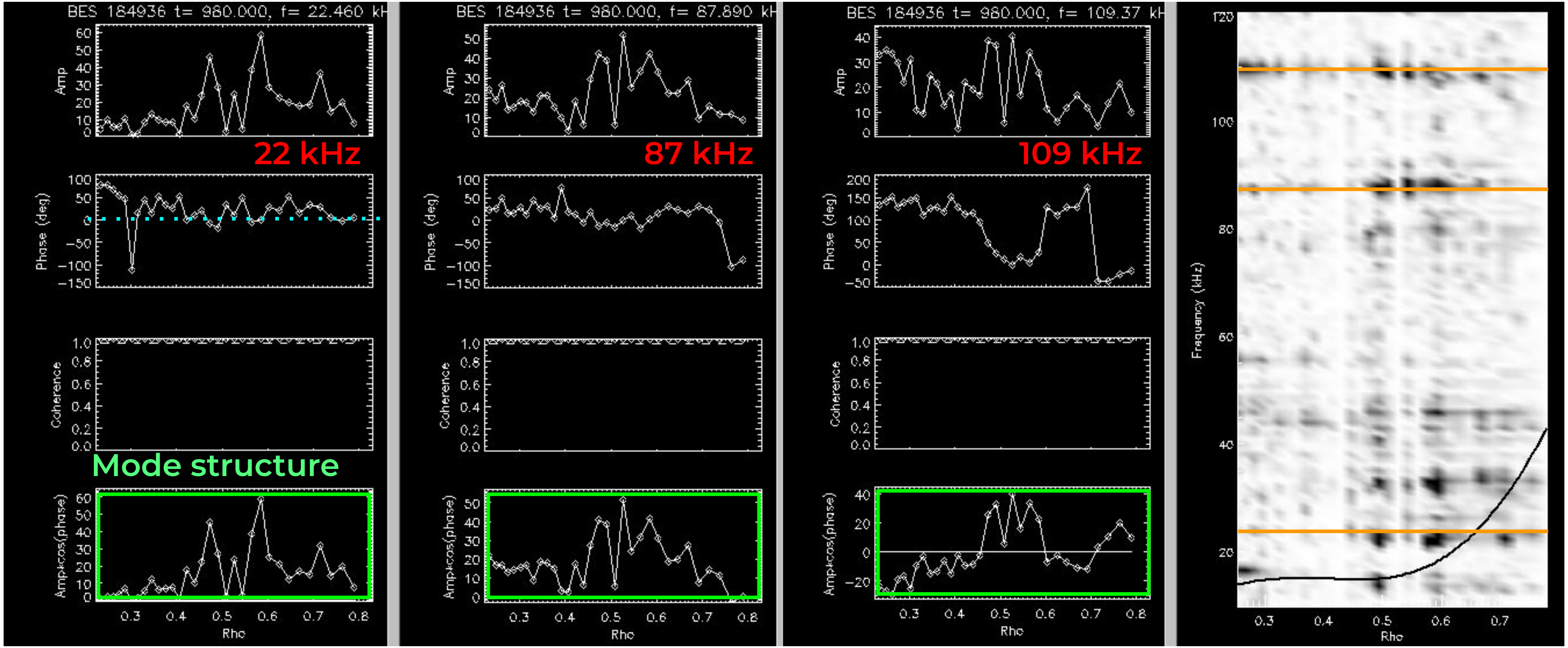
Prominent frequencies are consistent with coupled modes



BES channel location

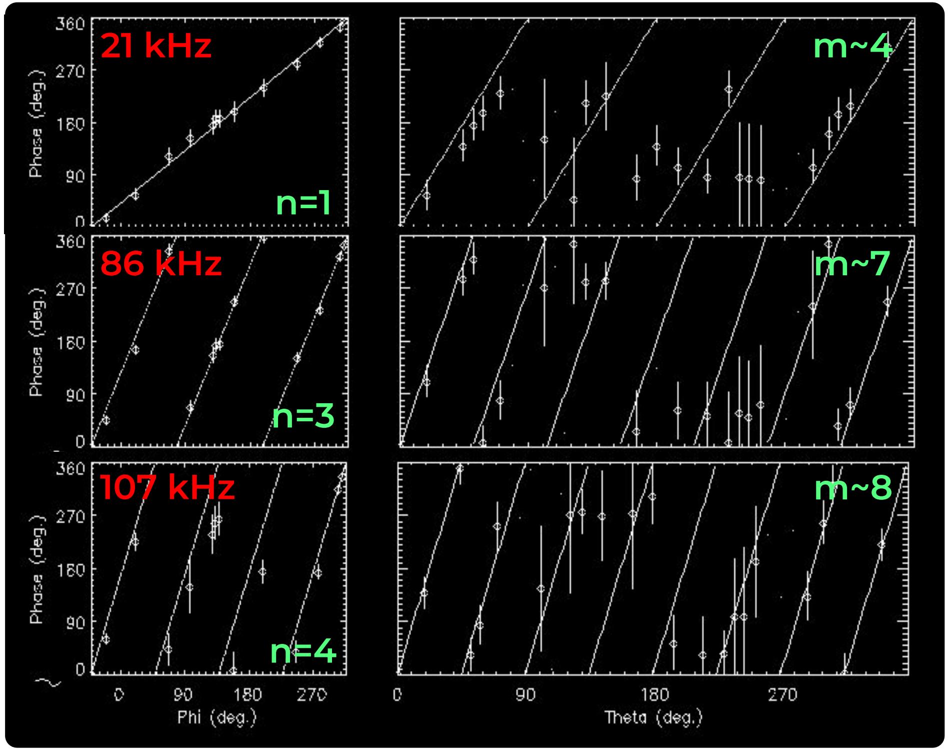
Multipow tool supports overlap of eigenmodes

t=[980,982] ms

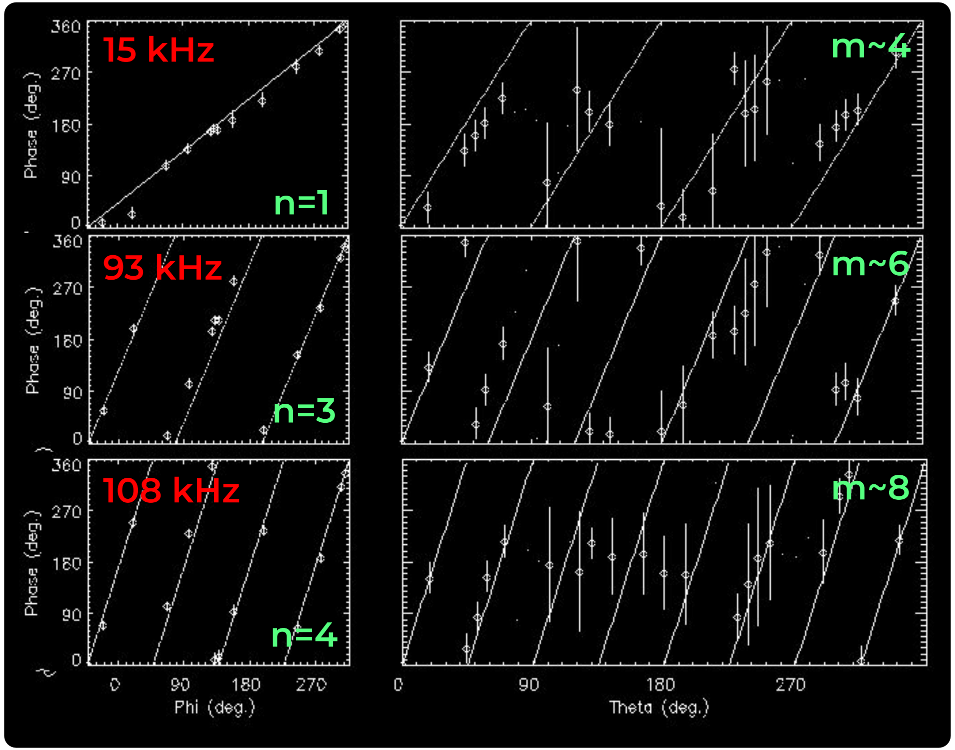


Mode determination confirmed by X^2 analysis [6]

t=[980,982] ms



t=[984,986] ms

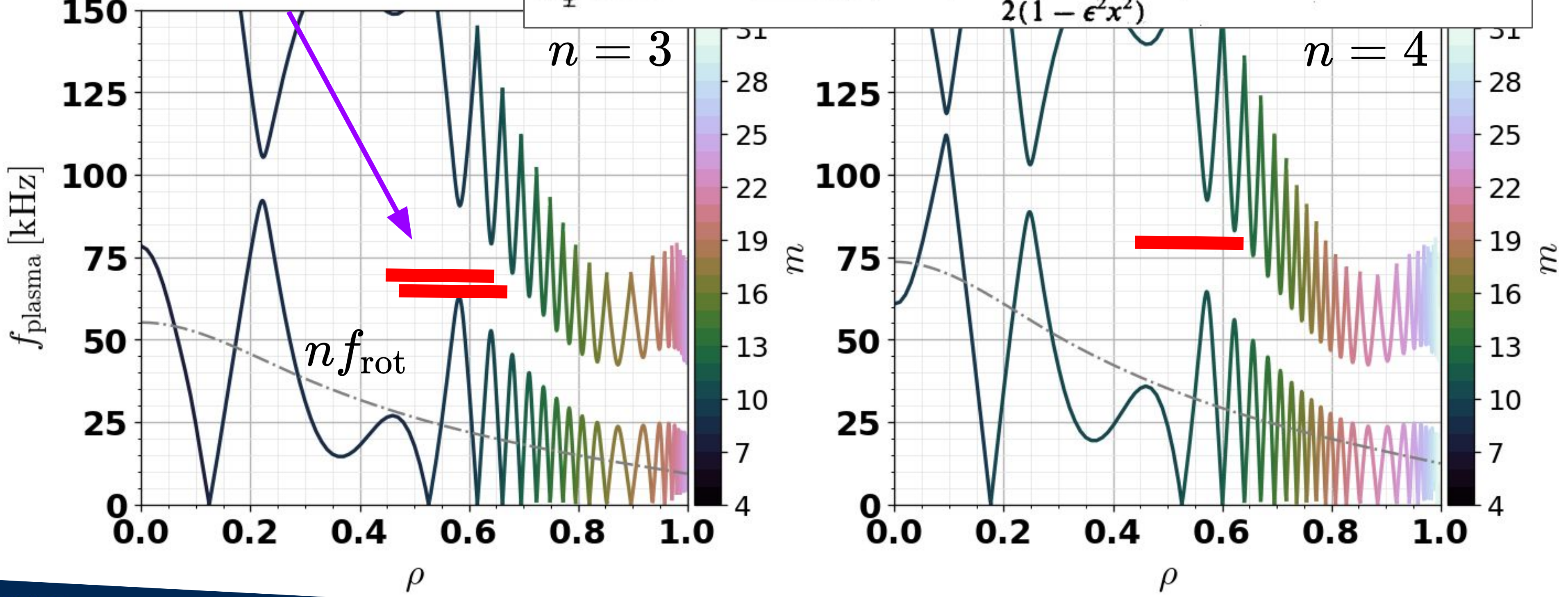


[6] Strait, E. 2006, RSI 77(2): 023502

Comparison with Alfven continuum reinforces results

Predicted freq. and radius agree with expt.

$$\omega_{\pm}^2 = \frac{k_{\parallel m}^2 v_A^2 + k_{\parallel m+1}^2 v_A^2 \pm \sqrt{(k_{\parallel m}^2 v_A^2 - k_{\parallel m+1}^2 v_A^2)^2 + 4\epsilon^2 x^2 k_{\parallel m}^2 v_A^2 k_{\parallel m+1}^2 v_A^2}}{2(1-\epsilon^2 x^2)}$$ [7]



[7] Fu, G. & Van Dam, J. 1989, Phys. Fluids B 1, 1949–1952

What have we found?

Quadratic coupling is consistently identified on sub-millisecond timescales

Onset of nonlinearity is **precipitated by growth of TAEs**

Subsequent changes in TAE amplitudes are **correlated with duration of quadratic nonlinearity**

Energy transfer to or from low-frequency modes is hypothesized to be facilitated by nonlinearities

Technique amenable to automation; may be relevant to machine learning-driven feedback control

What's next???

Submit paper (**PoP**), just passed R&A @ GA!

Determine **direction of energy transfer** during nonlinearity

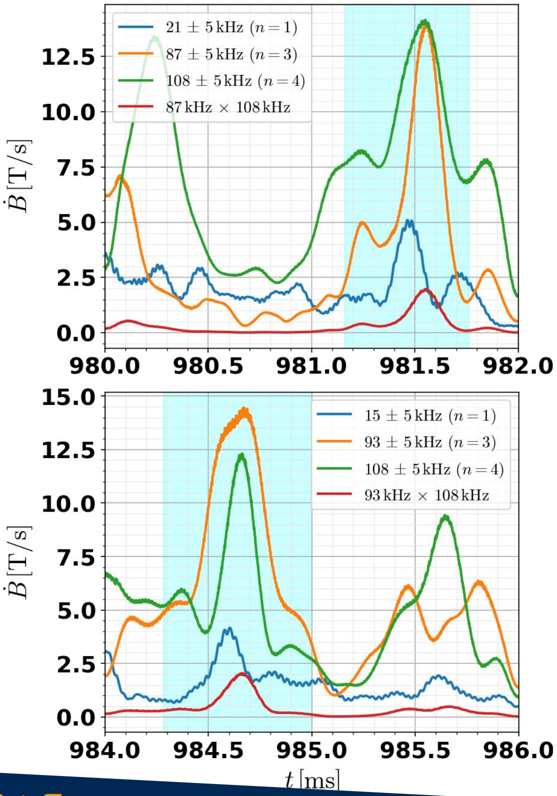
Quantify **role of nonlinear coupling** and energy transfer in mediating saturated amplitude of TAEs

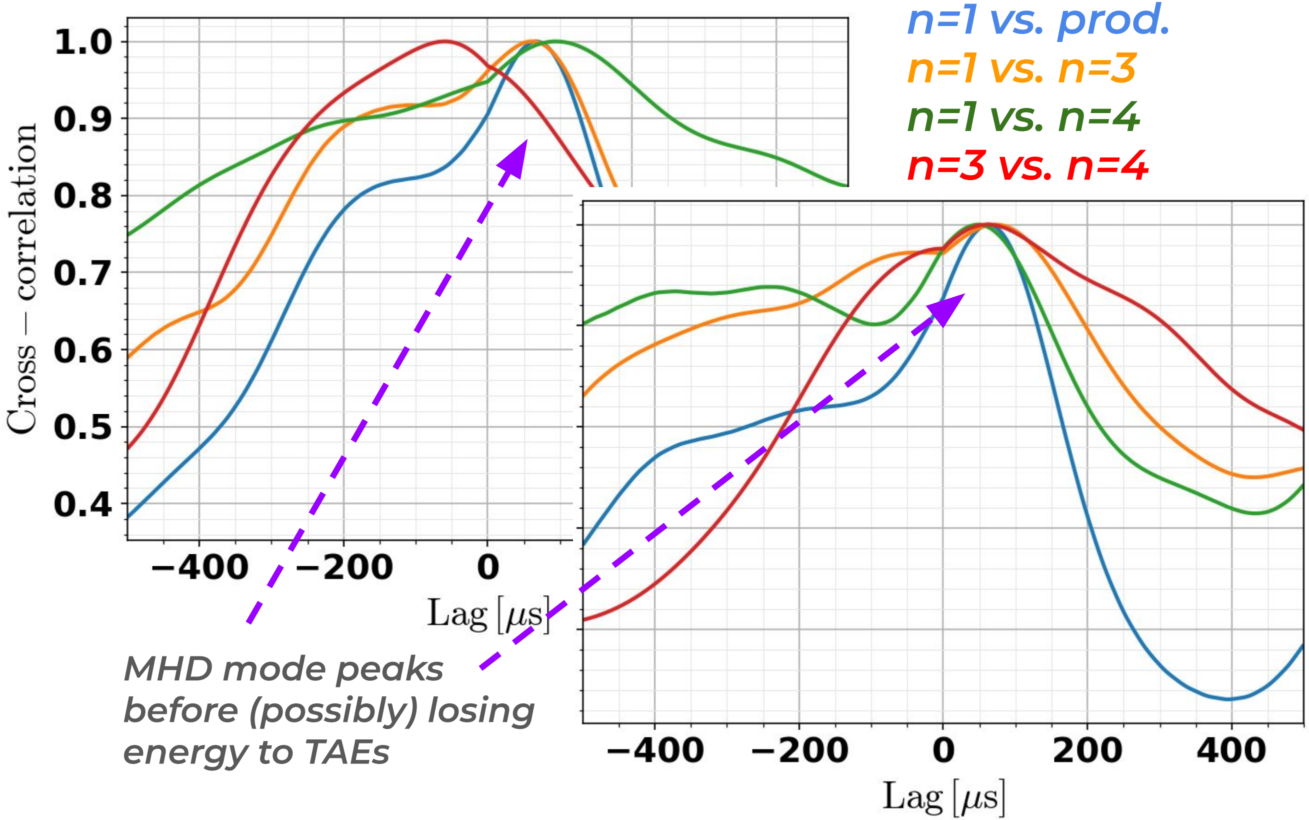
Quantify changes in TAE spectra and neutron rate in context of NBI program

Leverage **FAR3d and TRANSP simulations** to provide insight into wave-wave and wave-particle interactions

Correlate fluctuations in density and magnetic field with perturbations in fast-ion distribution function / Infer **transport**

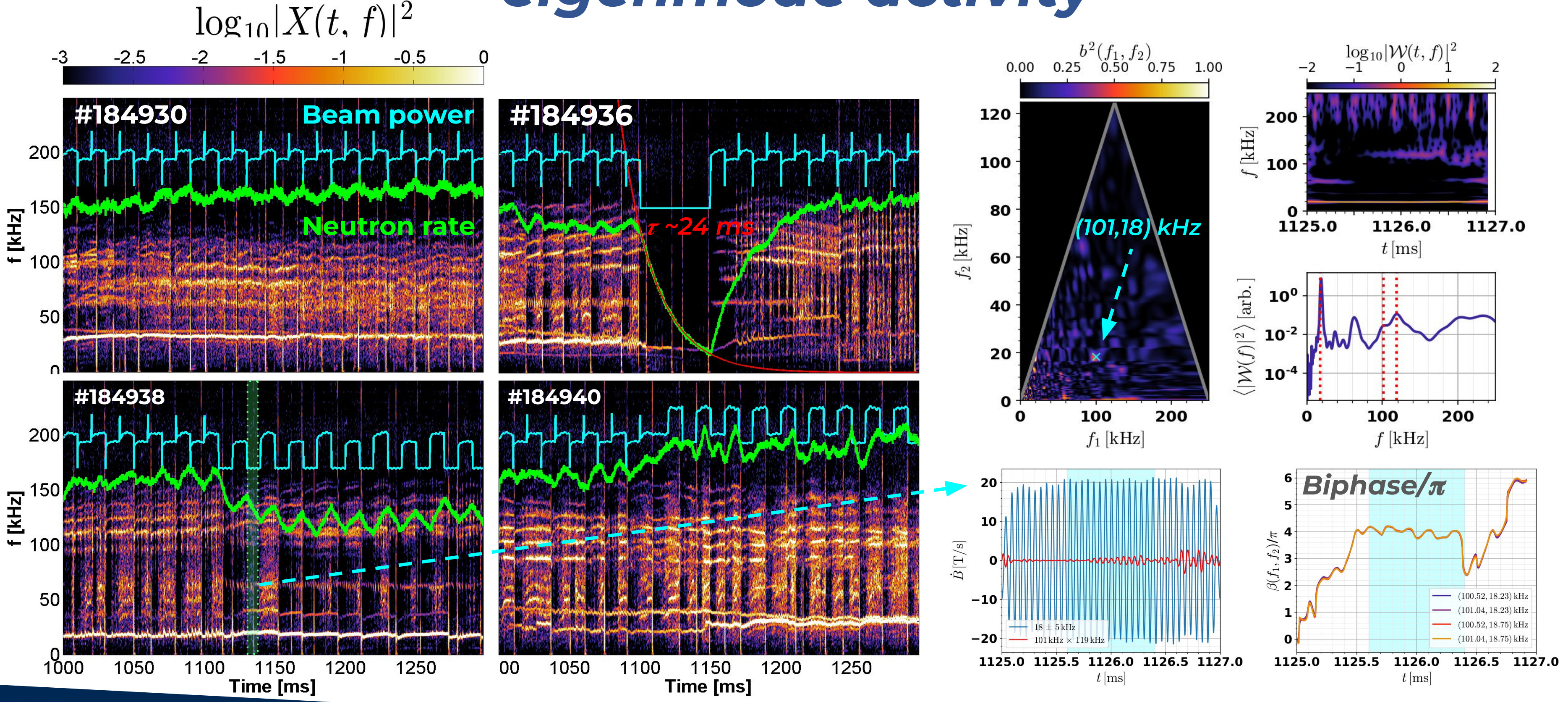
Causality determination attempted with cross-correlation analysis[8]





[8] Yamada, T., 2010 PoP, 17, 052313

Beam program conveniently samples range of eigenmode activity



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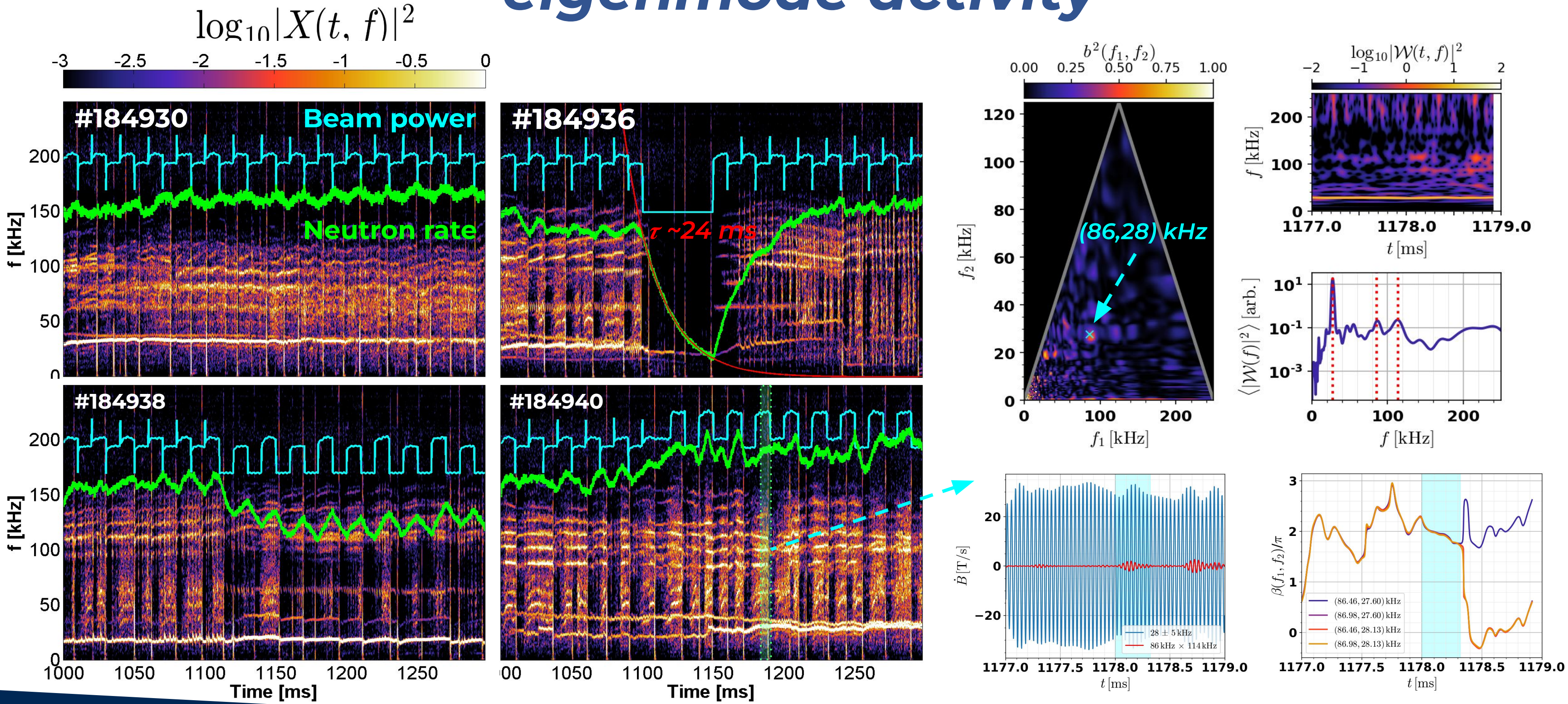
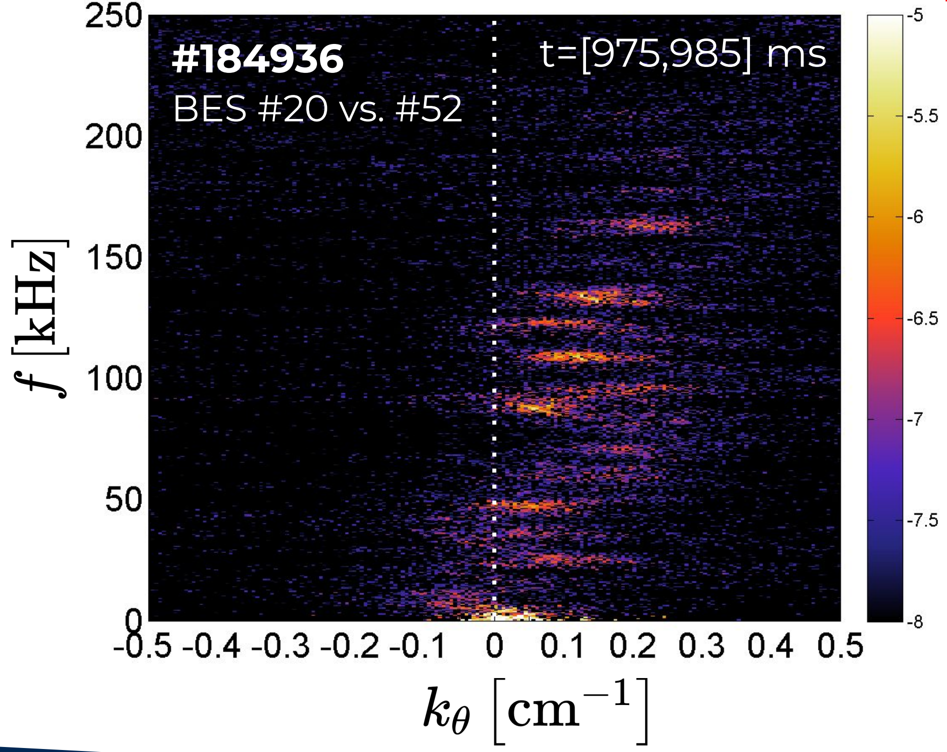


Local wavenumber-frequency spectrum estimated from BES fluctuations and cross-phase

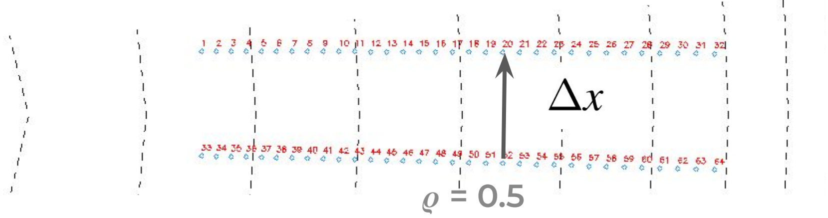


[9] Beall, J., Kim, Y., & Powers, E., 1982, Journal of Applied Physics, 53(6), 3933-3940

$$S(k,f) = \frac{1}{M}\sum_{i=1}^{M} I_{\Delta k}\left[k - k_i(f)\right] \times \frac{1}{2}\left[S_1^i(f) + S_2^i(f)\right]$$

$$I_{\Delta k}\left[k - k_i(f)\right] = \begin{cases} 1, & |k - k_i(f)| \leqslant \Delta k/2 \\ 0, & \text{elsewhere} \end{cases}$$

$$k_i(f) = \Delta\theta_{1,2}^i(f)/\Delta x$$

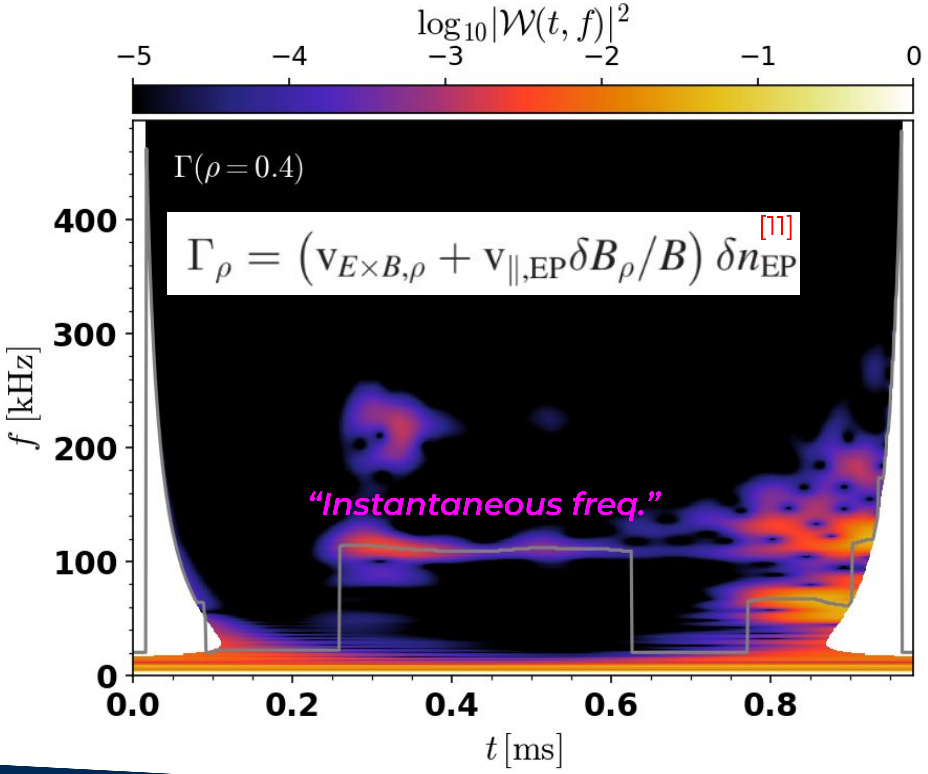


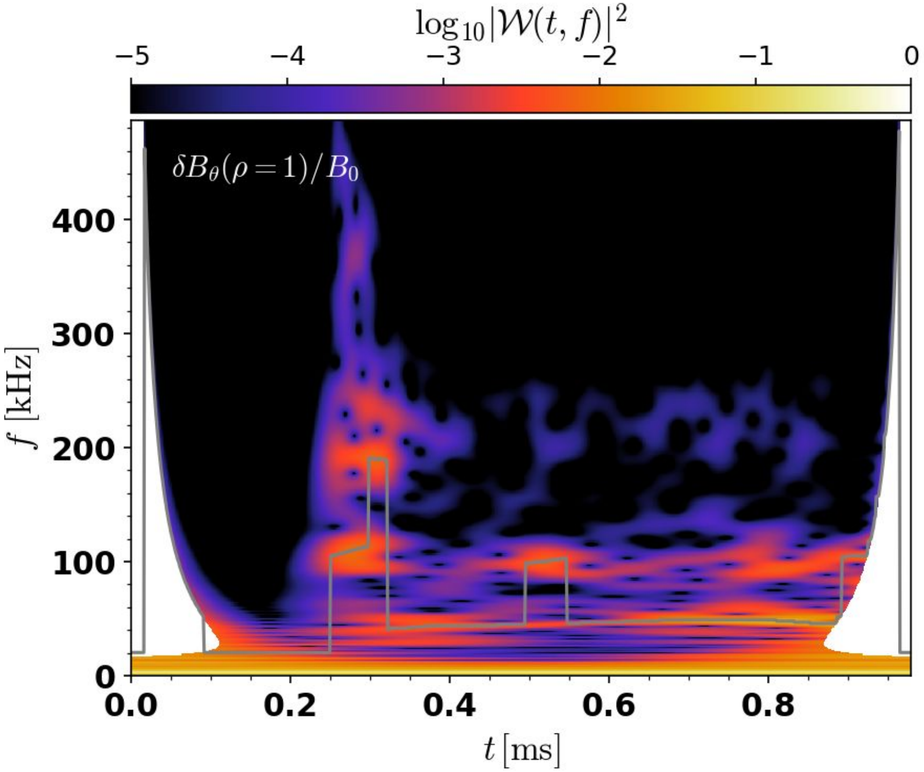
Theory predicts most unstable mode: [10]

$$k_\theta \rho_{\mathrm{EP}} \sim 1 \longrightarrow n \sim \frac{a B_0 Z_f e}{q\sqrt{2 E_f m_f}}$$

[10] Heidbrink, W., 2002 PoP, 9(5), 2113-2119

FAR3d simulations report similar fluctuations in radial particle flux, magnetics, and phi





[11] Spong, D., et al. 2021,NF 61, 116061

Thanks for your time!

email: gariggs@mix.wvu.edu

Questions?

What kind of plasma are we talking about?

Fundamentals

D ($m_i = 2m_p$)

$n_e \sim 2 \times 10^{13}\ \mathrm{cm}^{-3}$

$n_i \sim 2 \times 10^{13}\ \mathrm{cm}^{-3}$

$T_e \sim 1$ keV

$T_i \sim 1$ keV

B ~ 1.25 T

$I_p \sim 0.7$ MA

Velocities

$v_A \sim 4000$ km/s ~ 1.5% c

$v_{th,i} \sim 300$ km/s $\quad v_{fi} \sim 2500$ km/s

$v_{th,e} \sim 18{,}000$ km/s ~ 6% c

Characteristic scales

$\Omega_i/2\pi \sim 9.5$ MHz $\quad \omega_{p,i}/2\pi \sim 640$ MHz

$\Omega_e/2\pi \sim 35$ GHz $\quad \omega_{p,e}/2\pi \sim 40$ GHz

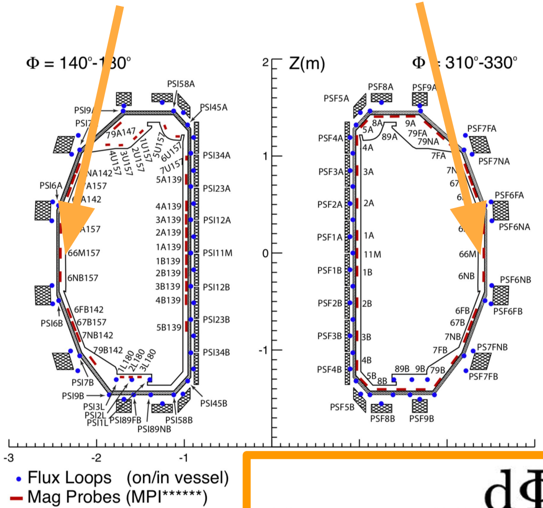
$\lambda_{D,i} \sim 50\ \mu$m $\quad \varrho_i \sim 4$ mm

$\lambda_{D,e} \sim 50\ \mu$m $\quad \varrho_e \sim 60\ \mu$m

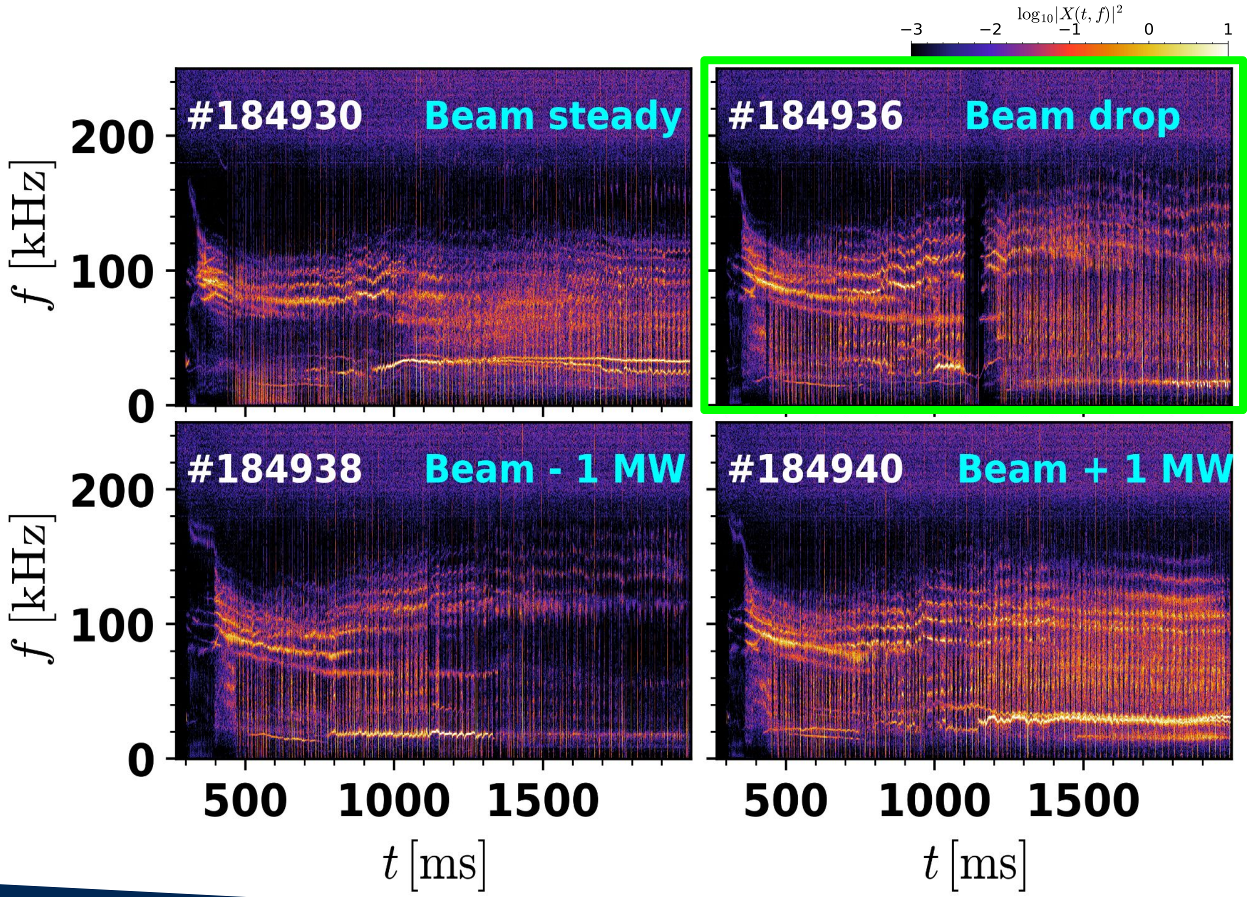
$\varrho_{fi} \sim 4$ cm

dB_θ/dt @ edge is measured via inductive coils

Toroidal array on outer midplane infers mode #



$$\mathcal{E} = -N\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$



Adapted from Lesur, M., 2010, Ecole Polytechnique