Fast Ion Effects on Zonal Flow Growth based on the extended Hasegawa-Mima Equation

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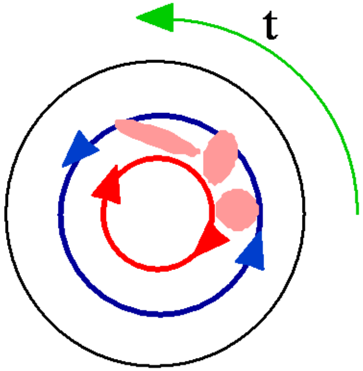
Outline

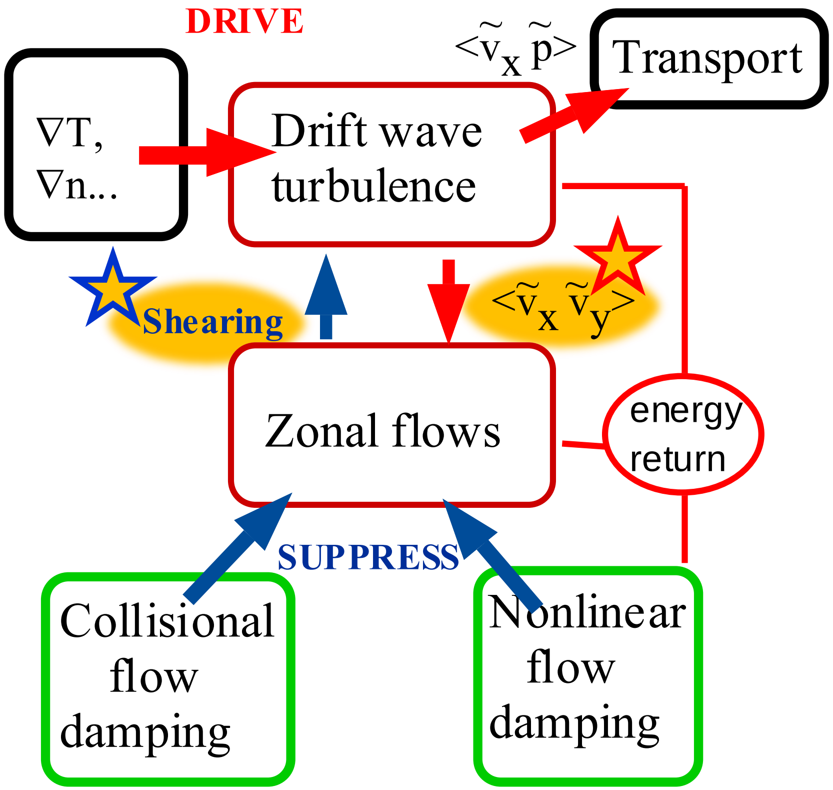
- Physics of Zonal Flows
- Fast Ion Effects on Tokamak Confinement
- Simple Model for Zonal Flow Generation
- Main Results including Fast Ions
- Discussion

Basic Physics of Zonal Flow

from [Diamond, Itoh, Itoh and Hahm, “Zonal Flows in Plasma – a Review” PPCF (2005)]



Suppression of DW by Shearing



DRIVE

∇T,
∇n...

Drift wave turbulence

$\langle \tilde{v}_x \tilde{p} \rangle$

Transport

Shearing

$\langle \tilde{v}_x \tilde{v}_y \rangle$

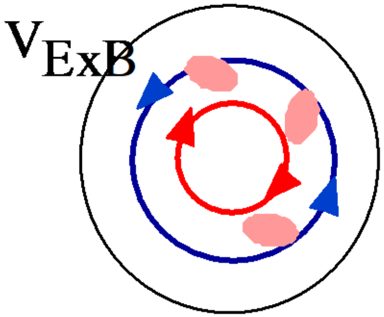
Zonal flows

energy return

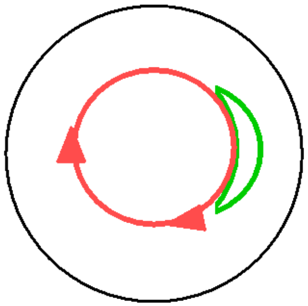
SUPPRESS

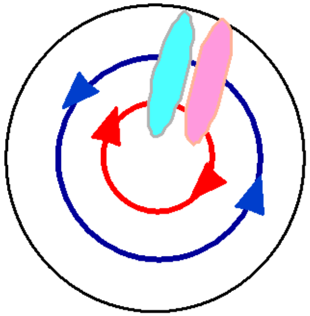
Collisional flow damping

Nonlinear flow damping



Generation by Vortex Tilting

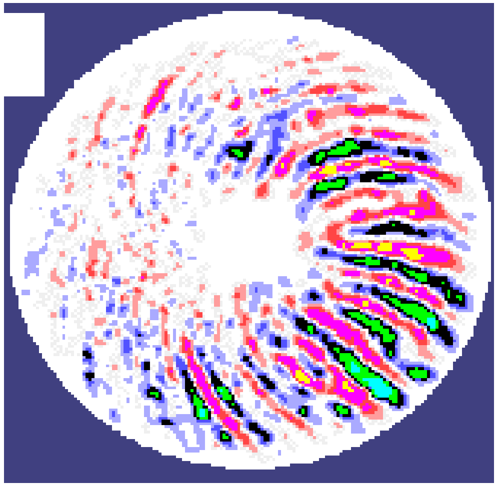




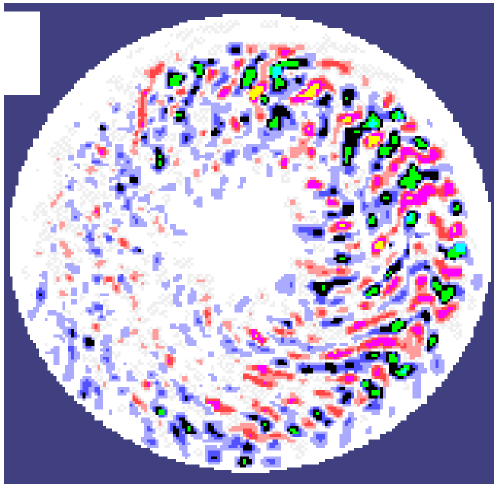
Damping by Collisions

Sheared Zonal Flow Regulates Turbulent Eddy Size and Transport

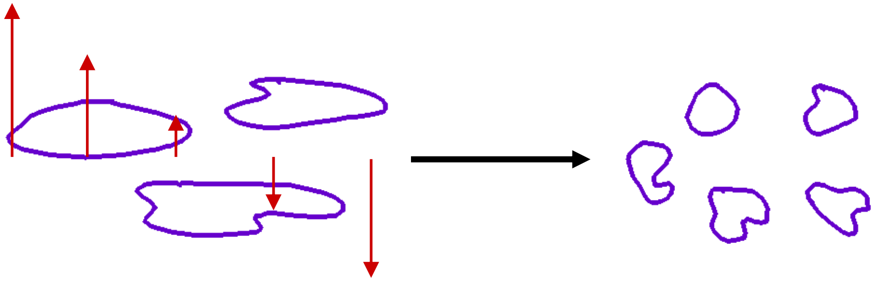
No flow



With flow



- Externally driven E×B Shear Flows were used before for the direct control of the turbulence. [H-mode, ITBs, …]
- Self-generated E×B zonal flow from turbulence reduces radial size of eddies.

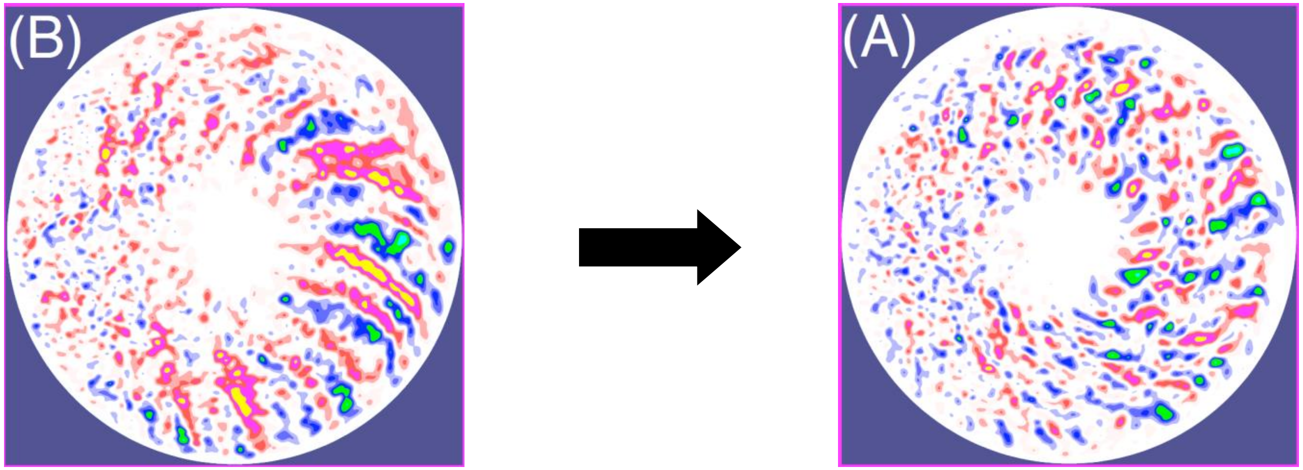


- Breakup of radially elongated structures reduces transport.

[Lin, Hahm, Lee, Tang and White, Science (1998)]

Duality of Flow Generation and Random Shearing of Eddies

Based on Conservation laws: Wave-kinetic equation for details.



$\omega_k \gg \omega_{\mathrm{ZF}}$ ➡ Drift Wave Action Density N_k is conserved.

(adiabatic invariant)

From $\omega_{\mathrm{DW}} = \dfrac{k_\theta v_*}{1 + k_\perp^2 \rho_s^2}$, shearing $\longrightarrow$ $k_r^2 \nearrow$ $\longrightarrow$ Drift Wave Energy $E_k = N_k \omega_k \searrow$

Since total energy is conserved between Zonal Flow and Drift Wave, energy for ZF generation is extracted from DWs.

[Diamond *et al*., IAEA-FEC ‘98]

Zonal Flow Generation via Modulation in Toroidal Geometry

[Chen, Lin and White, Phys. Plasmas (2000)]

$$\Phi_+(\vec{k}_+ = \vec{k} + \vec{q}_r) \propto \frac{\Phi_{\mathrm{DW}}}{D_+}\Phi_{\mathrm{ZF}}$$

$\Phi_{\mathrm{DW}}(k_r)$

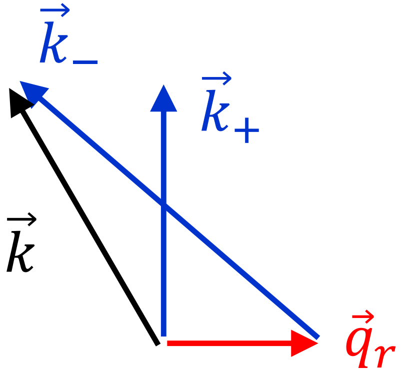
"Drift Wave"

from E×B nonlinearity

$$\Phi_-(\vec{k}_- = \vec{k} - \vec{q}_r) \propto \frac{\Phi_{\mathrm{DW}}}{D_-}\Phi_{\mathrm{ZF}}$$

Modulation via $\Phi_{\mathrm{ZF}}(q_r)$

"Side-bands" DW produced.



Φ_+

Φ_-

$\Phi_{\mathrm{DW}}(k_r)$

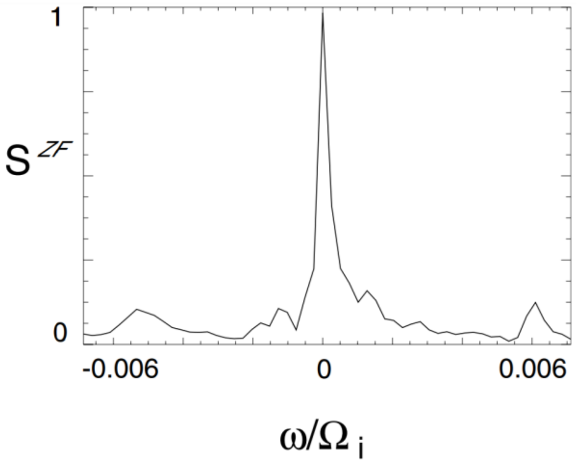
$$\frac{q^2}{\sqrt{\epsilon}}\gamma_{\mathrm{ZF}}\Phi_{\mathrm{ZF}} \propto \frac{\Phi_{\mathrm{DW}}^2}{D(\gamma_{\mathrm{ZF}})}\Phi_{\mathrm{ZF}}$$

Source term from Hasegawa-Mima nonlinearity

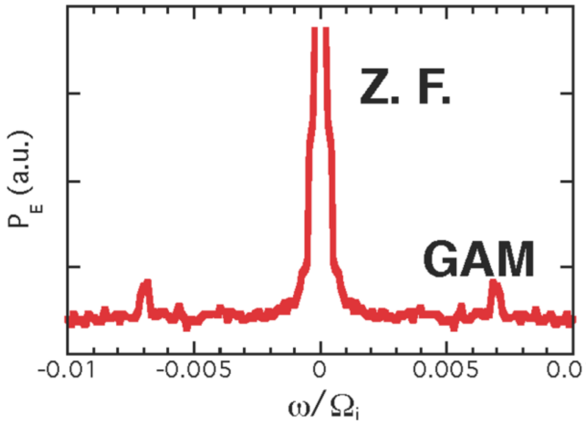
Neoclassically enhanced Polarization (vorticity) in long term [Rosenbluth-Hinton, PRL '98]

Characterization of Zonal Flow Properties from Simulations Motivated Experimental Measurements

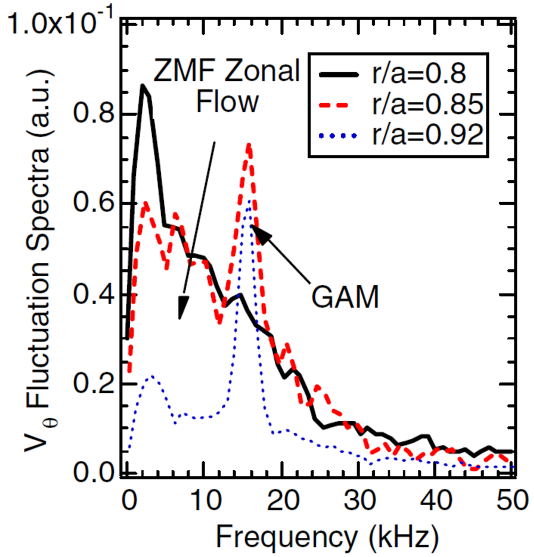
- Near Zero-frequency Zonal Flows are dominant in core plasmas.



from Gyrokinetic Simulation by Z. Lin *et al*.
[T.S. Hahm *et al*., PPCF ‘00]



[A. Fujisawa *et al*., PRL ‘04]
from Stellarator (CHS)



[D.K. Gupta, G. McKee *et al*., PRL ‘06]
from Tokamak (DIII-D)

- Rare example of theory leading experiment -

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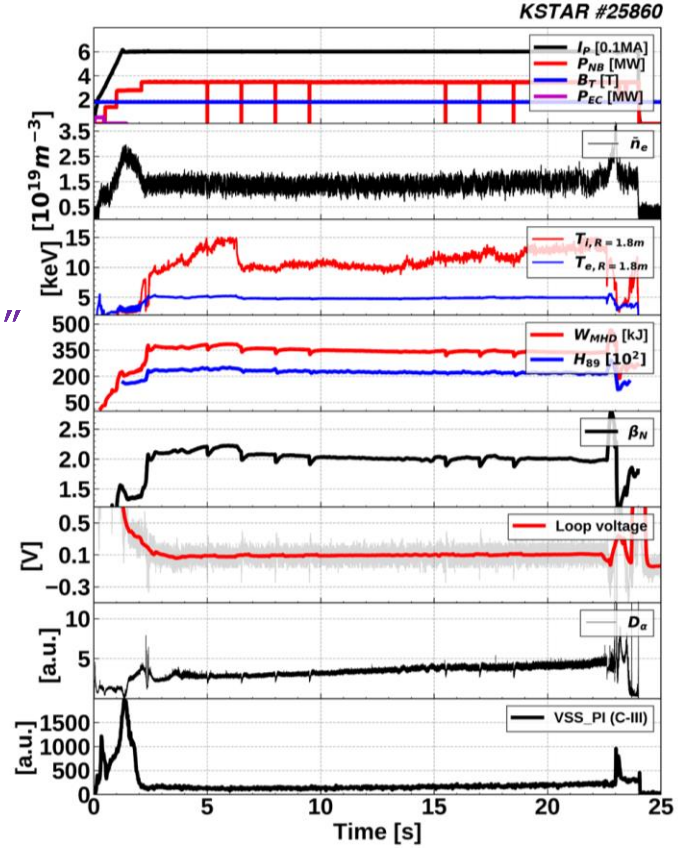
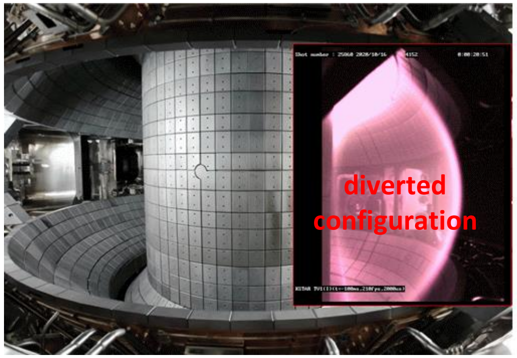
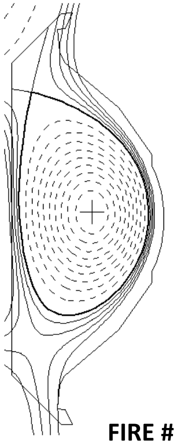
FIRE (Fast Ion Regulated Enhancement) mode

- **Stationary ITB discharges** have been established in a diverted configuration at $q_{95} \sim 4–5$ on KSTAR.
- L-H transition was avoided by keeping **low density and unfavorable ∇B single null configuration.**
- **Fast ions** have significant roles in this new regime, so it is coined to “Fast-Ion-Regulated Enhancement.”



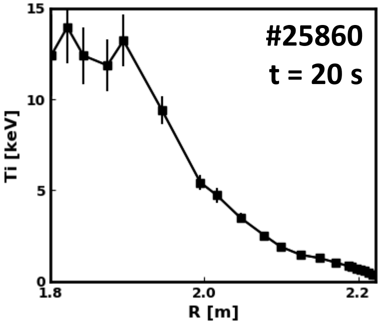
<Camera Image of KSTAR FIRE mode >





FIRE #25860
EFIT construction at 20 s

<Ion Temperature Profile of FIRE mode>



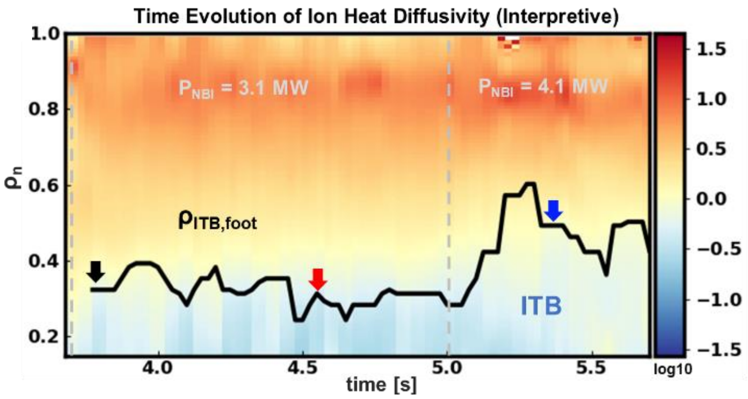
*H. Han, S.J. Park and Y.-S. Na et al. Nature **609** 269 (2022)*

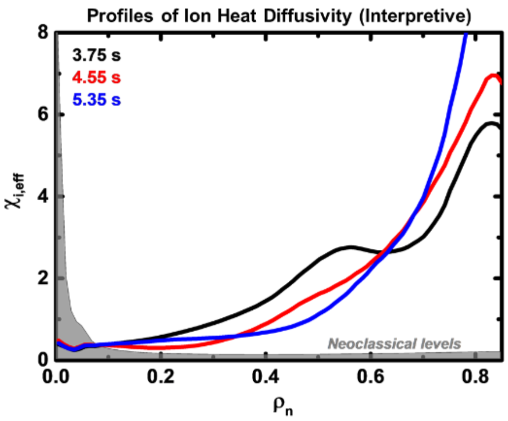
Courtesy: Y.-S. Na

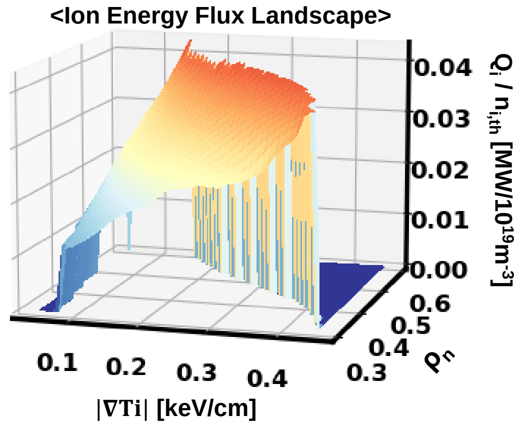
ITB characteristics – heat diffusivity and S-curve

- **Thermal Ion Heat Diffusivity and S-curve**

- The time evolution of the ion heat diffusivity was calculated from the power balance analysis.
- The **thermal ion heat diffusivity reduces** in time correlated **with the expansion of ITB** though it is still above the neoclassical level.
- The relation between the **ion energy flux** and the **ion temperature gradient** shows that there is a "S-curve" in the 3-D landscape* [P.H. Diamond *et al.*, PRL (1997)].
- The reduction of the energy flux while the gradient increases implies a transport bifurcation.


Time Evolution of Ion Heat Diffusivity (Interpretive)

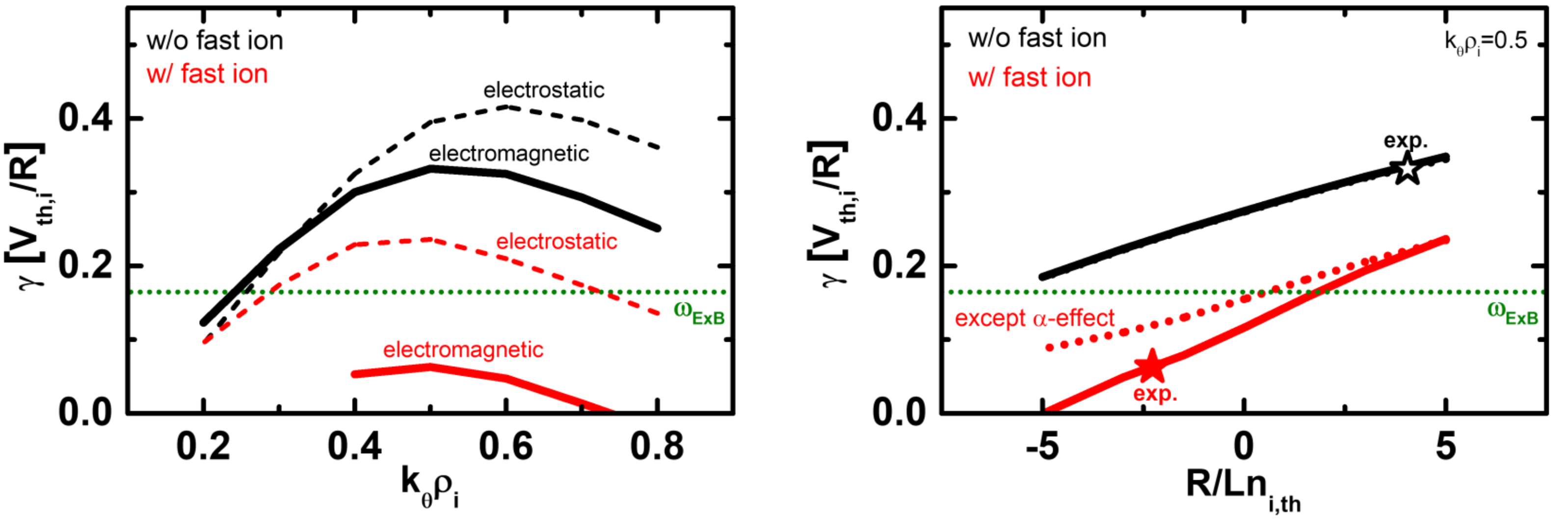

Profiles of Ion Heat Diffusivity (Interpretive)


<Ion Energy Flux Landscape>

Courtesy: Y.-S. Na

FIRE (Fast Ion Regulated Enhancement) mode

- **GKW Linear simulations for evaluating fast ion effects**

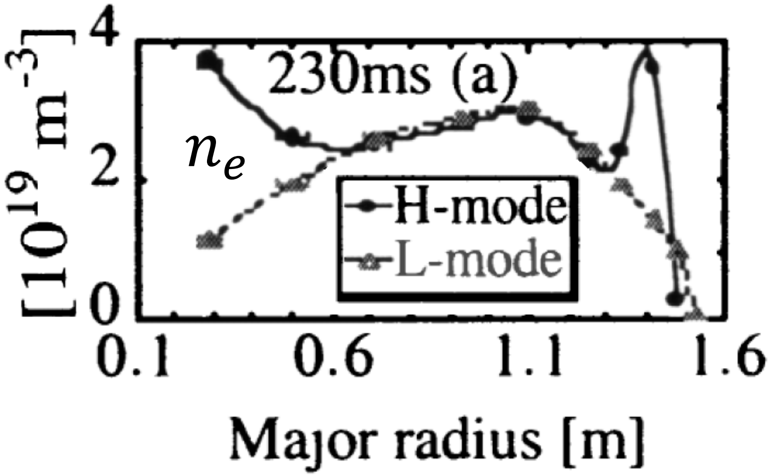


- **$R/L_{ni} < 0$ due to dilution by centrally peaked fast ions $R/L_{nf} \gg 1$**
  $\Rightarrow$ Strong ITG stabilization.
- ExB shear stabilization is also substantial.
  (But even in its absence, significant reduction of turbulence expected.)
- Shafranov shift and electromagnetic effects contribute to further stabilization.

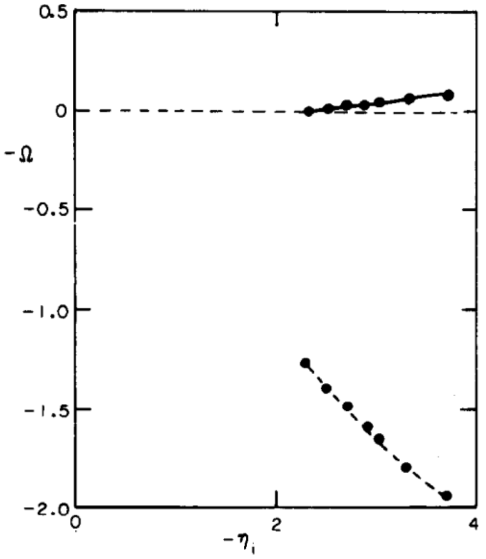
H. Han, S.J. Park and Y.-S. Na et al. Nature ***609*** *269 (2022)*

Courtesy: S.J. Park

Ion Temperature Gradient Mode becomes weaker for hollow density profiles



Experimental data from NSTX
[R. Maingi *et al.*, PRL (2004)]



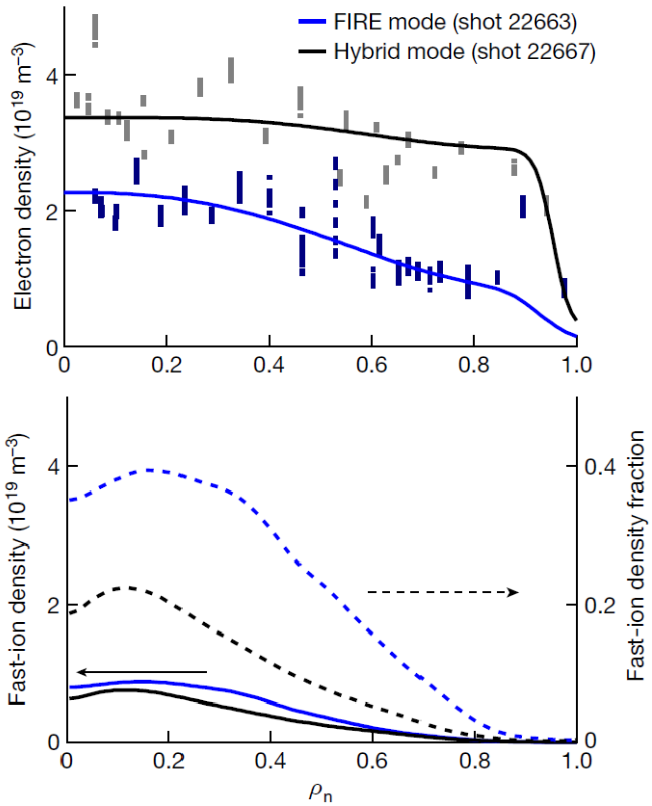
[Hahm-Tang, PFB (1989)]

FIG. 2. Plot of numerically computed Ω vs η_i for $L_n/L_s = -1$, $b_i = 0.1$, and $\tau = 1$. The solid and dashed lines correspond, respectively, to $\mathrm{Im}(-\Omega)$ and $\mathrm{Re}(-\Omega)$. Note that $\Omega = \omega/\omega_{*e} \propto -\omega$ since $L_n < 0$.

- n_e sometimes gets hollow in core of H-mode plasma during ELM-free period.

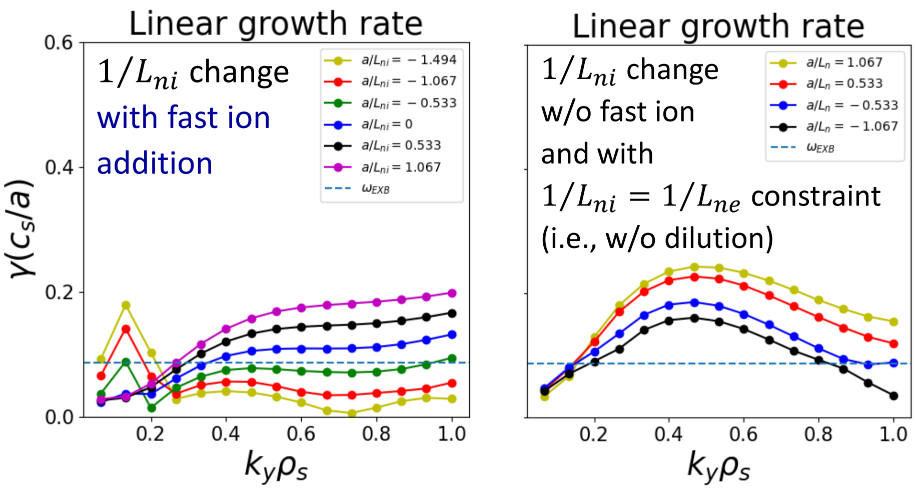
- "A particularly interesting new feature for the inverted density profile cases is that $\gamma \ll |\omega_r|$ is satisfied for η_i modes over a wide range of negative η_i values."
- More detailed toroidal analyses in
  - [Du, Jhang, Hahm *et al.*, PoP (2017)]

Hollow Density ITG becomes significantly weaker with fast ion induced dilution



[H. Han, S.J. Park *et al*., Nature (2022)]

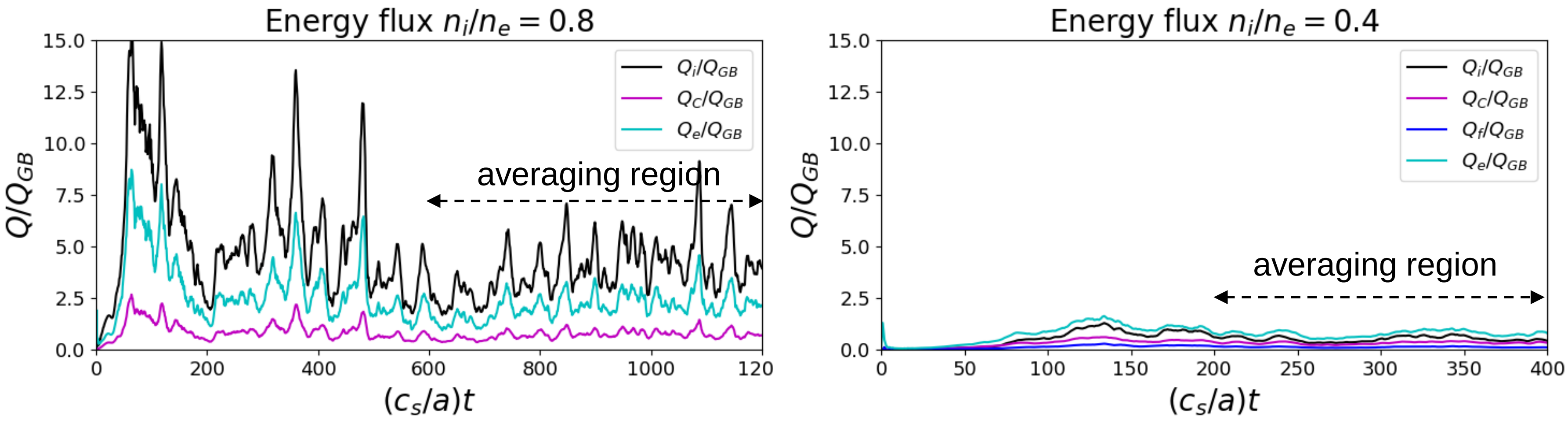
- Main ion density profile becomes hollow with centrally peaked fast ion density profile.



from CGYRO linear simulations
[D.U. Kim, C.K. Sung, S.J. Park *et al*.,
Submitted to Nuclear Fusion (2023)]

- “$1/L_{ni}$” is the key quantity determining ITG stability with fast ion induced dilution.

Dilution effect



- Energy flux level predicted by CGYRO nonlinear gyrokinetic simulation shows the reduction when effect of dilution is included.

  \- $Q_i[Q_{GB}] \sim 4.567 \rightarrow 0.836, Q_e[Q_{GB}] \sim 2.102 \rightarrow 0.769$

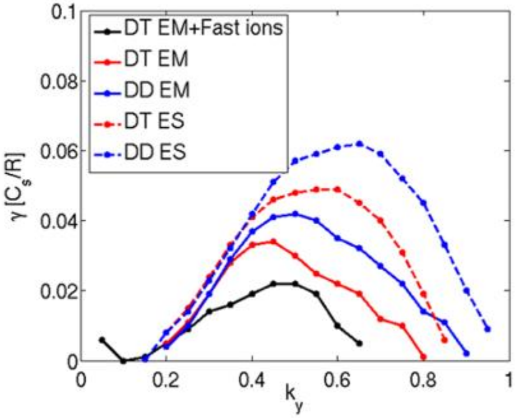
  \- But, energy flux is still higher than nonlinear run with fast ion ($Q_i[Q_{GB}] \sim 0.001, Q_e[Q_{GB}] \sim 0.067$).

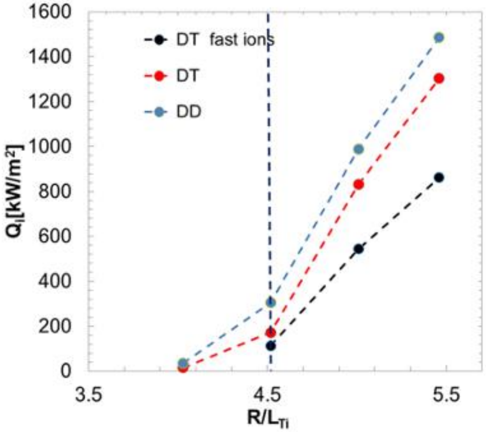
- Dilution contribute reduction of energy flux, but not sufficient to explain the reduction of energy flux due to fast ion.

Courtesy: D.U. Kim, C.K. Sung *et al*.

Variety of Fast Ion related Linear Stabilizers

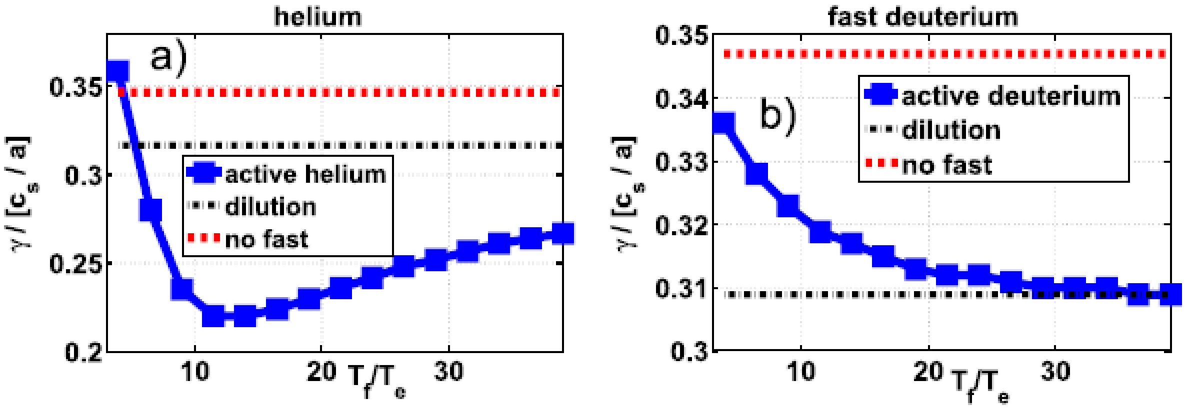
- Isotope and fast ions significantly reduce turbulence, correlated with EM effects.





[J. Garcia et al., PoP ‘18]
GENE simulations

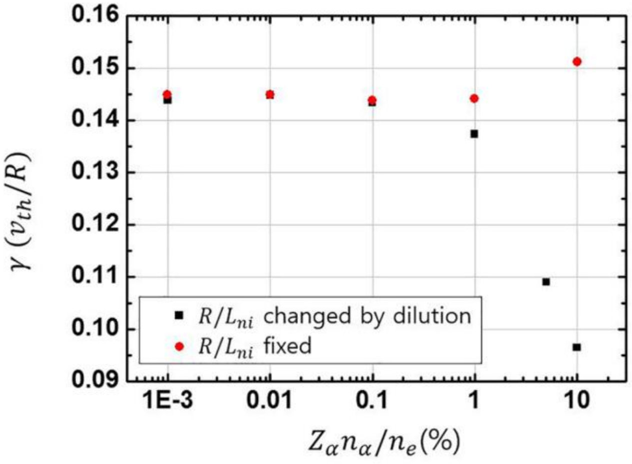
- Fast ions stabilize microinstabilities through wave-particle resonance interactions



[A. Di Siena, NF ‘18]
GENE simulations

Microturbulence-driven Energetic Particle Transport

from GKW simulations of TEM [S.M. Yang *et al*., PoP 25, 122305 (2018)]



- It can be significantly lower than thermal particle transport, due to Orbit averaging and Frequency detuning.
- Similar conclusions on ITG-driven EP Transport: [Estrada *et al*., PoP (2006)],
  [Zhang *et al*., PRL (2008)],
  [Angioni *et al*., NF (2009)],
  [Di Siena *et al*., PCS (2016)].

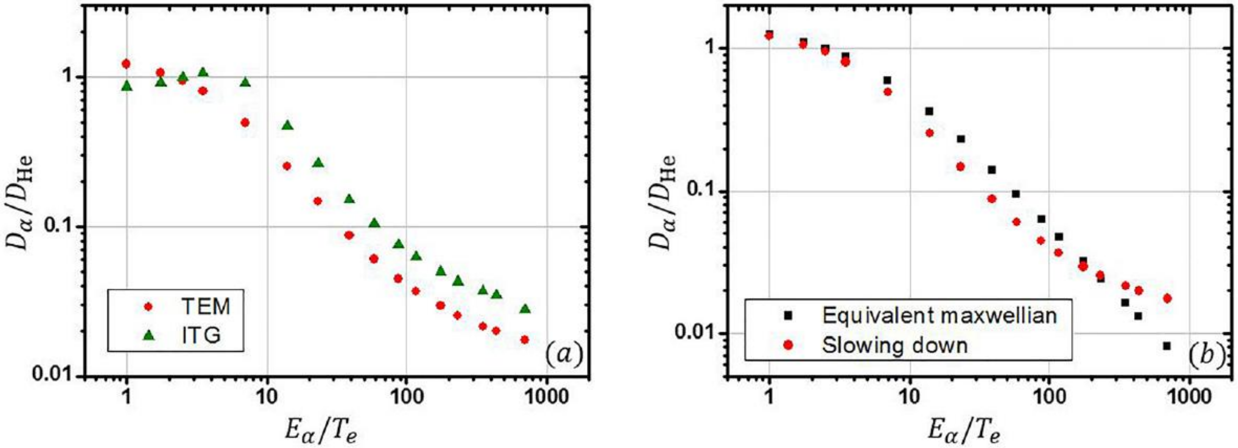


FIG. 3. The normalized diffusivity D_α/D_{He} of the α particles from linear GKW calculations for (a) the TEM (circles, red) and the ITG (triangles, green) case and (b) the equivalent Maxwellian (squares, black) and the slowing down (circles, red) distribution as a function of E_α/T_e.

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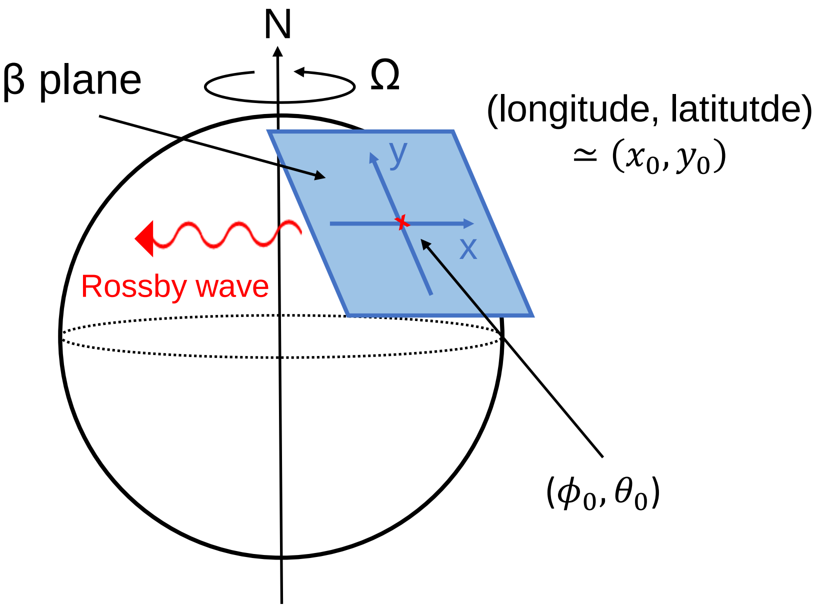
T. S. Hahm, G. J. Choi, S. J. Park and Y. S. Na.
Phys. Plasmas 30, 072501 (2023).

Spherical geometry and beta-plane approximation



Coriolis parameter (θ: latitude):

$$f = 2\Omega\sin\theta$$

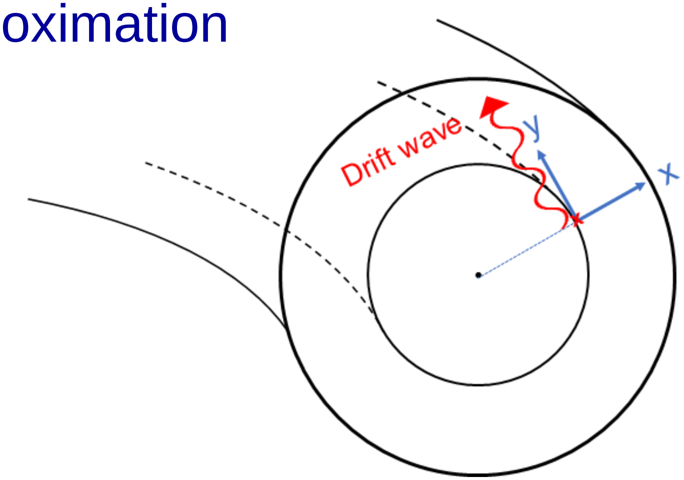
$$= 2\Omega\sin\theta_0 + 2\Omega\cos\theta_0 y + O(y^2)$$

$$\sim f_0 + \underline{\beta} y$$

β : beta parameter

Courtesy : K. Obuse

Toroidal Geometry and Slab Approximation (MFE)



Charney-Hasegawa-Mima Equation

- $$\frac{\partial}{\partial t}(\phi - \rho_R^2 \nabla_\perp^2 \phi) + [\phi, \phi - \rho_R^2 \nabla_\perp^2 \phi] + \beta \frac{\partial \phi}{\partial x} = 0$$

$$(x, y) = (longitude, latitude)$$

$\rho_R = \sqrt{gH_m}/\Omega$: Rossby radius of deformation

β : Local variation of Coriolis parameter

GFD

- $$\frac{\partial}{\partial t}(\phi - \rho_s^2 \nabla_\perp^2 \phi) + [\phi, \phi - \rho_s^2 \nabla_\perp^2 \phi] + \left(\frac{\partial n_0}{\partial x}\right) \frac{\partial \phi}{\partial y} = 0$$

$\rho_s = c_s/\Omega_{ci}$: Larmor radius at T_e

$\frac{\partial n_0}{\partial x} \propto v_{*e}$: diamagnetic flow

MFE

Potential Vorticity Conservation

- $$\frac{d}{dt}[\phi - \nabla_\perp^2 \phi - \beta y] = 0$$ GFD

- $$\frac{d}{dt}\left[\phi - \nabla_\perp^2 \phi + \left(\frac{\partial n_0}{\partial x}\right) x\right] = 0$$ MFE

* Inhomogeneous mixing of PV
→ Zonal Flow Generation

e.g. Dritschel and McIntyre (2008).

Decomposition of Hasegawa-Mima Equation

- For zonal component,

$$\frac{\partial}{\partial t}\underline{\nabla_\perp^2\langle\phi\rangle} - \underline{\left\langle \boldsymbol{\nabla}\phi_{\mathrm{DW}} \times \mathbf{b} \cdot \boldsymbol{\nabla}\nabla_\perp^2\phi_{\mathrm{DW}} \right\rangle} = 0$$

"Zonal Flow Shear"

- "Flux of DW vorticity"

≀ G.I. Taylor 1915

- "Divergence of Reynolds Stress"

- For drift waves,

$$\frac{\partial}{\partial t}\left(\phi_{\mathrm{DW}} - \nabla_\perp^2\phi_{\mathrm{DW}}\right)$$

$$-\boldsymbol{\nabla}\langle\phi\rangle \times \mathbf{b} \cdot \boldsymbol{\nabla}\left(\phi_{\mathrm{DW}} - \nabla_\perp^2\phi_{\mathrm{DW}}\right)$$

$$+\boldsymbol{\nabla}\phi_{\mathrm{DW}} \times \mathbf{b} \cdot \boldsymbol{\nabla}\nabla_\perp^2\langle\phi\rangle + \underline{\frac{\partial}{\partial y}\phi_{\mathrm{DW}}} = 0$$

"From $\partial n_0/\partial x \propto \omega_{*e}$ "

Gyrokinetic Interpretation of Hasegawa-Mima Eqn.

- $\frac{\partial}{\partial t} n_{i,\mathrm{gc}} - \frac{c}{B_0} \vec{\nabla}\delta\phi \times \hat{b} \cdot \vec{\nabla} n_{i,\mathrm{gc}} = 0,$ ExB advection of guiding center density

  where $n_{i,\mathrm{gc}}$ satisfies Quasi-neutrality,

$$\delta n_{i,\mathrm{gc}} + \underline{\rho_s^2 \nabla_\perp^2 \frac{|e|\delta\phi}{T_e}} = |e| \left(\frac{\delta\phi - \langle \delta\phi \rangle}{T_e} \right)$$

“Vorticity” in fluid context

≀

“Polarization density” (moment of gyrophase angle-dependent distribution function) in modern Gyrokinetics.

Related discussions :

e.g. Hahm-Lee-Brizard, Phys. Fluids, ’88

D. Strintzi and B. Scott, PoP, ‘04

T.S. Hahm, L. Wang, and J. Madsen, PoP, ’09

This is a long wavelength (or warm ion $T_i \ll T_e$) limit of “$\{1 - \Gamma_0(k_\perp^2 \rho_i^2)\} \frac{|e|\delta\phi}{T_i}$” in gyrokinetics.

Modulational Instability of Zonal Flows

[Diamond, Itoh, Itoh, Hahm, Review of ZF, PPCF, ’05]

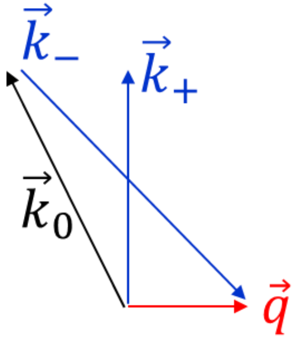
$$\Phi_{\mathrm{DW}}(\vec{k}_0,\omega_0) \longrightarrow \begin{cases} \Phi_+(\vec{q}+\vec{k}_0,\Omega+\omega_0) \propto \dfrac{\Phi_{\mathrm{DW}}}{\omega_0-\omega_+ +\Omega}\Phi_{\mathrm{ZF}} \\[2ex] \Phi_-(\vec{q}-\vec{k}_0,\Omega-\omega_0) \propto \dfrac{\Phi_{\mathrm{DW}}}{\omega_0+\omega_- -\Omega}\Phi_{\mathrm{ZF}} \end{cases}$$

“Drift Wave”

Modulation via $\Phi_{\mathrm{ZF}}(\vec{q},\Omega)$

“Side-bands” DW produced

from E×B nonlinearity



$$\Phi_+,\ \Phi_- \xrightarrow{\Phi_{\mathrm{DW}}(\vec{k}_0)} (\omega_0-\omega_+ +\Omega)(\omega_0+\omega_- -\Omega)\Phi_{\mathrm{ZF}}$$

$$\propto (M_{0,+}+M_{0,-})\Phi_{\mathrm{DW}}^2\,\Phi_{\mathrm{ZF}}$$

Reynolds stress from Hasegawa-Mima polarization nonlinearity

Earlier considerations : [P. Guzdar et al., PoP ’01; P. Shukla et al., Euro. Phys. D ’02]

Zonal Flow Growth Rate

from [Diamond, Itoh, Itoh, Hahm, PPCF, ’05]
toroidal kinetic extension [Chen, Lin, White, PoP ‘00]

$$\Gamma^2 = \underbrace{\gamma_{\mathrm{mod}}^2}_{\text{from Reynolds Stress Drive}} - \underbrace{\Delta_{\mathrm{mm}}^2}_{\text{from Frequency Mismatch}} {}^{*}$$

where

$$\gamma_{\mathrm{mod}}^2 \cong 2k_y^2 q_x^2 |\phi_{\mathrm{DW0}}|^2$$

$$\Delta_{\mathrm{mm}}^2 \equiv \left\{\frac{1}{2}\underline{\left((\omega_0 - \omega_+) + (\omega_0 + \omega_-)\right)}\right\}^2 \cong k_y^2 q_x^4$$

Frequency mismatch between the primary drift frequency and
the zonal flow modulated sideband drift wave’s characteristic frequency

* Same structure for modulational generation of nonlinear breather
[Y. Kosuga et al., PoP ‘22]

Inclusion of Fast Ions in Quasi-neutrality

"Vorticity" in fluid picture

⇕

"Polarization Density in Modern Gyrokinetics"

$$\frac{\delta n_{f,\mathrm{pol}}}{n_{f0}} = -\left(1-\Gamma_0(b_f)\right)\frac{Z_f|e|\delta\phi}{T_f}$$

Fast Ion guiding-center density from linear GK: $\omega \ll k_\parallel v_{T_f}$,

J. Garcia, '18
S.M. Yang, '18
D.Kim, '23

- Long wavelength: $\frac{\langle\delta n_f\rangle}{n_{f0}} = -q_x^2\rho_s^2\frac{|e|\langle\delta\phi\rangle}{T_e}$, $\frac{\delta n_{f\mathrm{DW}}}{n_{f0}} \simeq -\frac{Z_f|e|\delta\phi}{T_f}$

  from guiding-center density

- Intermediate wavelength: $\frac{\langle\delta n_f\rangle}{n_{f0}} = -\left(1-\Gamma_0(b_f)\right)\frac{Z_f|e|\langle\delta\phi\rangle}{T_f}$, negligible compared to main ion contribution.

  $$\frac{\delta n_{f\mathrm{DW}}}{n_{f0}} \simeq -\frac{Z_f|e|\delta\phi}{T_f}$$

Electron Drift Wave in the presence of Fast Ions

- Linearizing the Hasegaw-Mima equation for drift waves yields,

$$\omega = \frac{(1-f_h)(L_{ne}/L_{ni})}{1+(1-f_h)k_\perp^2\rho_s^2}\,\omega_{*e}$$

Here, $\omega_{*e} \equiv \frac{k_y\rho_s c_s}{L_{ne}}$, $\qquad f_h \equiv \frac{Z_f n_{f0}}{n_{e0}}$

- Both frequency and dispersion affected by the dilution
- Central peaking of fast ion density ⇒ L_{ne}/L_{ni} ↘

Long Wavelength Regime

e.g. KSTAR FIRE mode : T_f/T_e ~ 10

$$\left(k_\perp^2\rho_s^2 \ll k_\perp^2\rho_{T_f}^2 \ll 1, \qquad q_x^2\rho_s^2 \ll q_x^2\rho_{T_f}^2 \ll 1\right)$$

- For zonal component,

$$\underline{\frac{\partial}{\partial t}\nabla_\perp^2\langle\phi\rangle} - \left\langle \boldsymbol{\nabla}\phi_{\mathrm{DW}} \times \mathbf{b}\cdot\boldsymbol{\nabla}\underline{(1-f_h)\nabla_\perp^2\phi_{\mathrm{DW}}}\right\rangle = 0$$

“zonal vorticity not affected by fast ion induced dilution”

- For drift waves ,

$$\frac{\partial}{\partial t}\{\phi_{\mathrm{DW}} - (1-f_h)\nabla_\perp^2\phi_{\mathrm{DW}}\}$$

:“DW vorticity reduced”

$$-\boldsymbol{\nabla}\langle\phi\rangle \times \mathbf{b}\cdot\boldsymbol{\nabla}\{\phi_{\mathrm{DW}} - (1-f_h)\nabla_\perp^2\phi_{\mathrm{DW}}\}$$

$$+\boldsymbol{\nabla}\phi_{\mathrm{DW}} \times \mathbf{b}\cdot\boldsymbol{\nabla}\nabla_\perp^2\langle\phi\rangle + (1-f_h)\frac{L_{ne}}{L_{ni}}\frac{\partial}{\partial y}\phi_{\mathrm{DW}} = 0$$

Here,

$$f_h \equiv \frac{Z_f n_{f0}}{n_{e0}}, \qquad \phi_{\mathrm{DW}} \equiv \frac{L_{ne}}{\rho_s}\frac{|e|\delta\phi}{T_e} \;\; for\;\; k_\parallel \neq 0, \qquad \langle\phi\rangle \equiv \frac{L_{ne}}{\rho_s}\frac{|e|\langle\delta\phi\rangle}{T_e}$$

Zonal Flow Growth Rate
(Long Wavelength Regime)

$$\Gamma^2 = \underbrace{\gamma_{\mathrm{mod}}^2}_{\text{from Reynolds Stress Drive}} - \underbrace{\Delta_{\mathrm{mm}}^2}_{\text{from Frequency Mismatch}}$$

where

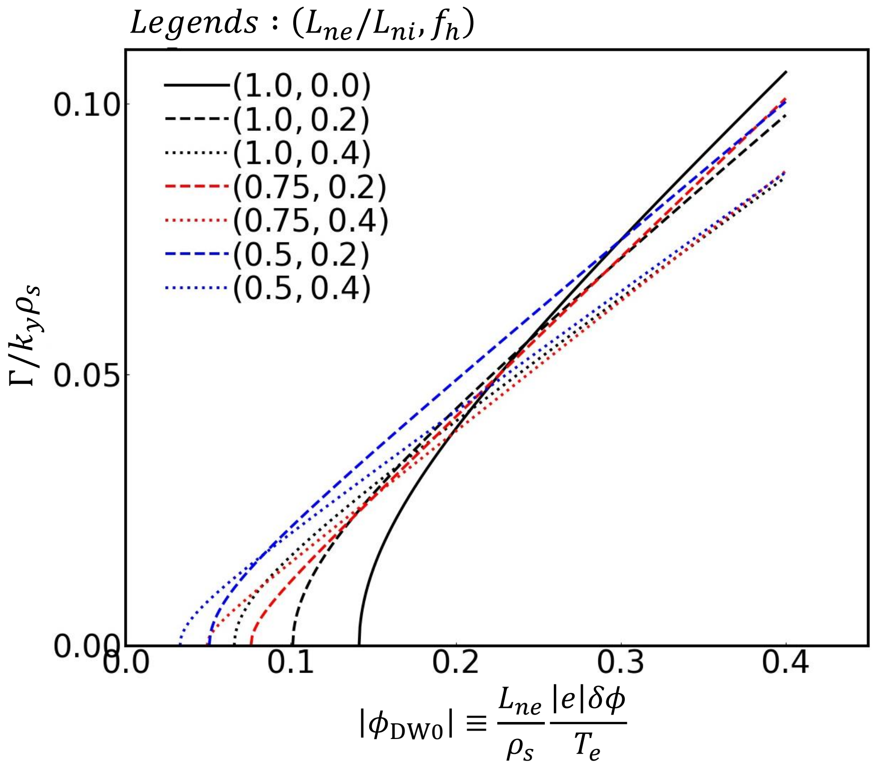
$$\gamma_{\mathrm{mod}}^2 \cong 2(1 - f_h) k_y^2 q_x^2 |\phi_{\mathrm{DW0}}|^2$$

$$\Delta_{\mathrm{mm}}^2 \equiv (1 - f_h)^4 \left(\frac{L_{ne}}{L_{ni}}\right)^2 k_y^2 q_x^2$$

- Reduction of only the DW vorticity → a factor $(1 - f_h)$ in γ_{mod}^2
- Fast ion-induced frequency downshift and a reduction of dispersion → a factor $(1 - f_h)^4 \left(\frac{L_{ne}}{L_{ni}}\right)^2$ in Δ_{mm}^2

Fast Ions make Zonal Flow Generation easier

- Presence of fast ions ($f_h \neq 0$) and reduction of the main ion density gradient $|L_{ne}/L_{ni}|$ lower the threshold for the zonal flow generation.



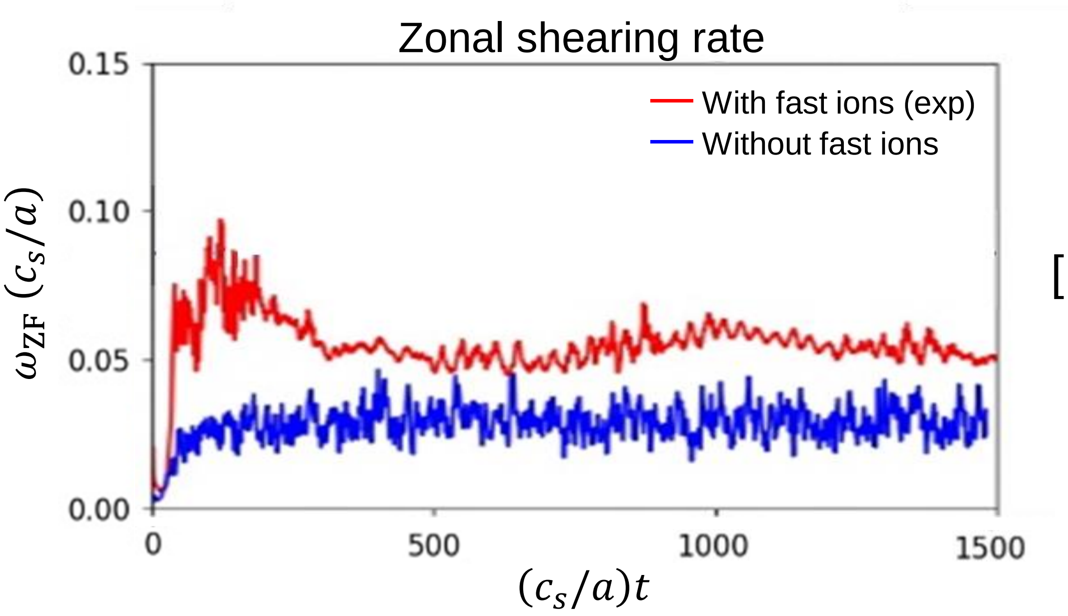
($k_\perp^2\rho_s^2 \ll q_x^2\rho_s^2 \ll 1$ limit)

Normalized Zonal flow growth rate $\Gamma/k_y\rho_s$
in the unit of $\mathrm{c_s}/L_{ne}$ for $q_x\rho_s = 0.2$

Gyrokinetic Simulations show feeble turbulence, but appreciable zonal flows

- Considerable zonal flow amplitude despite of very low microturbulence fluctuation level.

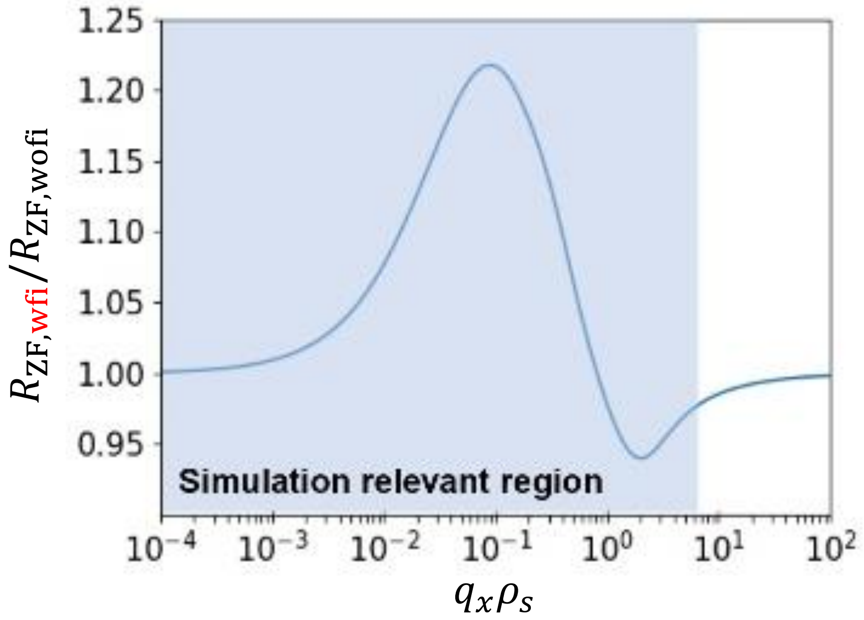
(CGYRO simulation for FIRE mode of KSTAR with fast ions $T_f/T_e \sim 10$)



[D. Kim , S.J. Park, C. Sung et al., ICPP, 2022]

Rosenbluth-Hinton Residual Zonal Flow Level is higher with Fast Ions

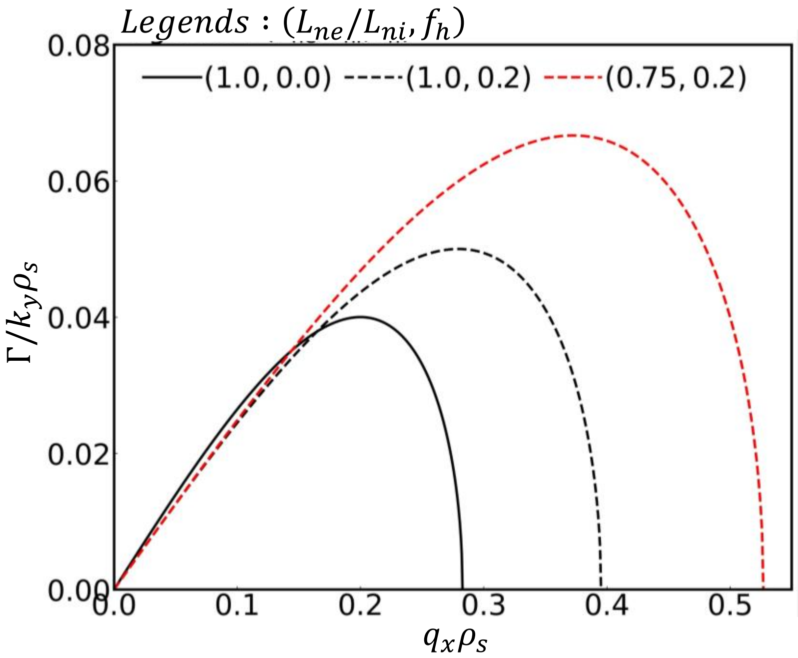
Changes in Rosenbluth-Hinton Zonal Flow Level



for KSTAR FIRE mode parameters
using an analytic formula from [Y.W. Cho and T.S. Hahm, NF, 59.066026 (2019)]

Zonal Flows get generated in broader range of q_x due to fast ions

- Zonal flow generation possible for a wider range of wavenumbers $q_x\rho_s$ due to the threshold behavior with respect to the drift wave amplitude.



Normalized Zonal flow growth rate $\Gamma/k_y\rho_s$ in the unit of c_s/L_{ne} for $\frac{|e|\delta\phi}{T_e} = 0.2\rho_s/L_{ne}$

c.f. ZF growth rate from Wave-Kinetic approach considering CTEM

[M.S. Hussain, W. Guo, and L. Wang NF '22]

related to the linear growth rate (not the drift wave amplitude)

covering $\gamma_{\mathrm{lin}} \gg \Gamma$

Intermediate Wavelength Regime

$$\left(k_\perp^2\rho_s^2 \ll 1 \ll k_\perp^2\rho_{Tf}^2, \qquad q_x^2\rho_s^2 \ll 1 \ll q_x^2\rho_{Tf}^2\right)$$

With fusion product α-particles in ITER : $\frac{T_f}{T_e} \sim 10^2$

Recent JET experiments with ICRH : $\frac{T_f}{T_e} \sim 30$

- For zonal component, $$\frac{\langle \delta n_f \rangle}{n_{f0}} = -\left(1 - \Gamma_0(b_f)\right)\frac{Z_f|e|\langle\delta\phi\rangle}{T_f},$$
- For drift waves, $$\frac{\delta n_{f\mathrm{DW}}}{n_{f0}} \simeq -\frac{Z_f|e|\delta\phi}{T_f}$$

⇒ Both insignificant compared to main ion contribution

Intermediate Wavelength Regime - 2

$$\left(k_\perp^2\rho_s^2 \ll 1 \ll k_\perp^2\rho_{T_f}^2, \qquad q_x^2\rho_s^2 \ll 1 \ll q_x^2\rho_{T_f}^2\right)$$

- For zonal component,

$$(1-f_h)\frac{\partial}{\partial t}\nabla_\perp^2\langle\phi\rangle - \left\langle \boldsymbol{\nabla}\phi_{\mathrm{DW}} \times \mathbf{b}\cdot\boldsymbol{\nabla}(1-f_h)\nabla_\perp^2\phi_{\mathrm{DW}}\right\rangle = 0$$

- For drift waves,

"Both Zonal and DW's vorticity reduced"

$$\frac{\partial}{\partial t}\{\phi_{\mathrm{DW}} - (1-f_h)\nabla_\perp^2\phi_{\mathrm{DW}}\}$$

$$-\boldsymbol{\nabla}\langle\phi\rangle \times \mathbf{b}\cdot\boldsymbol{\nabla}\{\phi_{\mathrm{DW}} - (1-f_h)\nabla_\perp^2\phi_{\mathrm{DW}}\}$$

$$+(1-f_h)\boldsymbol{\nabla}\phi_{\mathrm{DW}} \times \mathbf{b}\cdot\boldsymbol{\nabla}\nabla_\perp^2\langle\phi\rangle + (1-f_h)\frac{L_{ne}}{L_{ni}}\frac{\partial}{\partial y}\phi_{\mathrm{DW}} = 0$$

Here,

$$f_h \equiv \frac{Z_f n_{f0}}{n_{e0}}, \qquad \phi_{\mathrm{DW}} \equiv \frac{L_{ne}}{\rho_s}\frac{|e|\delta\phi}{T_e} \;\; for \;\; k_\parallel \neq 0, \qquad \langle\phi\rangle \equiv \frac{L_{ne}}{\rho_s}\frac{|e|\langle\delta\phi\rangle}{T_e}$$

Zonal Flow Growth Rate
(Intermediate Wavelength Regime)

$$\Gamma^2 = \underbrace{\gamma_{\mathrm{mod}}^2}_{\text{from Reynolds Stress Drive}} - \underbrace{\Delta_{\mathrm{mm}}^2}_{\text{from Frequency Mismatch}}$$

where

$$\gamma_{\mathrm{mod}}^2 \cong 2k_y^2 q_x^2 |\phi_{\mathrm{DW0}}|^2$$

$$\Delta_{\mathrm{mm}}^2 \equiv (1 - f_h)^4 \left(\frac{L_{ne}}{L_{ni}}\right)^2 k_y^2 q_x^2$$

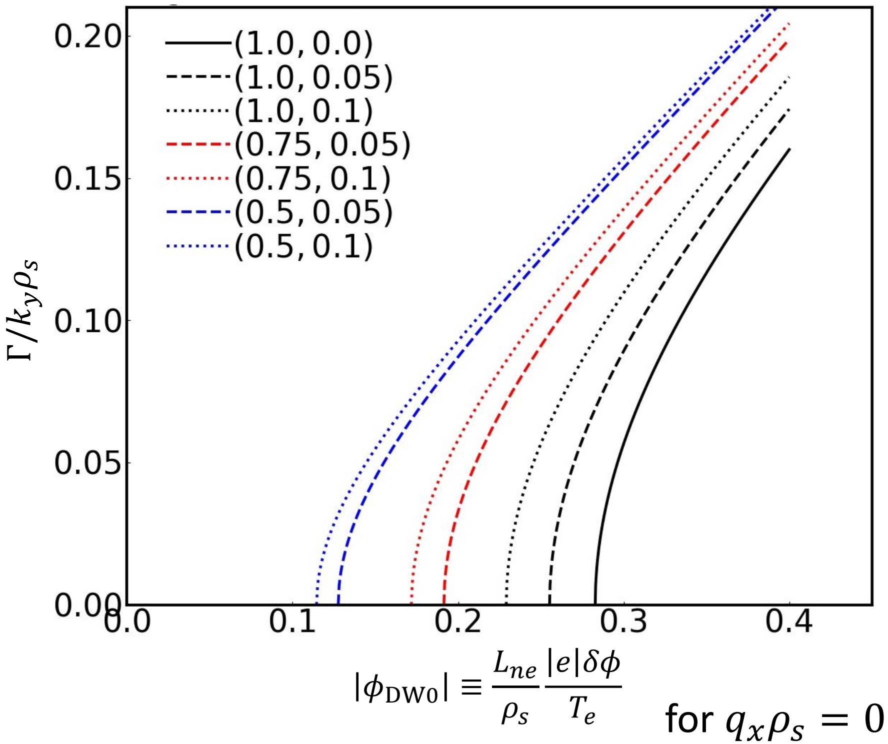
- Both zonal and DW vorticity are reduced → γ_{mod}^2 does not change
- Fast ion-induced frequency downshift and a reduction of dispersion

→ a factor $(1 - f_h)^4 \left(\frac{L_{ne}}{L_{ni}}\right)^2$ in Δ_{mm}^2

Zonal Flow Growth for weak turbulence amplitude

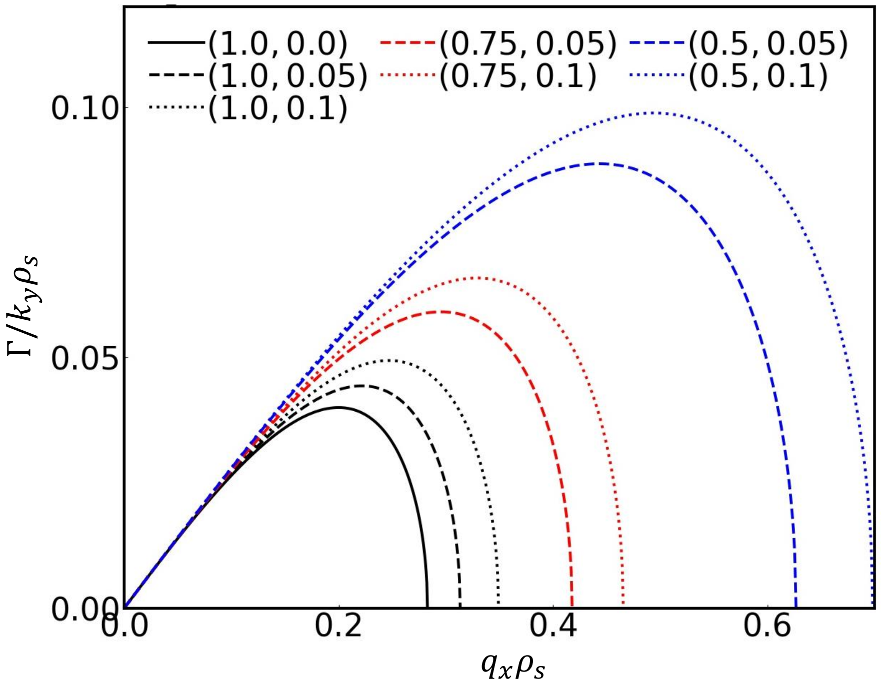
- Presence of fast ions ($f_h \neq 0$) and reduction of the main ion density gradient $|L_{ne}/L_{ni}|$ lower the threshold for the zonal flow generation and make possible for a wider range of wavenumbers $q_x\rho_s$

$Legends : (L_{ne}/L_{ni}, f_h)$



for $q_x\rho_s = 0.4$

$Legends : (L_{ne}/L_{ni}, f_h)$



for $\frac{|e|\delta\phi}{T_e} = \frac{0.2\rho_s}{L_{ne}}$

Its dependence is stronger compared to the long wavelength case

Conclusions (on our work)

- Nonlinear mode coupling analyses based on modified Hasegawa-Mima equation with **fast ions' density response** show its favorable impact on zonal flow generation.

- **In long wavelength regime**

  - Threshold amplitude for zonal flow modulational instability is reduced due to the reduced frequency mismatch despite Reynolds stress drive for zonal flow is reduced .

- **In intermediate wavelength regime**

  - Threshold amplitude for zonal flow modulational instability is reduced due to the reduced frequency mismatch. (stronger than a long wavelength case)

- Our finding could contribute to a more in-depth understanding behind numerous recent results on tokamak plasma confinement enhancement caused by the fast ions.

Possible Applications and Related Results from Others

- **Long wavelength regime**

  ~ FIRE mode of KSTAR with 100 keV range of fast ion temperature ($T_f/T_e \sim 10$).

  [H. Han et al, Nature, '22]

- **Intermediate wavelength regime**

  ~ ITER parameters with fusion product α-particles ($T_\alpha/T_e \sim 10^2$)

  [F.M. Poli et al., NF, '12] [S.H. Kim et al., NF '16]

  ~ experiments of JET with 1 Mev range fast ions ($T_\mathrm{f}/T_e \sim 30$).

  [S. Mazzi et al., Nat. Phys. '22]

- **Fast ion driven Alfvenic instabilities**' role in generating zonal flow deserves further theoretical research as highlighted in S. Mazzi's paper.

[L. Chen and F. Zonca, PRL, '12]
[Z. Qiu, L. Chen and F. Zonca, PoP, '16]
[A. Ishizawa, K. Imadera et al., NF, '21]

Extensions and Future Work

- **Desirable extensions to toroidal geometry**
  - Neoclassically enhanced polarization
  - Toroidal mode structure
  - Kinetic effects

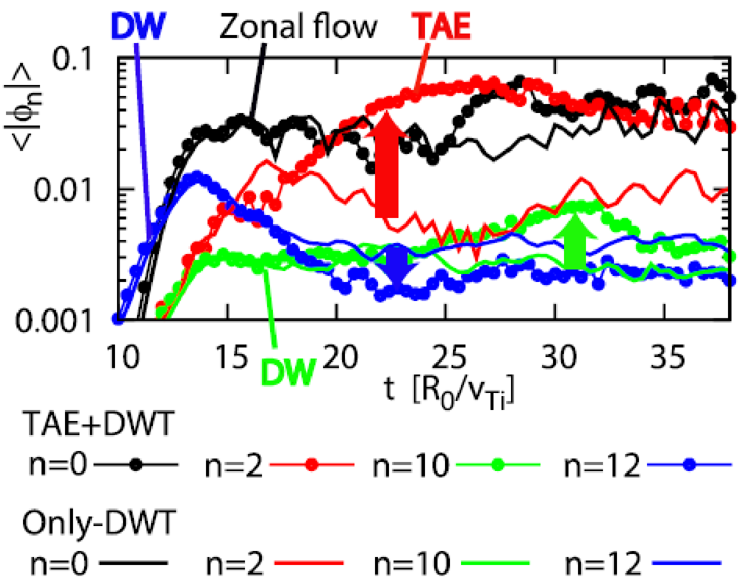
[L. Chen, Z. Lin, and R. White, PoP, '00]
[L. Wang and T.S. Hahm, PoP, '09]
[Y.W. Cho and T.S. Hahm, NF, '17]

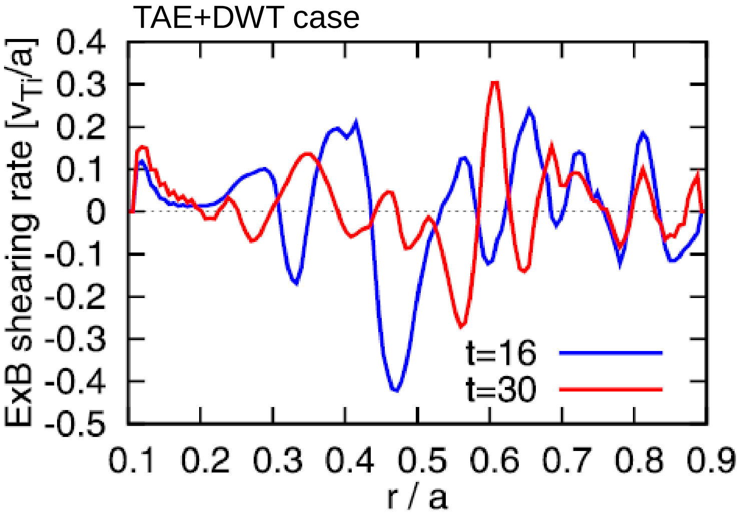
- **Validations against KSTAR results on FIRE mode**
  - CGYRO flux-tube nonlinear simulations are consistent with ITB formation, but too optimistic [H. Han et al., Nature '22]
  - Observation of low-n magnetic fluctuations from Mirnov coils and CGYRO simulations

→ "Global gyrokinetic simulations needed for proper validations."

[GTC simulation initiated;
S.J.Park, G.J.Choi et al.]

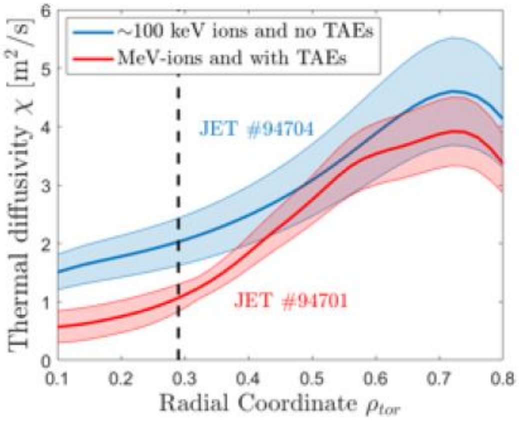
Zonal Flow grows further in the presence of TAE.

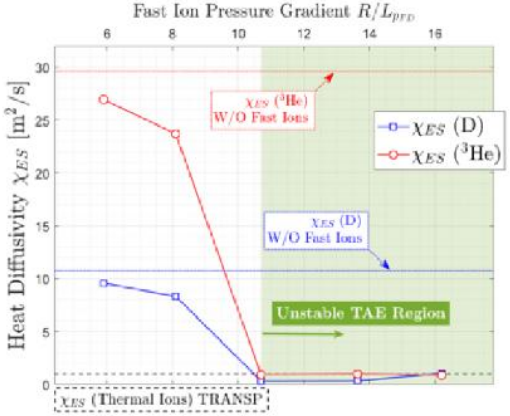


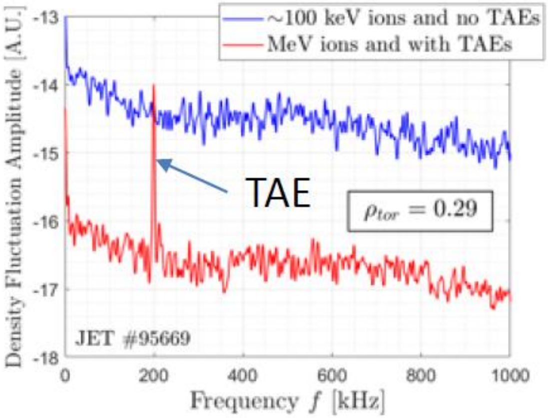


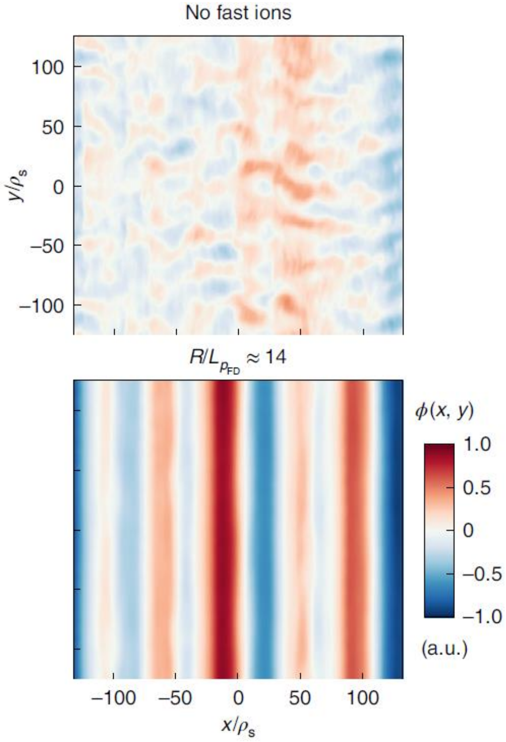
Is it due to EP or due to TAE ?

Fast ion induced Turbulence Suppression in JET









- Calculation of turbulent heat flux by scanning FI pressure gradient at rho=0.3 in GENE. (Otherwise same input from interpretive TRANSP in #9569.)

- Presence of ~MeV ions from ICRH

- At the FI pressure gradient above a certain threshold,
  - Reduction of turbulent heat flux
  - TAE observed
  - Zonal flow produced by TAE significant

[S. Mazzi *et al.*, Nat. Phys. (2022)]

Courtesy: H.T. Kim

Zonal Flow Generation via AEs

[L. Chen and F. Zonca, PRL, '12]

- Zonal structure spontaneous excitation is more easily induced by TAEs including proper trapped-ion responses.

$$|\delta B_r/B_0|^2_{thres} \sim \rho_i^2/4\epsilon_0(qR_0)^2$$

threshold amplitude
for modulational instability

Here, $\epsilon_0 = 2r_0/R_0 + \Delta'$ (Δ' : Shafranov shift)

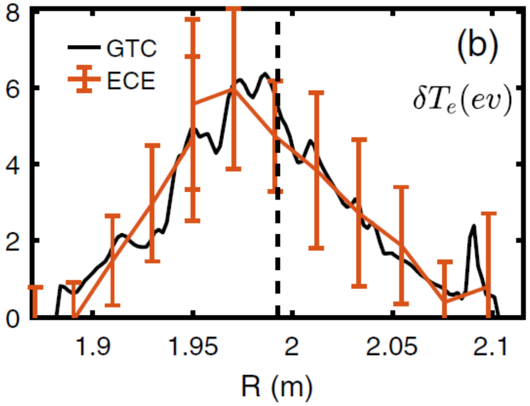
[Z. Qiu, L. Chen and F. Zonca, PoP, '16]

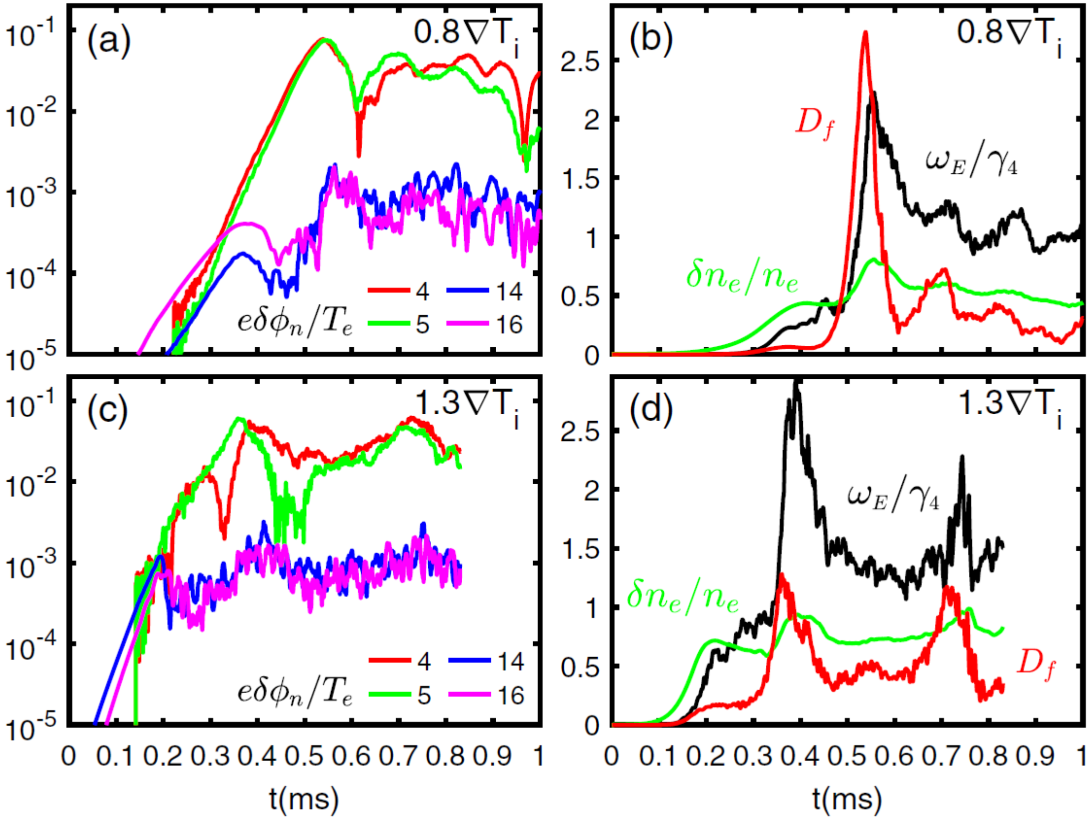
- Forced generation of Zonal Flow can happen during linear growth phase of TAE through a mechanism related to resonant EP nonlinearity.

$$\delta\phi_Z = i\frac{\pi}{8}\frac{k_\perp^2}{k_Z}\frac{\widehat{K}\widehat{G}}{\gamma_L\hat{\chi}_{iZ}}\left|\hat{A}_0\right|^2\sum_m|\Phi_0|^2 e^{2\gamma_L t}$$

forced driven zonal flow
(threshold-less)

AE-driven fast ion transport regulated by turbulence



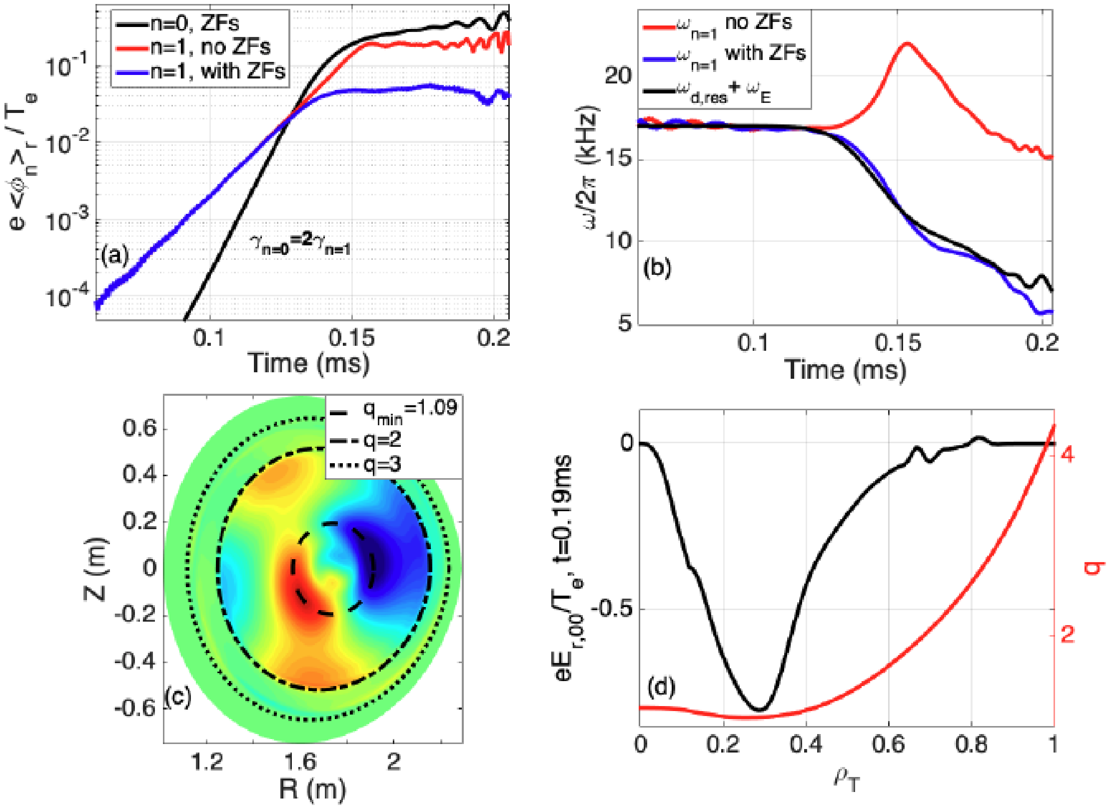


[P. Liu *et al.*, PRL (2022)]

- Profile of δT_e from GTC global nonlinear GK simulations of RSAE with ITG in DIII-D #159243.
- Single burst of fast ion flux by unstable RSAE dominant with no/weak turbulence.
- Strong turbulence makes qualitative change of fast ion transport to a level of experimental relevance: from single large burst to quasi-periodic smaller bursts (“quasi-steady state”).

Fast Ion driven Fishbone Produces Zonal Flows

- Fishbone-induced zonal flows saturating fishbone are found in GTC simulations of DIII-D experiment parameters.



(a) the volume-averaged perturbed electrostatic potential $e\phi/T_e$ (n=0,1)

(b) the n=1 mode frequency $\omega_{n=1}$ and the linearly resonant processional frequency $\omega_{d,res}$ plus the zonal ExB frequency ω_E at $q_{\min}$

(c) $e\phi/T_e$ in the poloidal plane

(d) zonal electric field $eE_{r,00}/T_e$ after saturation at t=0.19 ms.

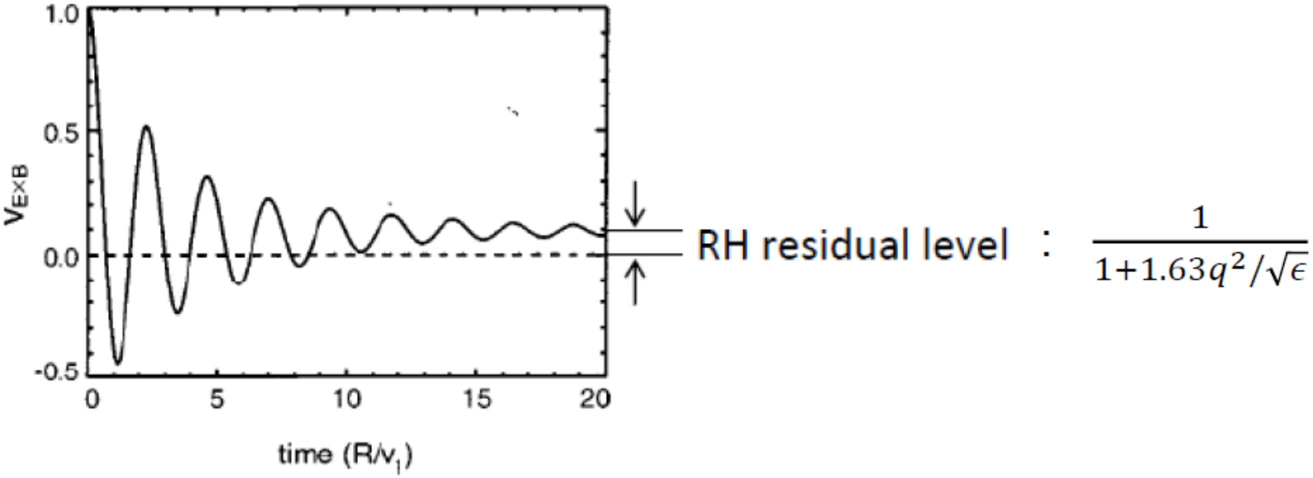
[G. Brochard et al., PRL submitted]

Backup

Residual Zonal Flows in Toroidal Geometry

- Based on gyro-Landau-fluid closure (up to mid 90's), ZF is completely damped even in collisionless plasmas.

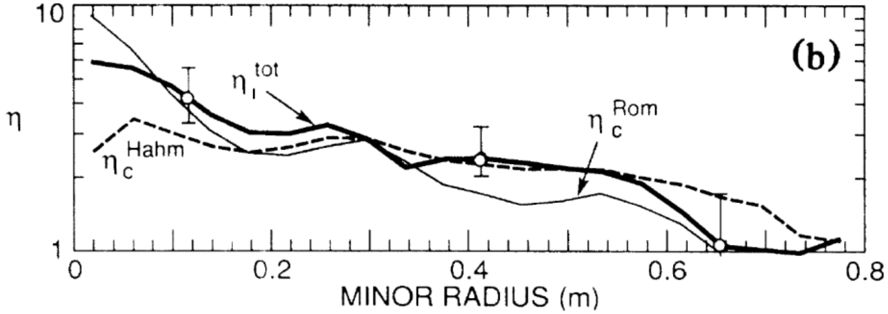
- Rosenbluth-Hinton [PRL '98] ZF undamped from Gyrokinetic theory

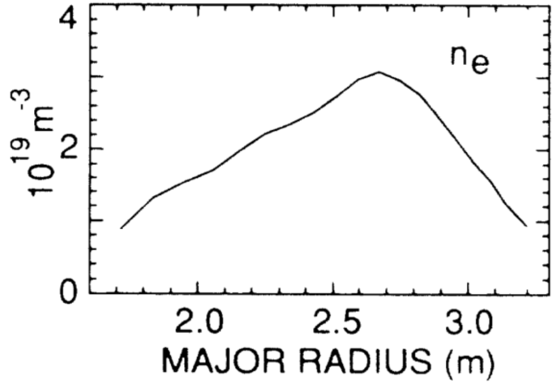


RH residual level : $\frac{1}{1+1.63q^2/\sqrt{\epsilon}}$

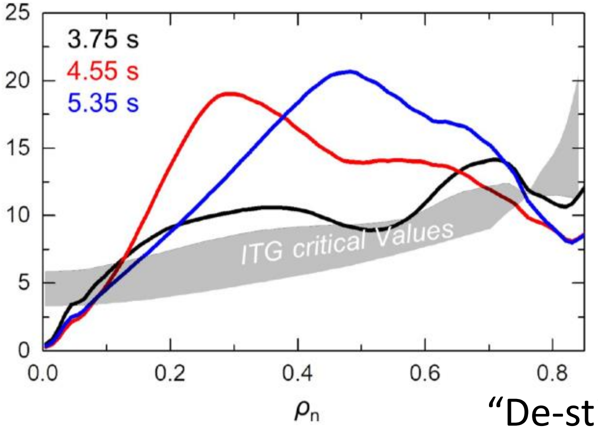
- Gyrokinetic codes are now benchmarked against the analytic results!

Hot Ion Mode vs FIRE Mode

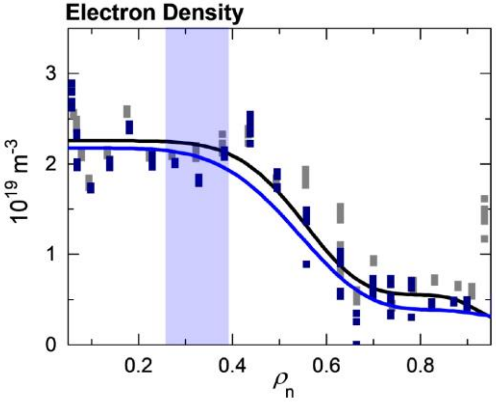




“Enhancement of $R/L_{T,\text{crit}}$ with T_i/T_e”



“De-stiffening with Fast Ions”



[S. Scott *et al.*, PRL (1990)] TFTR

[H. Han, *et al.*, Nature (2022)] KSTAR

- Overall peaking in $T_i(r)$

- ITB in $T_i(r)$

Supershot/Hot Ion Mode vs FIRE Mode

Similarity

- **NBI heating low density (1)**

< FIRE >	< Superprshot >
P_{NBI} ~ 3 - 4 MW	P_{NBI} ~ 20 – 30 MW
$\overline{n_e}$ ~ 0.5 - 1.5 x 10^{19} m^{-3}	$\overline{n_e}$ ~ 1 - 4 x 10^{19} m^{-3}

- **High core T_i (1)**

< FIRE >	< Superprshot >
$T_{i,0}$ ~ 10 keV	$T_{i,0}$ ~ 20 - 30 keV

- **No ELMs (1)**

- **High fraction of fast ions $n_{fast}/n_e \lesssim 50\%$ (2)**

Difference

- **Stationarity**

< FIRE >	< Superprshot >
$\tau_{\text{pulse}} \sim 10\ \text{s}$	$\tau_{\text{pulse}} \sim 1\ \text{s}$

- **Impurity issues (3)**

< FIRE >	< Superprshot >
No sign of impurities	degradation due to carbon influx

- **Transport property (2)**

< FIRE >	< Superprshot >
$\frac{R}{L_{Ti}} > \left(\frac{R}{L_{Ti}}\right)_{\text{cri}}$	$\frac{R}{L_{Ti}} \sim \left(\frac{R}{L_{Ti}}\right)_{\text{cri}}$

- **ExB shearing model effectively works in Supershots (4)**

---

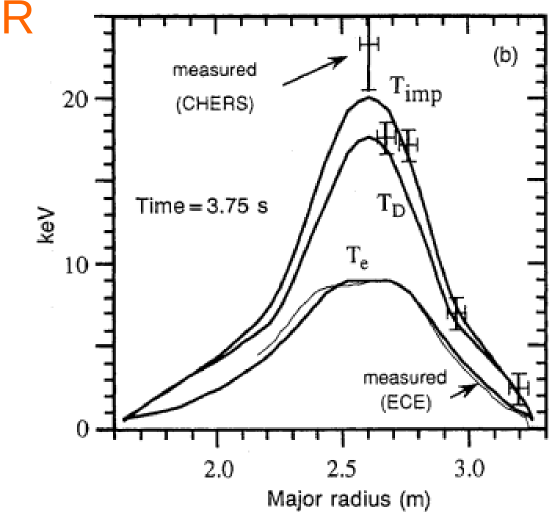
1) J.D Strachan, PRL, ’87
2) S.D. Scott, PRL, ‘89
3) A.T. Ramsey, ‘91
4) D.R. Ernst, PLR, ‘98

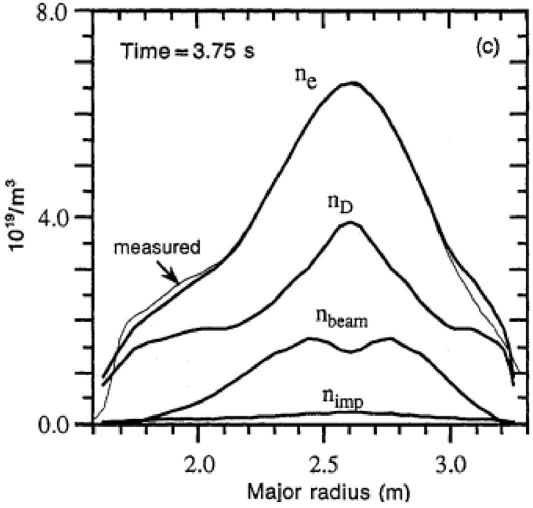
Courtesy: Y.-S. Na

High Fraction of fast ions in Supershot/FIRE

[R.V. Budny NF ’94]

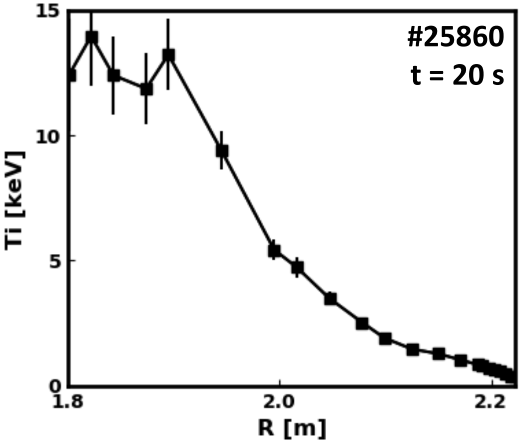
TFTR



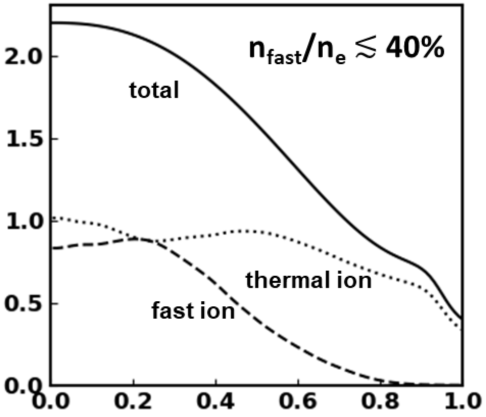


KSTAR

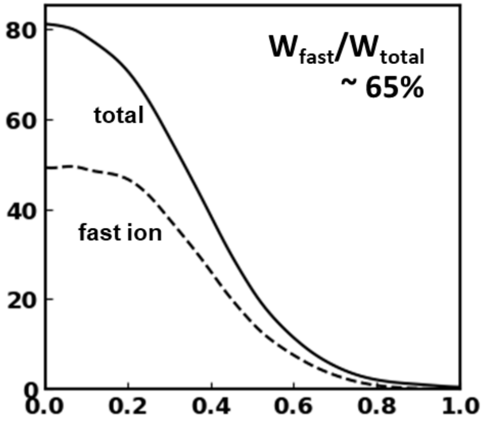
< Typical Ion Temperature Profile >



< Density profile in FIRE mode >



< Pressure profile in FIRE mode >



different density peaking

Courtesy: Y.-S. Na

Further Research Progress on Residual Zonal Flows

All considered Maxwellian F_0.

(exception :
Z.X. Lu et al., PPCF (2019)
Y.W. Cho and T.S. Hahm, NF (2019))

- Rosenbluth and Hinton dealt with long wavelength zonal flows only with $k_r \rho_{\theta i} \ll 1$
- Extensions to shorter wavelength regime
  F. Jenko et al., PoP (2000), E.J. Kim, P.H. Diamond et al. PRL (2003)
  Y.Xiao et al. PoP (2006), O. Yamagishi et al., PPCF (2018)
- Stellarators : H. Sugama and T. Watanabe, PRL(2005), PoP (2006)
  P. Monreal et al. PPCF (2016)
- **Modern gyrokinetic/bouncekinetic approach for all wavelength regime**
  - L. Wang and T.S. Hahm, PoP (2009)
- **More accurate procedure outlined in**
  - F.X. Duthoit, A. Brizard and T.S. Hahm PoP (2014)

- Applications to
  - Isotopic dependence of confinement
  T.S. Hahm et al., NF (2013)
  - Impurity Effects
  W.X. Guo, L. Wang et al., NF (2017)
  - Effects of RMP on H-mode transition
  G.J. Choi and T.S. Hahm, NF (2018)

α-particle Effects on Confinement of Burning Plasmas

- α-particle effects on Alfvenic energetic particle modes :
  L. Chen and F. Zonca, RMP (2016), Y. Todo, RMPP (2018)

- α-particle transport due to micro turbulence
  W. Zhang, Z. Lin and L. Chen et al., PRL (2008)
  C. Angioni and A. Peeters, PoP (2008)
  S. Yang, C. Angioni, T.S. Hahm et al., PoP (2018)

- α-particle effects on mean plasma rotation :
  (e.g. α-particle's large orbit loss) found to be insignificant for ITER:
  M.N. Rosenbluth and F.L. Hinton, NF (1996)

※ No previous works on α-particle effects on Residual zonal flows

(c.f. preliminary attempt : K.P. Lee and T.S. Hahm, Proceeding of M&C 2017)

Highlights of Results with Practical Interests

- α-particles enhance residual zonal flows, effect is maximum at $k_r\rho_{i,\mathrm{eff}} \sim 10^{-1}$

  (for D 50%, T 50% mixture, $\rho_{i,\mathrm{eff}} = \sum_a cm_a v_{Ta}/Z_a|e|B$).
- For 10% concentration, ~ 10% enhancement at $k_r\rho_{i,\mathrm{eff}} \sim 10^{-1}$ is expected.
- Effects can be considerable for ITER, and significant for DEMO and reactors.

- $R_{ZF}\left(k_r\rho_{i,\mathrm{eff}}\right) = \frac{\chi_{cl}}{\chi_{Neo}+\chi_{cl}}$

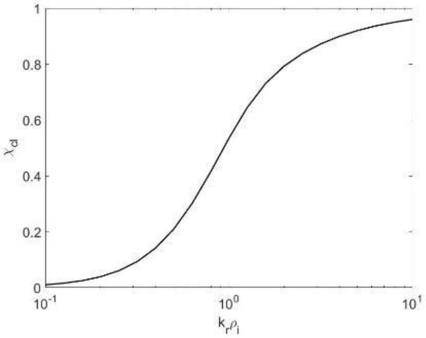
  i) χ_{cl} is a monotonically increasing in k_r (~ $tanh$-like shape)

  Transition occurs at lower k_r in the presence of energetic α's

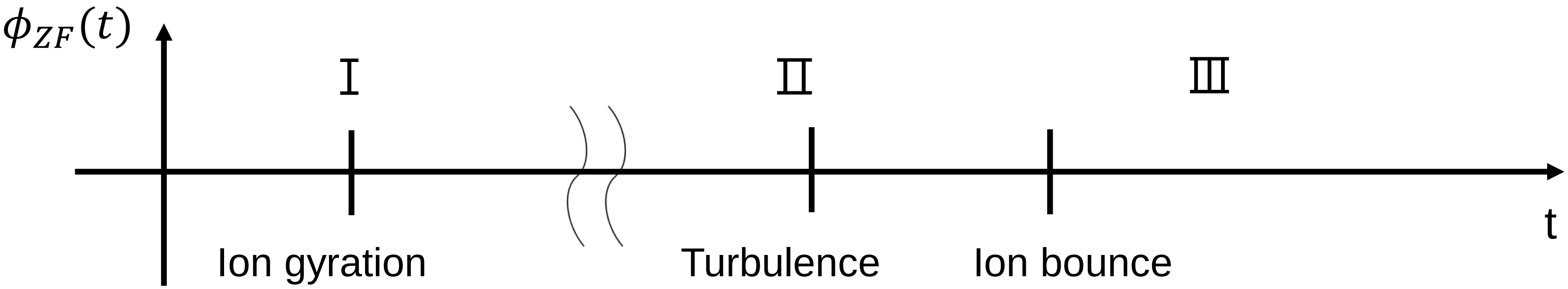
  ($k_r\rho_{i,\mathrm{eff}} \sim 10^{-1}$, $k_r\overline{\rho_\alpha} \sim 1$)

  ii) χ_{nc} peaks at similar k_r value and decreases as a function of k_r for higher k_r.

  i, ii) ⇒ R_{ZF} is enhanced for $k_r\rho_{i,\mathrm{eff}} \sim 10^{-1}$



Disparate Temporal Scales in Residual Zonal Flow Problem



Ion gyration period

Turbulence time scale

Ion bounce time scale

Ⅰ. Quasi-neutrality : $0 = n_e(\vec{x}) - n_i(\vec{x})$

Ⅱ. Polarization Shielding (Finite Larmor Radius effect) : from Gyrokinetics,

$t \gg \Omega_{ci}^{-1}$
$(\omega \ll \Omega_{ci})$

$$\chi_{\mathrm{cl}} \frac{e\Phi_{ZF}(0)}{T_i} = \frac{n_e(\vec{x}) - n_{i,gc}(\vec{x})}{n_0}$$

Ⅲ. Neoclassical enhancement of polarization shielding (Finite Banana Orbit Width effect) :

from Bouncekinetics,

$t \gg \omega_{\mathrm{b}i}^{-1}$
$(\omega \ll \omega_{\mathrm{b}i})$

$$(\chi_{\mathrm{Neo}} + \chi_{\mathrm{cl}}) \frac{e\Phi_{ZF}(\infty)}{T_i} = \frac{n_e(\vec{x}) - n_{i,\mathrm{bc}}(\vec{x})}{n_0}$$

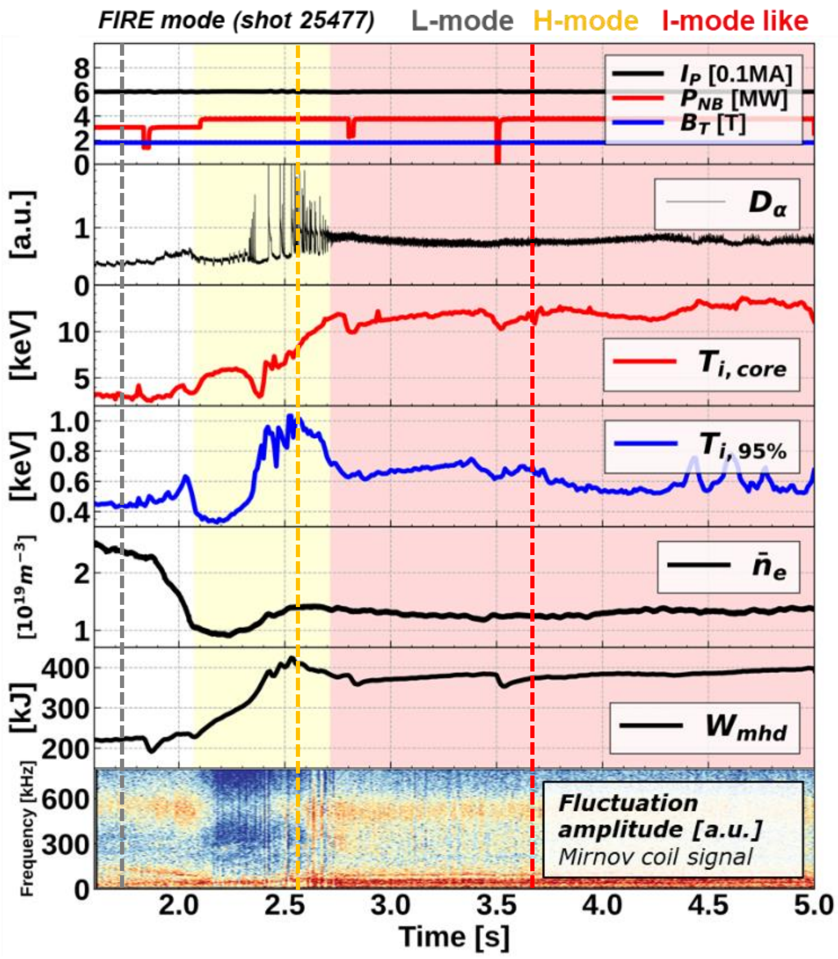
Neoclassical Polarization Density from Bounce-kinetic Approach [Fong & Hahm, PoP (1999)]

- Generalized Polarization Shielding :

$$\chi_{\mathrm{total}} = \chi_{\mathrm{Neo}} + \chi_{\mathrm{cl}} \text{ (Polarizability; Susceptibility)}$$

FIRE mode with I-mode like Edge

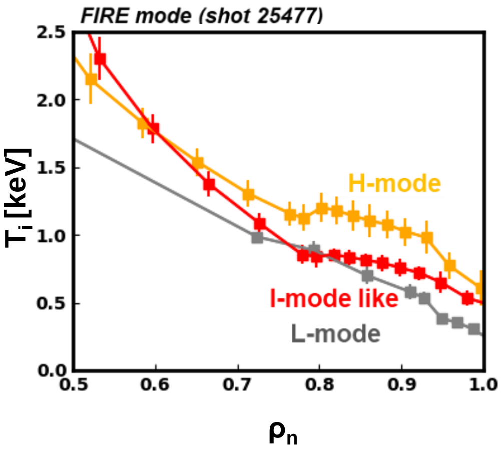
<Time Evolution of 0d Parameters >



- Some of FIRE modes have ETB is formed only in the energy channel not in the particle channel like **I-mode.**

- They shows a high ion temperature gradient at the edge region and **no clear barrier in the density profile.**

<Ion Temperature profile>

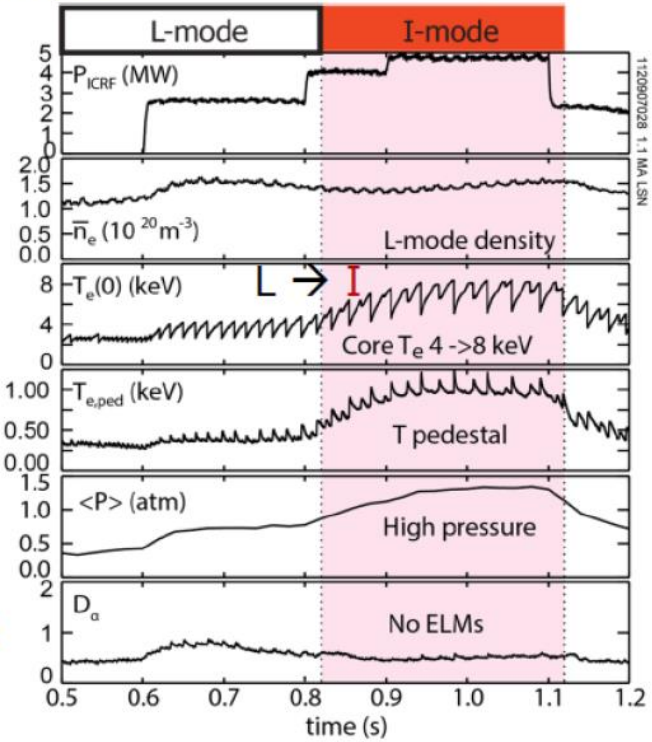


The absence of the particle transport barrier can enhance the fraction of fast ions by reducing thermalization of fast ions with a low density.

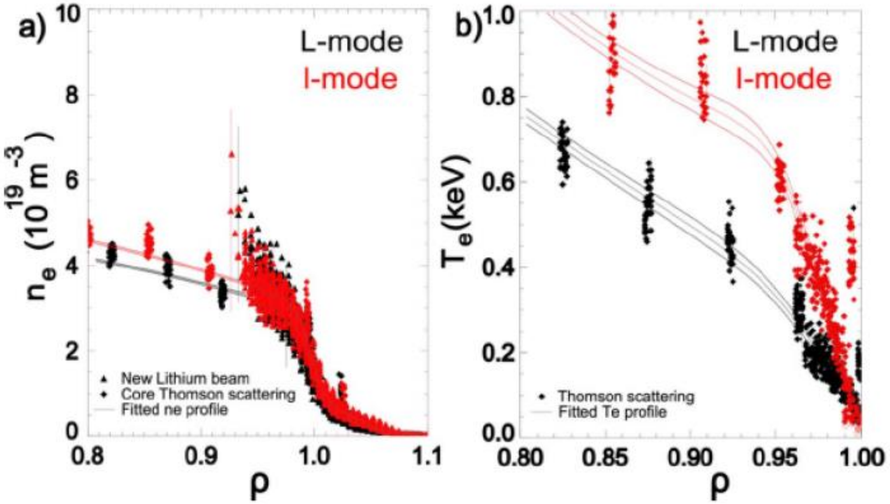
Courtesy: Y.-S. Na

I-mode is a stationary, high energy confinement regime, without particle barrier

Defining feature of **I-mode*** is a temperature pedestal, *without* a density pedestal.



C-Mod
Hubbard
IAEA 2012



ASDEX Upgrade (AUG)
Manz Nucl. Fusion 2015

Also, [McDermott *et al*., PoP (2009)]

** Obligatory clarification: This is NOT the same as the Limit Cycle Oscillation phase between L and H-mode, sometimes known as "I-phase"*