

# Role of isotopes in microturbulence from linear to saturated Ohmic confinement regimes

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Presented at the Theory Seminar PPPL, Mar. 7, 2024

# Outlines

- I. Introduction: isotopic dependence of energy confinement
- II. Quantitative agreement between gyrokinetic simulations and experiments
- III. Physical mechanisms behind the isotopic dependence of confinement
- IV. Discussions and conclusions

# Introduction

- Isotopes are pivotal for fusion energy: fusion plants, ITER
- **Experiments:** empirical scaling of the isotopic dependence of energy confinement

$$\tau_E \propto M_i^\sigma$$

**$\sigma$  ranges roughly between 0.2 to 0.5**

(*M. Bessenrodt-Weberpals et al. NF 1993; F. Wagner and U. Stroth PPCF 1993; ITER Phys. Expert Group on Confinement and transport, NF 1999; J. G. Cordey et al. NF 1999; H. Urano et al. PRL 2012; P. A. Schneider et al. PPCF 2021; K. Ida, Rev. RMPP 2023 .....*)

- L-mode confinement scaling ITER89-P (*P. N. Yushmanov et al., Nucl. Fusion 30, 1999 (1990)*)

$$\sigma = 0.5$$

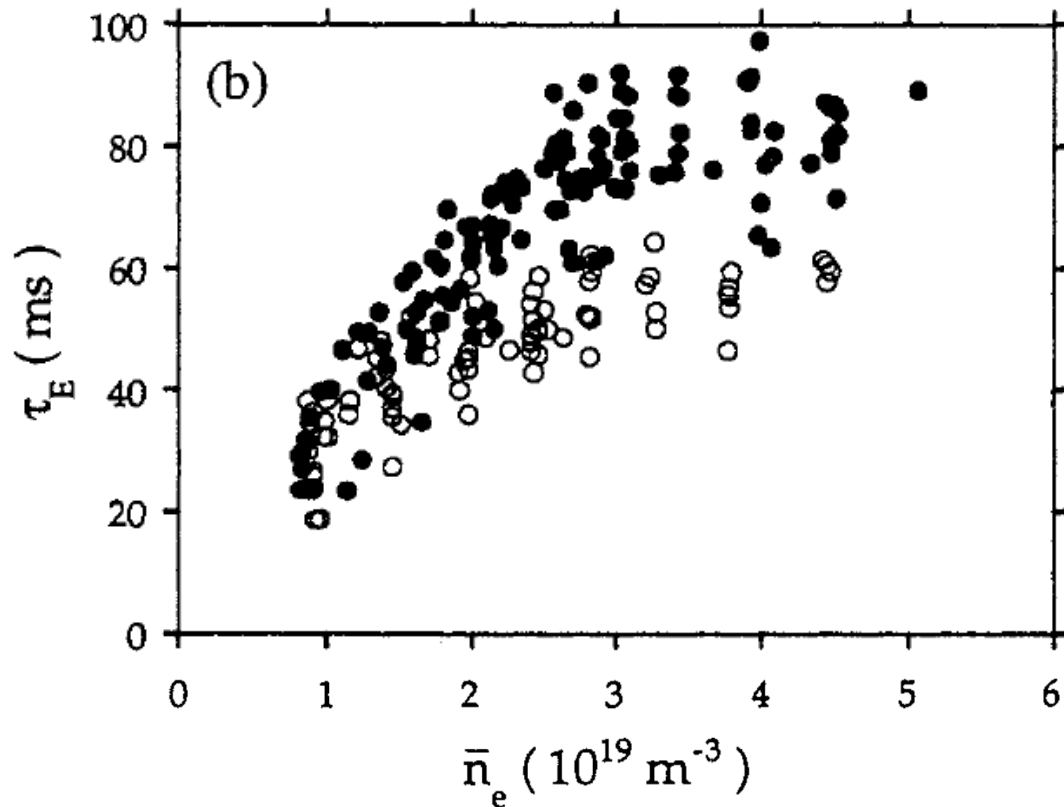
- H-mode confinement scaling ITER-IPB(y) (*ITER Phys. Expert Group on Confinement and transport, Nucl. Fusion 39, 2175 1999*)

$$\sigma = 0.2$$



# Introduction

➤ From linear Ohmic confinement (LOC) to saturated Ohmic confinement (SOC) regimes.



**ASDEX** (open circle: hydrogen; solid circle: deuterium) (Figure from Ref: M. Bessenrodt-weberpals et al., Nucl. Fusion 33, 1205 (1993))

Low density, LOC regime:

$$\sigma = 0.31$$

High density, SOC regime:

$$\sigma = 0.5$$

Similar results observed in

**FT-2** (*D. V. Kouprienko et al., NF 2022*)

**TCV** (*K. Tanaka, IAEA-FEC2023, London*)

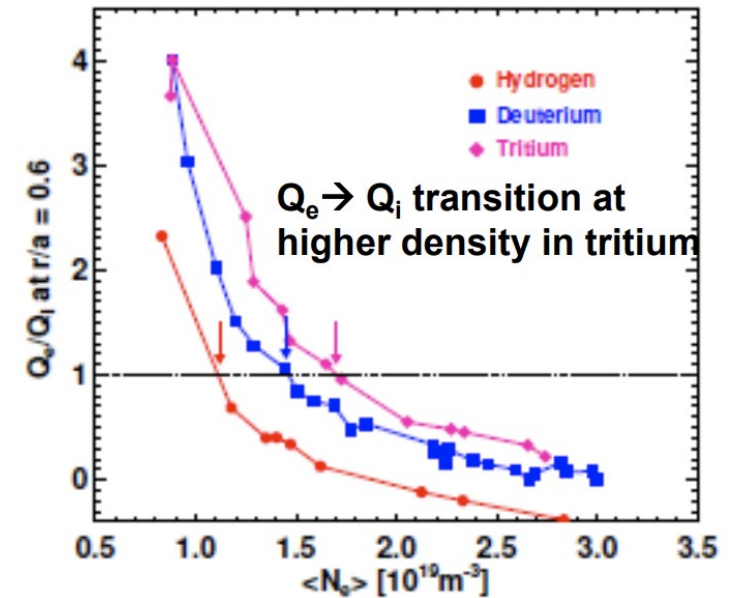
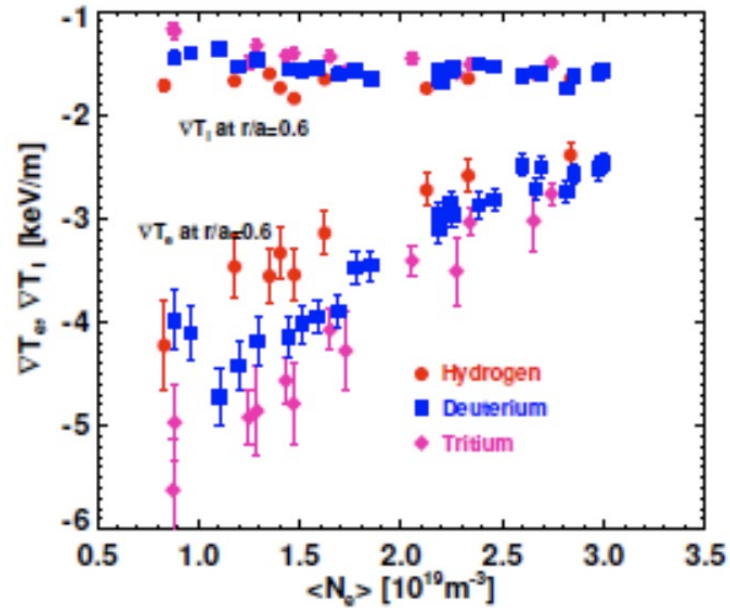
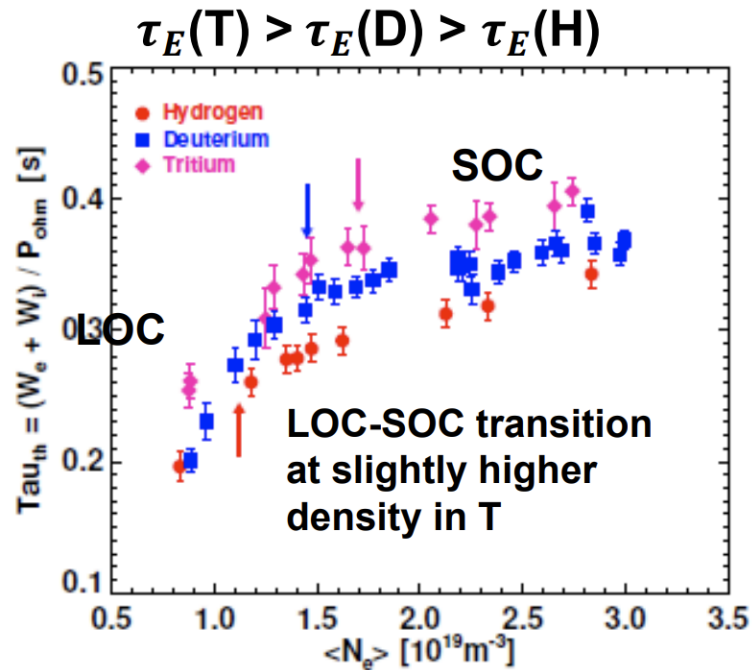
**JET-ILW** (*Delabie E. et al., IAEA-FEC2023, London*)

...

...

# Introduction

Similar results observed in  
**JET-ILW** (*Delabie E. et al., IAEA-FEC2023, London*)



# Introduction

➤ **Theoretical** understanding is **challenging**.

- **Mixing length estimation** with the linear growth rate  $\gamma$  and gyro-Bohm wave number scaling shows:

$$\chi = \gamma/k_{\perp}^2 \propto M_i^{0.5}$$

- **Gyro-Bohm diffusivity** :

$$\chi_{GB} = \rho_i^2 V_{Ti}/a \propto M_i^{0.5}$$

Both predict  $\tau_E \propto 1/\chi \propto M_i^{-0.5}$  **contradicting** to the experimental observations.

➤ Significant progress investigating isotope effects have been achieved.

(*I. Pusztai et al. PoP 2011, T. S. Hahm et al. NF 2013, Y. Xu et al. PRL 2013, A. Bustos et al. PoP 2015, M. Nakata et al. PRL 2017, J. Garcia et al. NF2017, Y. Idomura PoP 2019, E. A. Belli et al., PRL 2020 etc. ...* ...)

➤ However, understanding of the isotope effects is still insufficient. **Pursuit of a quantitative agreement with experiments is extremely challenging.**

# Quantitative agreement between GK simulations and experiments

➤ Simulation program: gKPSP,  $\delta f$ , global, gyrokinetic ion and bounce-averaged kinetic trapped electron, particle-in-cell code. (L. Qi et al., PoP 2016; J. M. Kwon et al., CPC 2017)

- Typical Ohmic and L-mode plasmas (based on Cyclone)

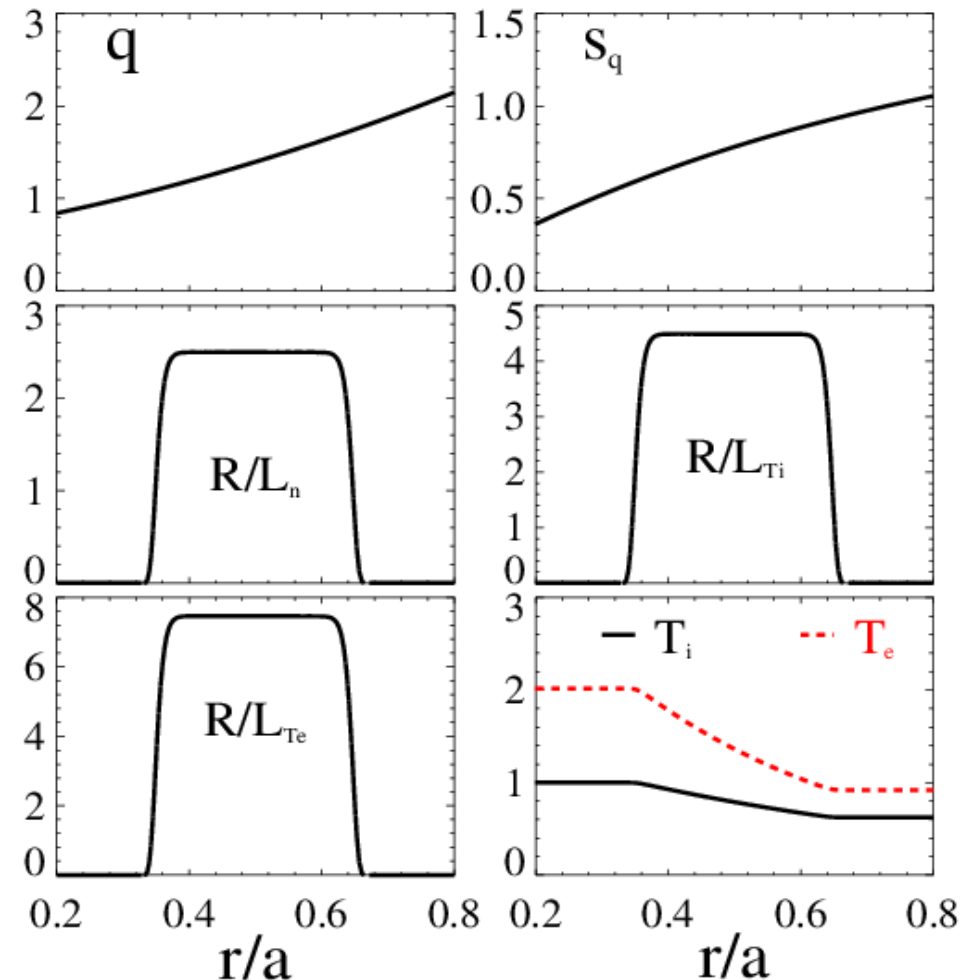
$$R/L_n = 2.5$$

$$R/L_{Ti} = 4.5$$

$$R/L_{Te} = 7.5$$

All parameters are fixed, but the density is varied.

- *ion-ion collisions*: Coulomb collision operator
- *electron-ion collisions*: Lorentz pitch-angle scattering operator



# Quantitative agreement between GK simulations and experiments

Estimation of energy confinement from simulations:

$$\tau_{eff} = \frac{1}{\chi_{eff}}$$

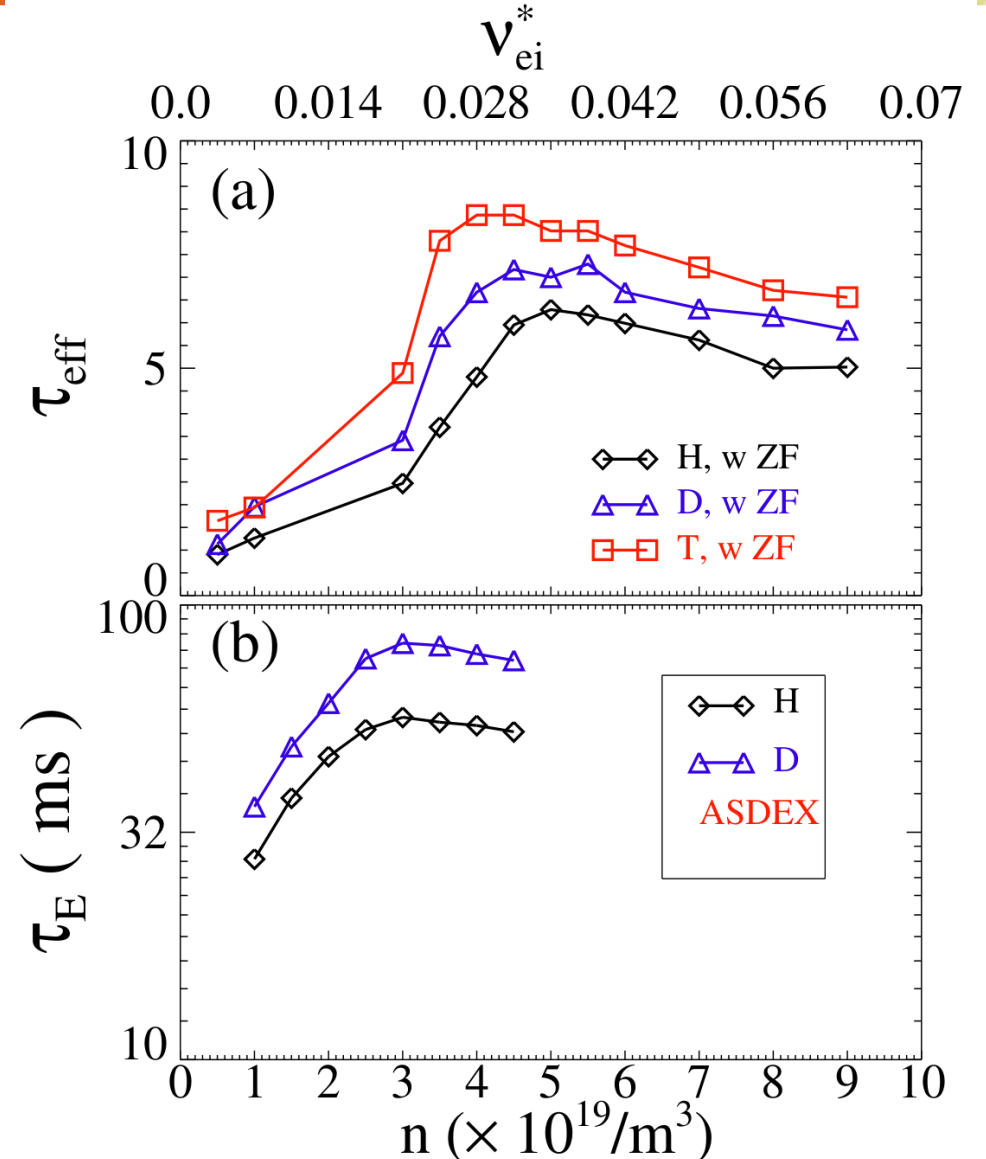
$$\chi_{eff} \equiv \frac{\chi_e n_e \frac{\partial T_e}{\partial r} + \chi_i n_i \frac{\partial T_i}{\partial r} + D_e T_e \frac{\partial n_e}{\partial r} + D_i T_i \frac{\partial n_i}{\partial r}}{n_e \frac{\partial T_e}{\partial r} + n_i \frac{\partial T_i}{\partial r} + T_e \frac{\partial n_e}{\partial r} + T_i \frac{\partial n_i}{\partial r}}$$

Figure (a) is from gKPSP simulations.

Figure (b) is ASDEX results, adapted from Fig. 1b in *M. Bessenrodt-weberpals et al., Nucl. Fusion 33, 1205 (1993)*

**agreement in trend!**

Isotopes improve the energy confinement time from LOC to SOC regimes.





# Quantitative agreement between GK simulations and experiments

## Background turbulence properties:

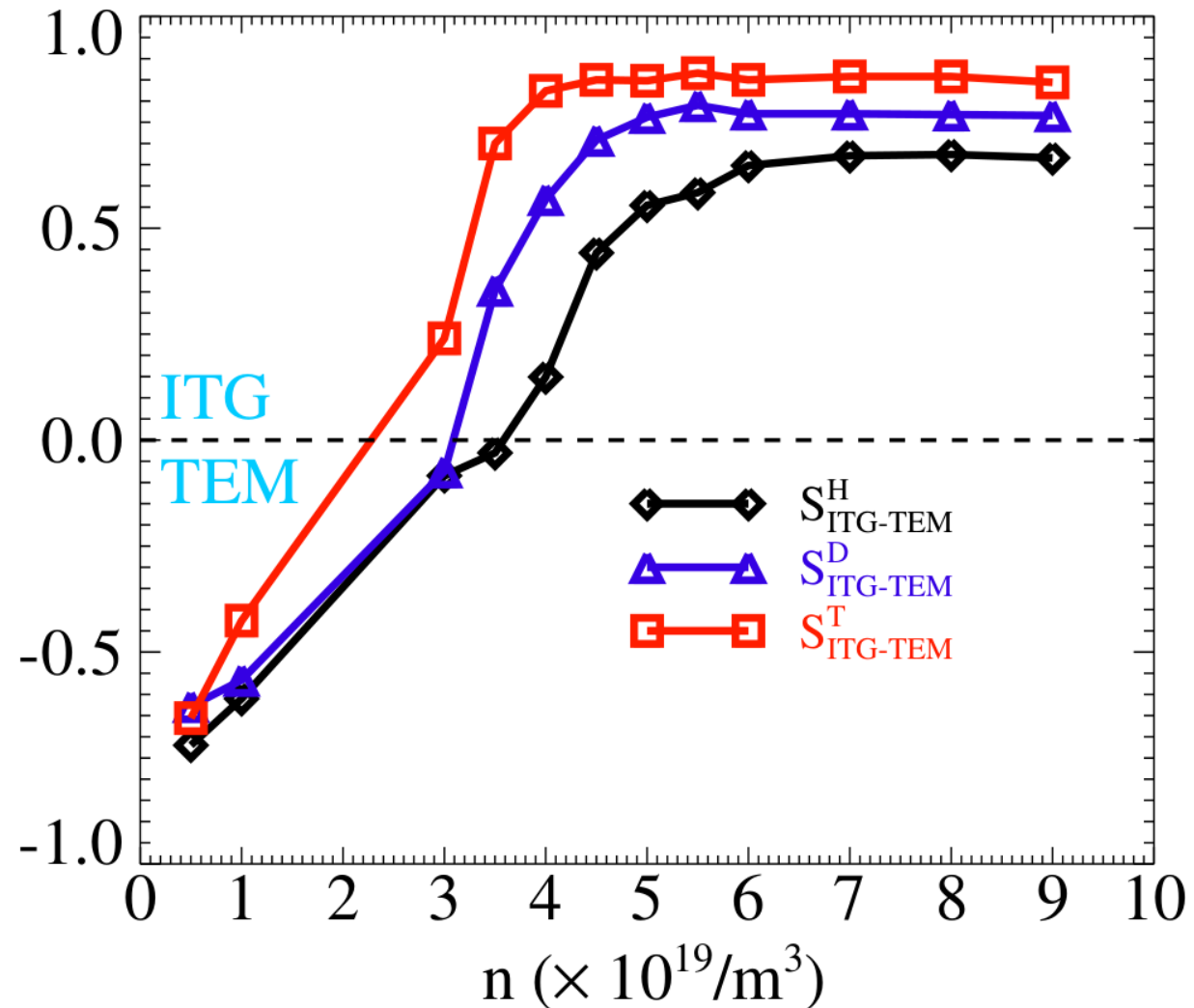
ITG-TEM turbulence strength coefficient

$$S_{ITG-TEM} \equiv \frac{S_{ITG} - S_{TEM}}{S_{tot}}$$
$$\equiv \frac{\sum_{\omega>0, k_\theta>0} |\phi(\omega, k_\theta)| - \sum_{\omega<0, k_\theta>0} |\phi(\omega, k_\theta)|}{\sum_{\omega>0, k_\theta>0} |\phi(\omega, k_\theta)| + \sum_{\omega<0, k_\theta>0} |\phi(\omega, k_\theta)|}$$

$S_{ITG-TEM} < 0$ : TEM dominant

$S_{ITG-TEM} > 0$ : ITG dominant

TEM dominant to ITG dominant along with  
LOC-SOC transition. (*J. Citrin PPCF2017, L. Qi  
NF2022, Y. Idomura PoP2023*)



# Quantitative agreement between GK simulations and experiments

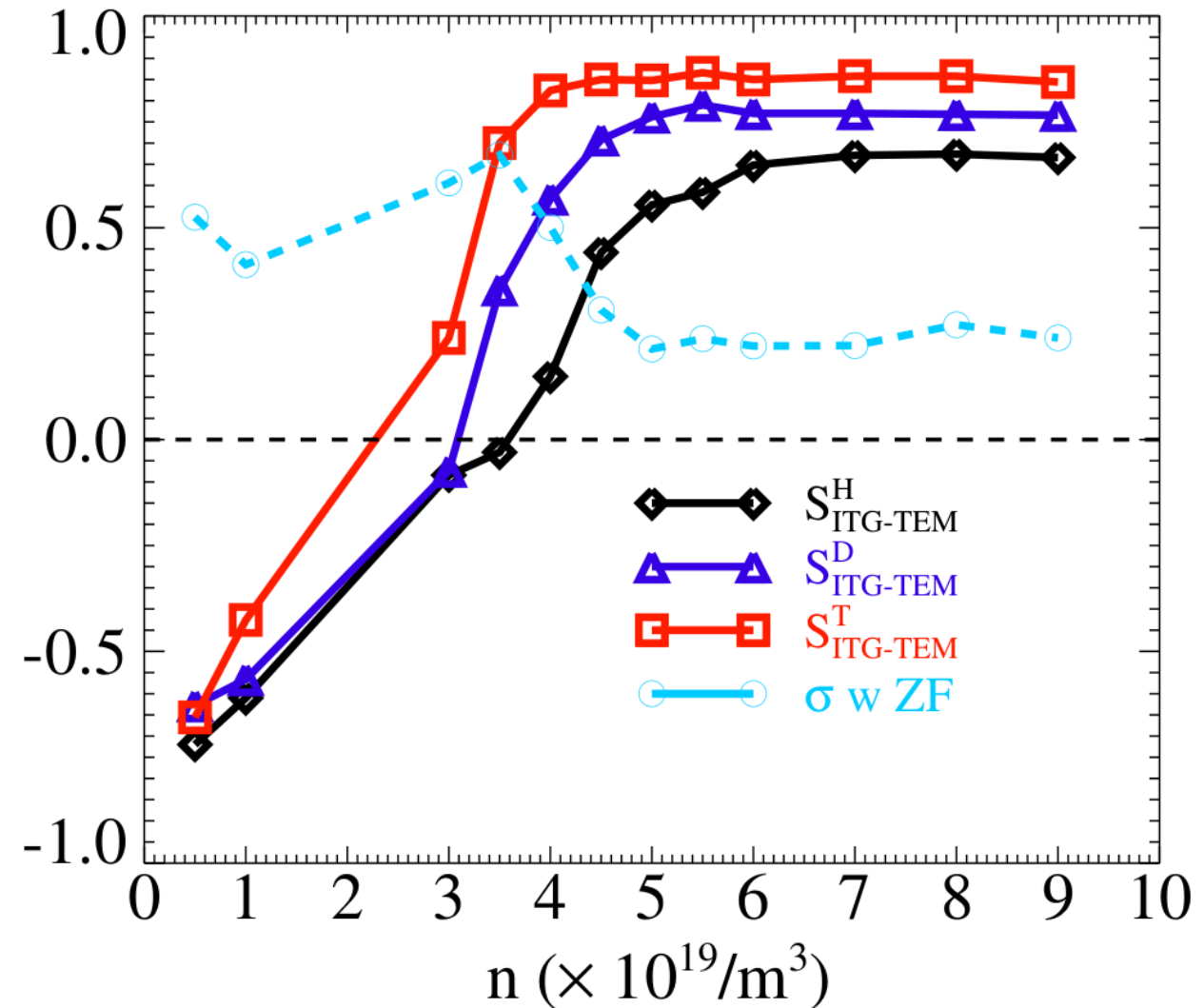
## Isotopic scaling:

✓  $\tau_{eff} \propto M_i^\sigma$

✓ TEM dominant LOC regime:  $\sigma \approx 0.5$

✓ ITG dominant SOC regime:  $\sigma \approx 0.25$

**A reasonable quantitative agreement in the range  $\sigma \in [0.2, 0.5]$ !**

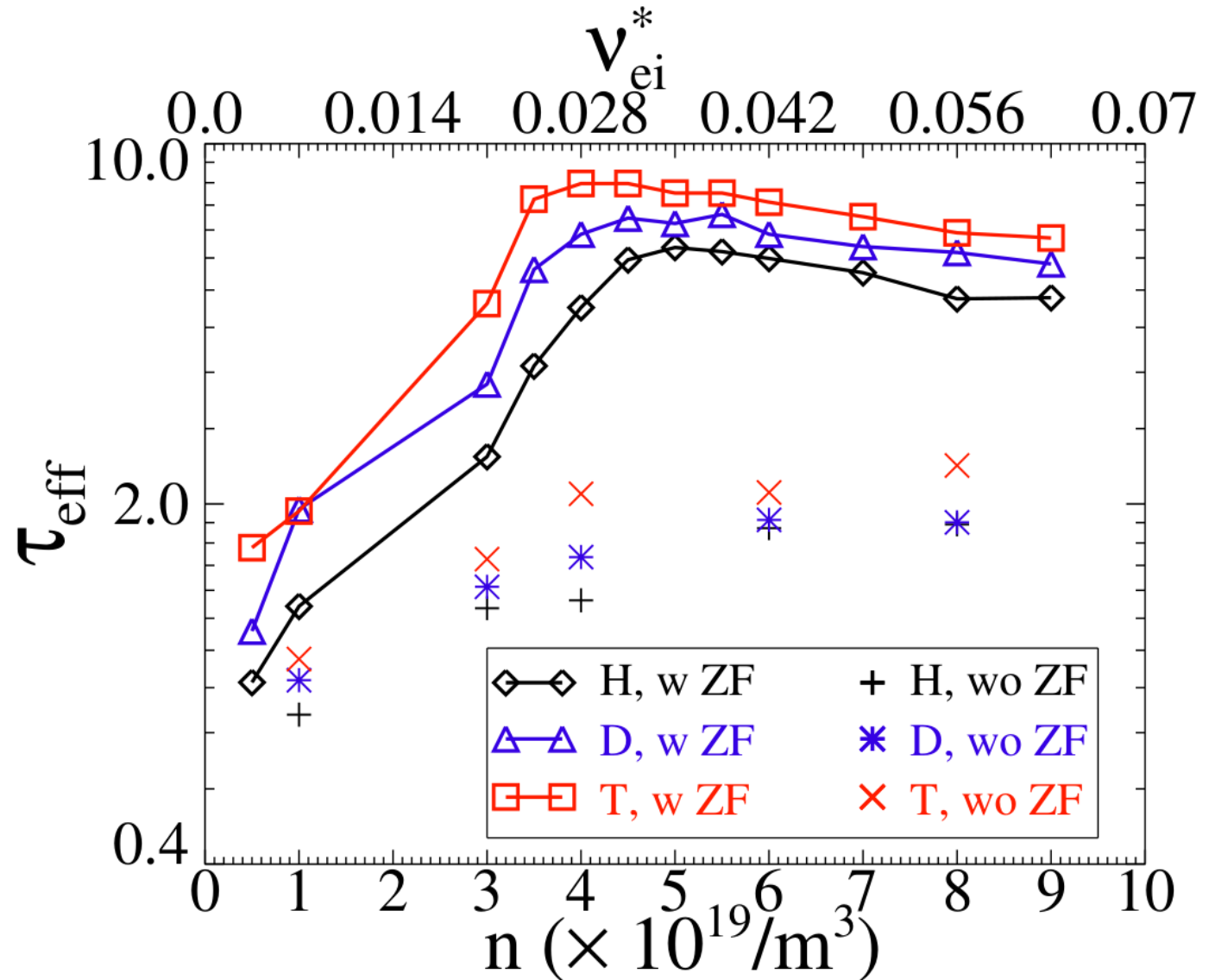


# Physical mechanisms behind the quantitative agreement ——— zonal flow

## Zonal flow:

**Discrete symbols:** scenarios without zonal flows.

- ✓ Isotope effects remain !
- ✓ Removal of zonal flow leads to a degradation of confinement in all cases.

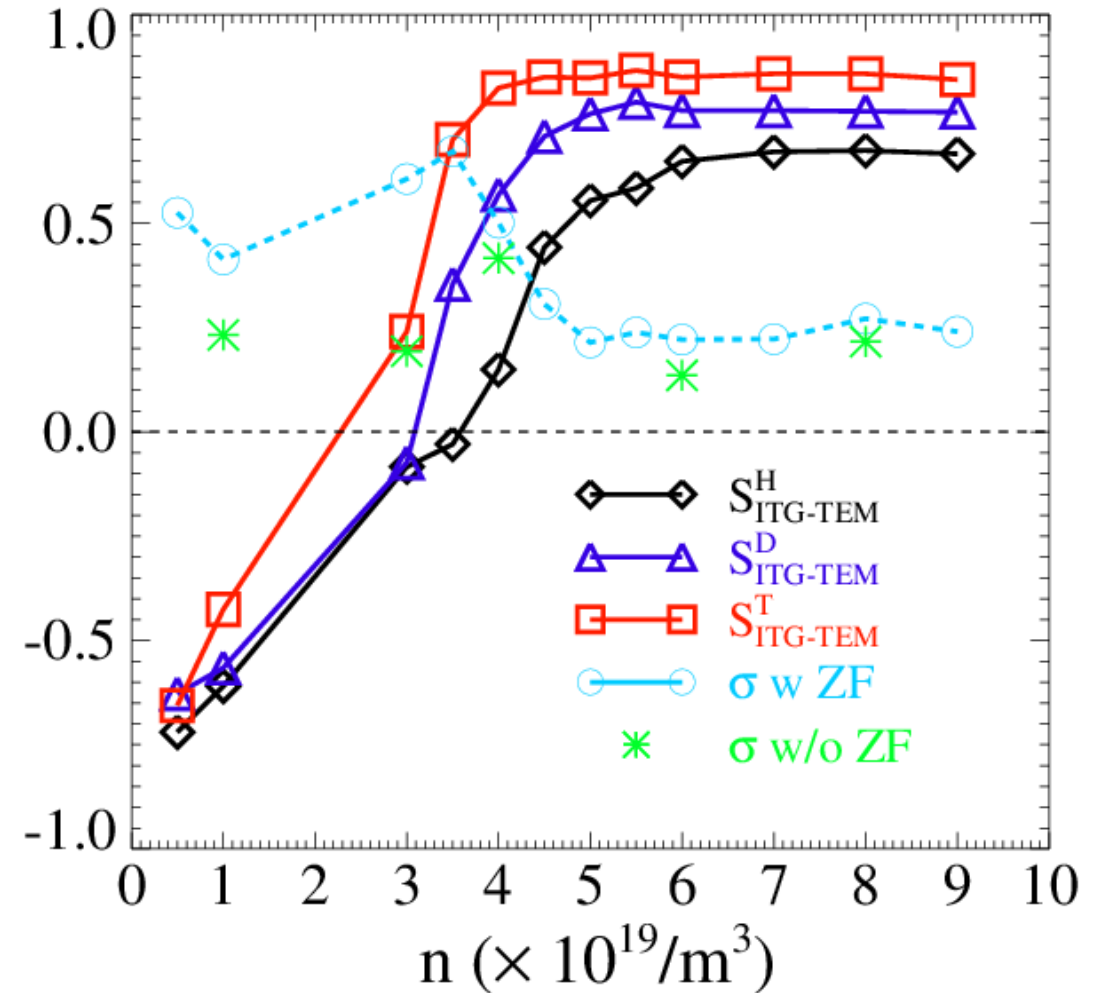


# Physical mechanisms behind the quantitative agreement ——— zonal flow

## Zonal flow:

**Green discrete star symbols:**  $\sigma$  without zonal flows

Zonal flow enhances the isotopic dependence of energy confinement, as it increases the scaling factor  $\sigma$ .





# Physical mechanisms behind the quantitative agreement ——— turbulence energy

## Turbulence energy analysis from spectrum:

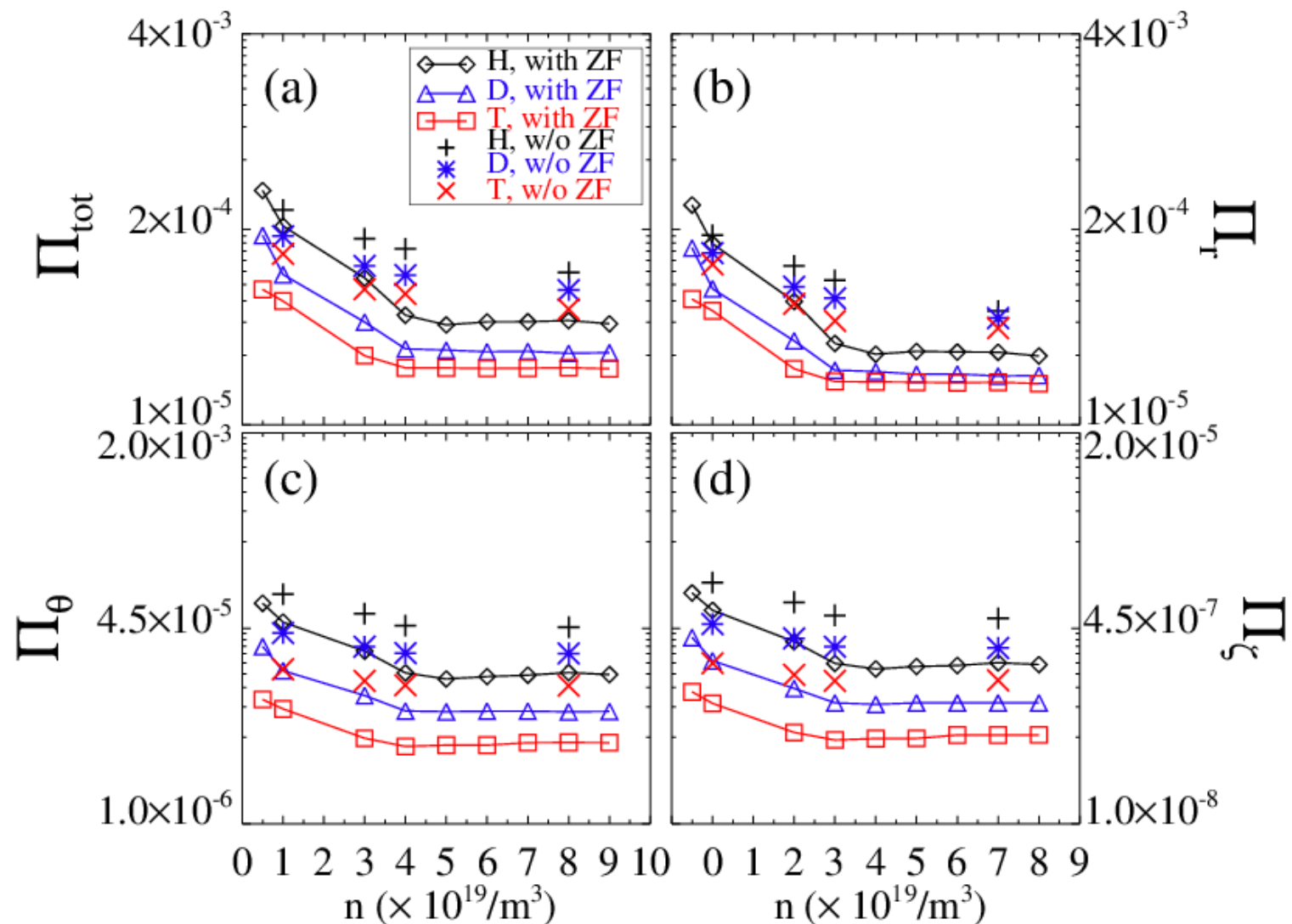
$$\Pi_{tot} = \Pi_r + \Pi_\theta + \Pi_\zeta$$

$$\Pi_r = |\delta E_r|^2 = \sum_{k_r, k_\theta, k_\zeta} k_r^2 |\phi(k_r, k_\theta, k_\zeta)|^2$$

$$\Pi_\theta = |\delta E_\theta|^2 = \sum_{k_r, k_\theta, k_\zeta} k_\theta^2 |\phi(k_r, k_\theta, k_\zeta)|^2$$

$$\Pi_\zeta = |\delta E_\zeta|^2 = \sum_{k_r, k_\theta, k_\zeta} k_\zeta^2 |\phi(k_r, k_\theta, k_\zeta)|^2$$

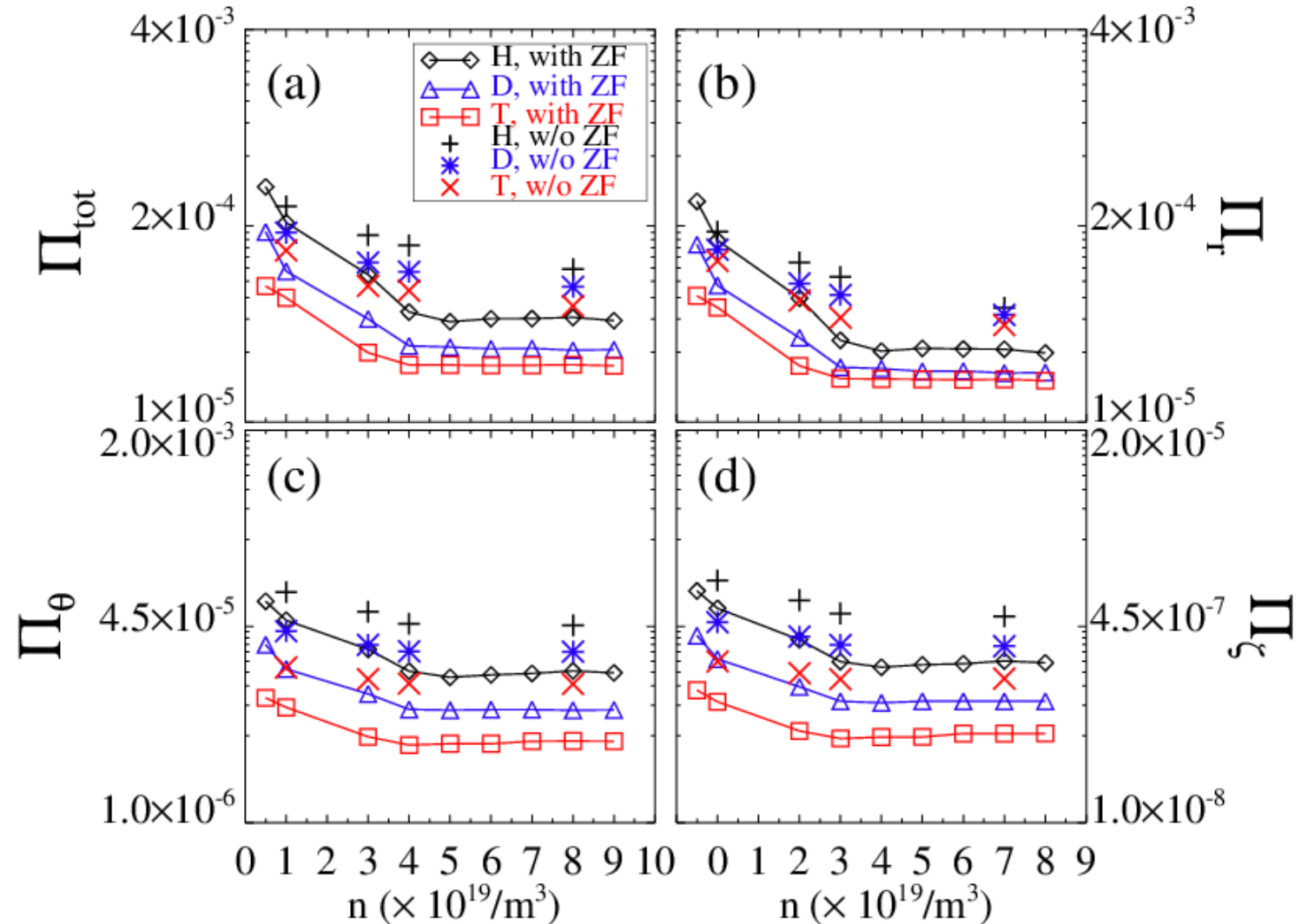
Note the radial electric field energy is also related to the poloidal  $\delta \mathbf{E} \times \mathbf{B}$  fluctuating velocity and vice versa.



# Physical mechanisms behind the quantitative agreement ——— turbulence energy

## Turbulence energy analysis from spectrum:

- Symbols and solid lines: scenarios with zonal flows
- Discrete symbols: scenarios without zonal flows
- **Presence of isotopes results in suppression of radial, poloidal and toroidal electric field energy for both scenarios with and without zonal flows.**



# Physical mechanisms behind the quantitative agreement ——— turbulence fluctuation properties

## Turbulence properties analysis from spectrum:

- Radial correlation length  $l_{cr} = 2\pi/\overline{k_r}$  and poloidal correlation length  $l_{c\theta} = 2\pi/\overline{k_\theta}$

$$\overline{k_r} = \sqrt{\Pi_r / \sum_{k_r, k_\theta, k_z} |\phi(k_r, k_\theta, k_z)|^2}$$

$$\overline{k_\theta} = \sqrt{\Pi_\theta / \sum_{k_r, k_\theta, k_z} |\phi(k_r, k_\theta, k_z)|^2}$$

- Turbulence eddy turn over rate  $\omega_T \propto k_\theta k_r \phi$
- Turbulence decorrelation rate  $\omega_c = 2\pi/t_c$ ,  $t_c$  is the turbulence correlation time.
- Selected cases:  $n = [1.0, 3.0, 6.0, 8.0] \times 10^{19}/m^3$ , both with and without zonal flows for isotopes: hydrogen, deuterium and tritium

# Physical mechanisms behind the quantitative agreement ——— **turbulence fluctuation properties**

## Turbulence properties analysis from spectrum:

### With zonal flow:

- $l_{cr} \propto M_i^{0.11}$  strongly deviates from gyro-Bohm scaling and  $l_{c\theta} \propto M_i^{0.53}$  gyro-Bohm scaling.  
← **Anisotropic**
- Decorrelation rate  $\omega_c \propto M_i^{-0.76}$  and eddy turn over rate  $\omega_T \propto M_i^{-0.87}$ . **Considerably different from the characteristic linear frequency  $\omega_{*e} \propto c_s/a \propto M_i^{-0.5}$**

**Mixing length estimations:**  $\tau \propto \frac{1}{\chi} \propto \frac{1}{l_c^2 \omega_c}$

- $\tau \propto M_i^{0.54}$ , using radial correlation length  $l_{cr} \propto M_i^{0.11}$
- $\tau \propto M_i^{-0.3}$ , using poloidal correlation length  $l_{c\theta} \propto M_i^{0.53}$
- **Radial electric field** rather than the poloidal one plays a key role in isotope effects.
- Strong deviation of  $l_{cr} \propto M_i^{0.11}$  from the gyro-Bohm scaling could be a key clue in understanding isotopic dependence.



# Physical mechanisms behind the quantitative agreement ——— turbulence fluctuation properties

## Turbulence properties analysis from spectrum:

### Without zonal flow:

- Radial correlation length:  $l_{cr}^{(0)} \propto M_i^{0.10}$
- Turbulence decorrelation rate:  $\omega_c^{(0)} \propto M_i^{-0.61}$
- Energy confinement time scaling:  $\tau^{(0)} \propto M_i^{0.41}$

	with zonal flow	without zonal flow
$l_{cr}$	$\propto M_i^{0.11}$	$\propto M_i^{0.10}$
$\omega_c$	$\propto M_i^{-0.76}$	$\propto M_i^{-0.61}$
$\tau$	$\propto M_i^{0.54}$	$\propto M_i^{0.41}$

Zonal flow enhances the isotopic dependence of energy confinement mainly through **reinforcing the inverse dependence of turbulence decorrelation rate on isotope mass.**

# Physical mechanisms behind the quantitative agreement ——— deviation from gyro-Bohm scaling

## ➤ Three contributors in isotope effects:

- Deviation from gyro-Bohm scaling

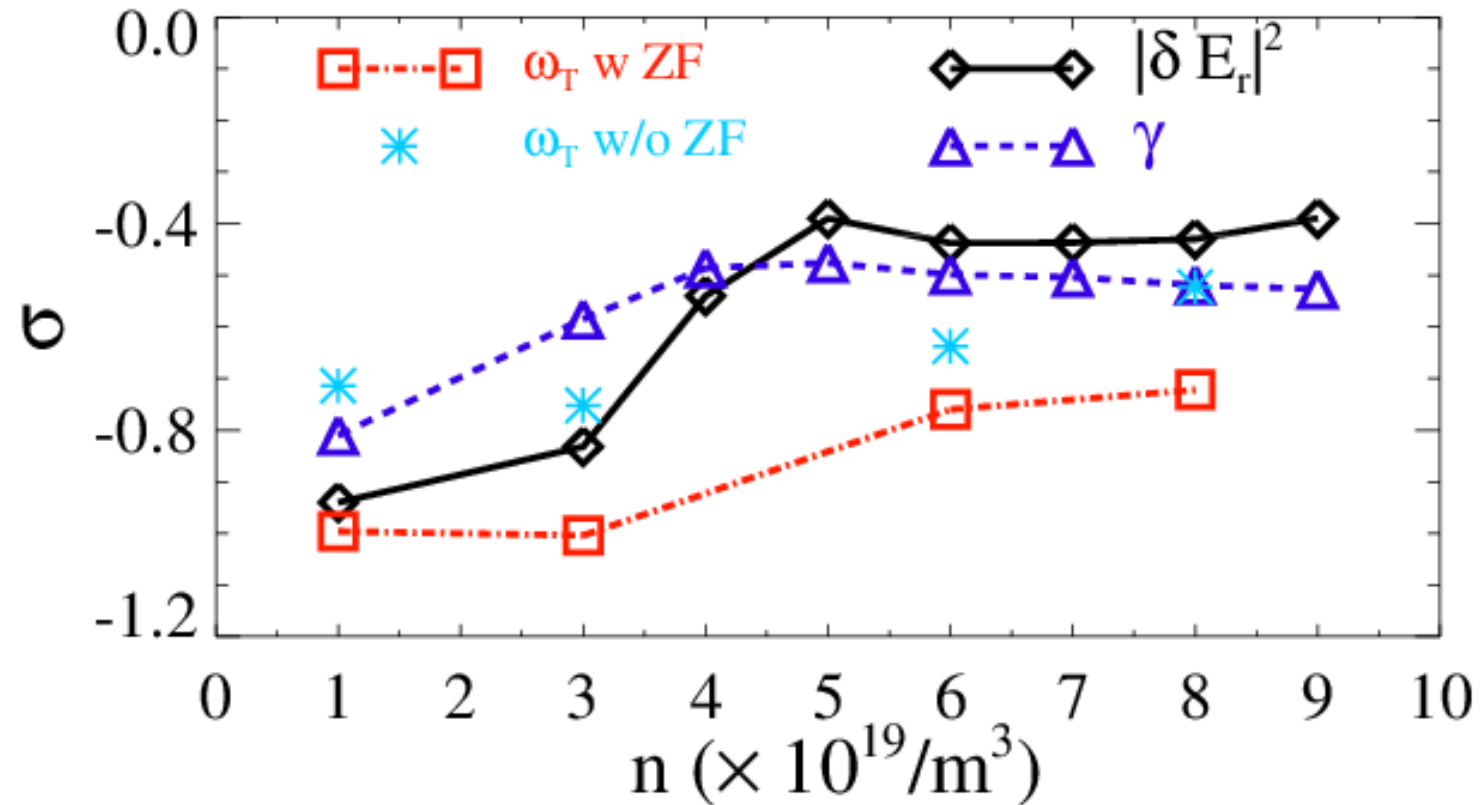
- Zonal flow

- TEM stabilization

- ✓  $|\delta E_r|^2$ : turbulence radial electric field intensity

- ✓  $\gamma$ : linear growth rate

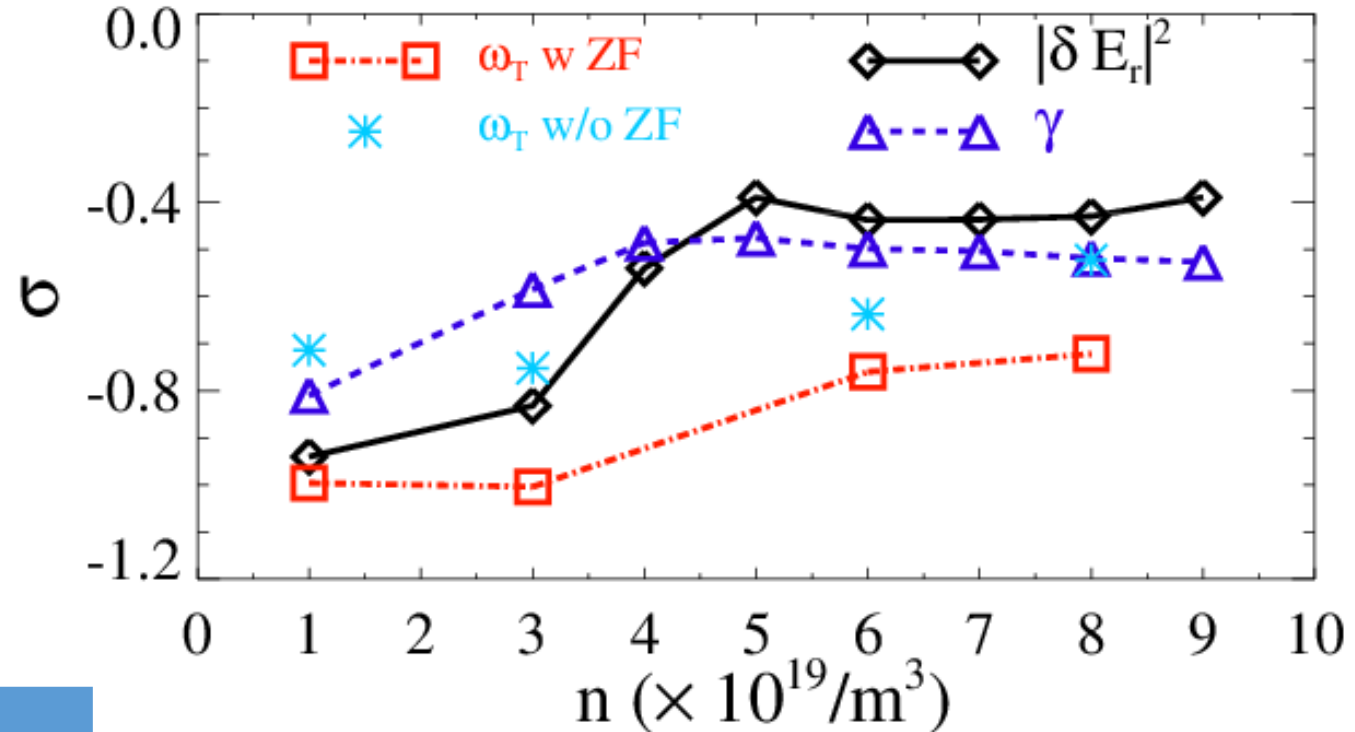
- ✓  $\omega_T$ : turbulence eddy turn over rate



# Physical mechanisms behind the quantitative agreement ——— deviation from gyro-Bohm scaling

## Deviation from gyro-Bohm scaling

- Characterized by  $l_{cr} \propto M_i^{0.11}$
- ITG dominant case,  $n = 8.0 \times 10^{19}/m^3$ 
  - ✓  $\gamma \propto M_i^{-0.5}$
  - ✓  $|\delta E_r|^2 \propto M_i^{-0.43}$
  - ✓ Mixing length estimation:
    - $\chi \propto \gamma / \bar{k}_r^2 \propto \gamma l_{cr}^2 \propto M_i^{-0.28}$
  - ✓  $\tau \propto \frac{1}{\chi} \propto M_i^{0.28}$



GK Simulations	Gyro-Bohm assumption
$l_{cr} \propto M_i^{0.11}$	$l_{cr} \propto M_i^{0.5}$
$\tau \propto M_i^{0.28}$	$\tau \propto M_i^{-0.5}$

# Physical mechanisms behind the quantitative agreement ——— TEM stabilization

## TEM stabilization by electron-ion collision

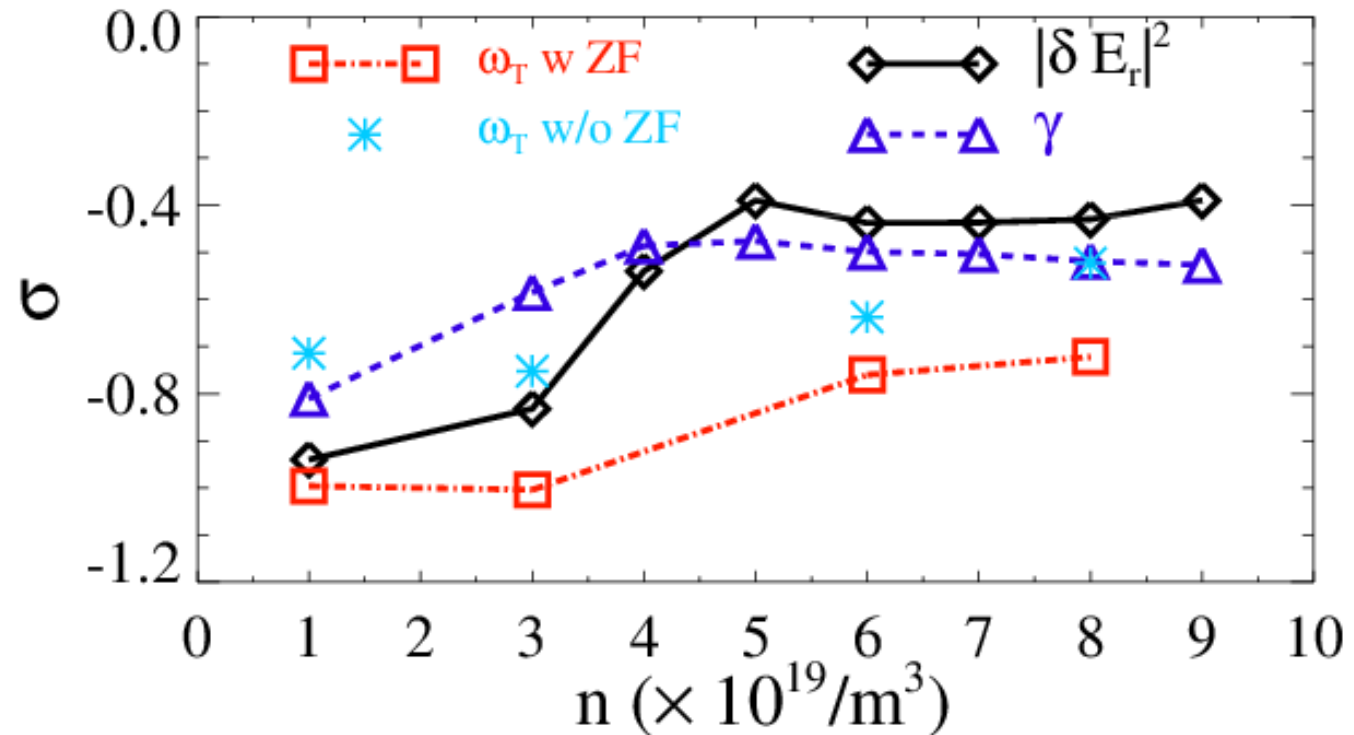
TEM dominant case,  $n = 1.0 \times 10^{19} / m^3$

$$\checkmark \gamma \propto M_i^{-0.8}$$

$$\checkmark |\delta E_r|^2 \propto M_i^{-0.94}$$

$$\checkmark \tau \propto \frac{1}{\chi} \propto 1/\gamma l_{cr}^2 \propto M_i^{0.58}$$

✓ Evidently, TEM stabilization introduces enhanced isotope effects **adding to the deviation from gyro-Bohm scaling.**





# Physical mechanisms behind the quantitative agreement ——— **TEM stabilization**

TEM stabilization corroborates early works by *Pusztai et al., PoP2011, Nakata, et al., PRL2017*

However, mechanism behind?

electron-ion collision  $\nu_{ei}$  is independent of ion mass.

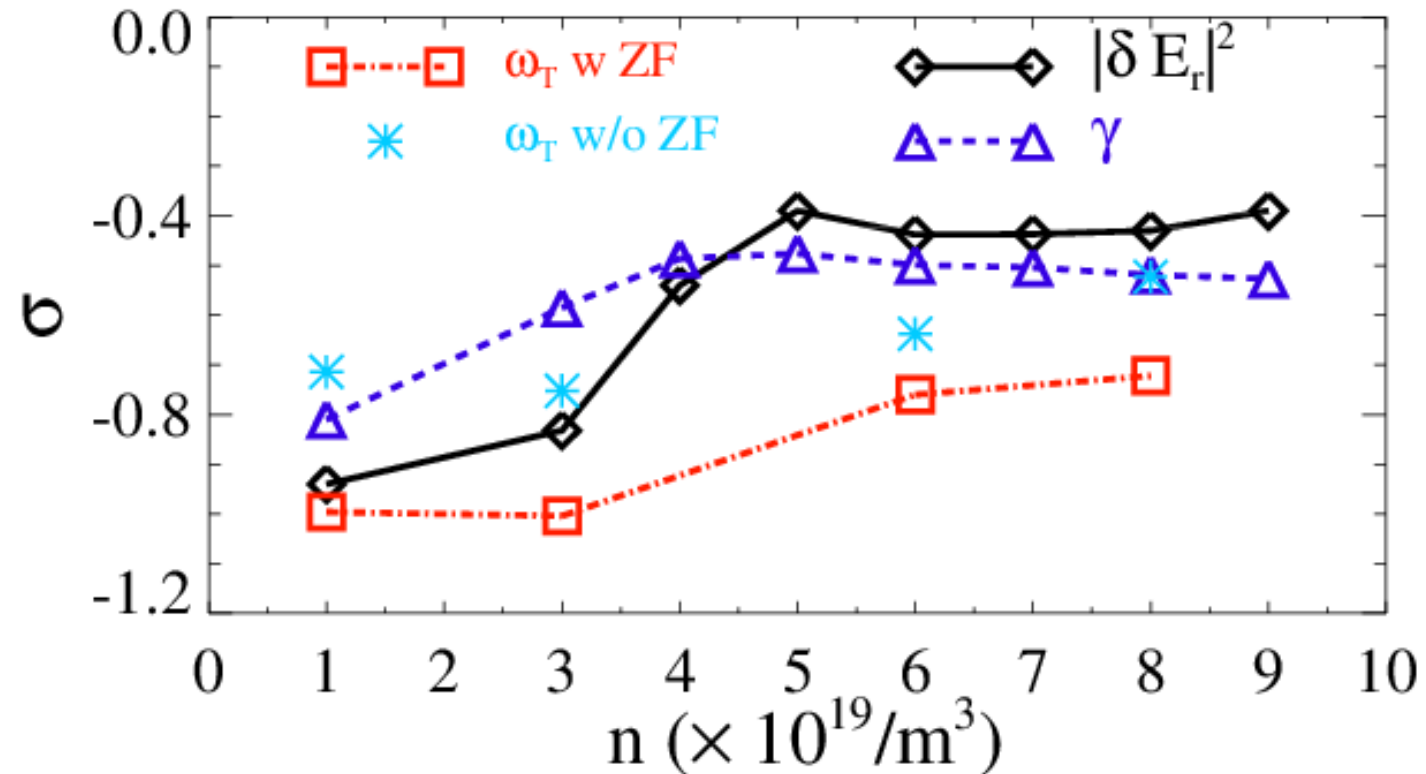
- ✓ **Collisional broadening of resonance:** from TEM dispersion relation, which requires the resonance condition:  $\omega - \overline{\omega_{de}} + i\nu_{ei}/\varepsilon$ ,  $\omega - \overline{\omega_{de}} \propto M_i^{-0.5}$ , while  $\nu_{ei} \propto M_i^0$ . Therefore, larger isotope mass, stronger collisional broadening.
- ✓ **The detrapping rate  $f_{dt}$**  defined as a percentage of detrapping trapped electrons per unit time is found a key measure of the detrapping effects (*L. Qi et al., NF 2020*). From simulations,  $f_{dt} \propto M_i^{0.6}$ , which means isotope mass increases the trapped electrons detrapping rate, destroys the trapped electron  $\nabla B$  drift resonance (*J.C. Adam, PoF1976*) faster and leads to reduced TEM turbulence

# Physical mechanisms behind the quantitative agreement ——— **Zonal flow**

## Zonal flow:

Zonal flow reinforces the inverse dependence of **turbulence decorrelation rate** (turbulence turn over rate  $\omega_T$ ) on isotope mass.

	with zonal flow	without zonal flow
$l_{cr}$	$\propto M_i^{0.11}$	$\propto M_i^{0.10}$
$\omega_c$	$\propto M_i^{-0.76}$	$\propto M_i^{-0.61}$
$\tau$	$\propto M_i^{0.54}$	$\propto M_i^{0.41}$



Early works showed the importance of zonal flow in isotope effects.

(Y. Xu PRL2013; Hahm NF2013; Nakata PRL2017; Garcia NF2017; Belli PRL2020 etc...)

# Physical mechanisms behind the quantitative agreement ——— **3 contributors**

Quantify the contributions from the three participators:

- **Deviation from gyro-Bohm scaling**
- **Zonal flow**
- **TEM stabilization**

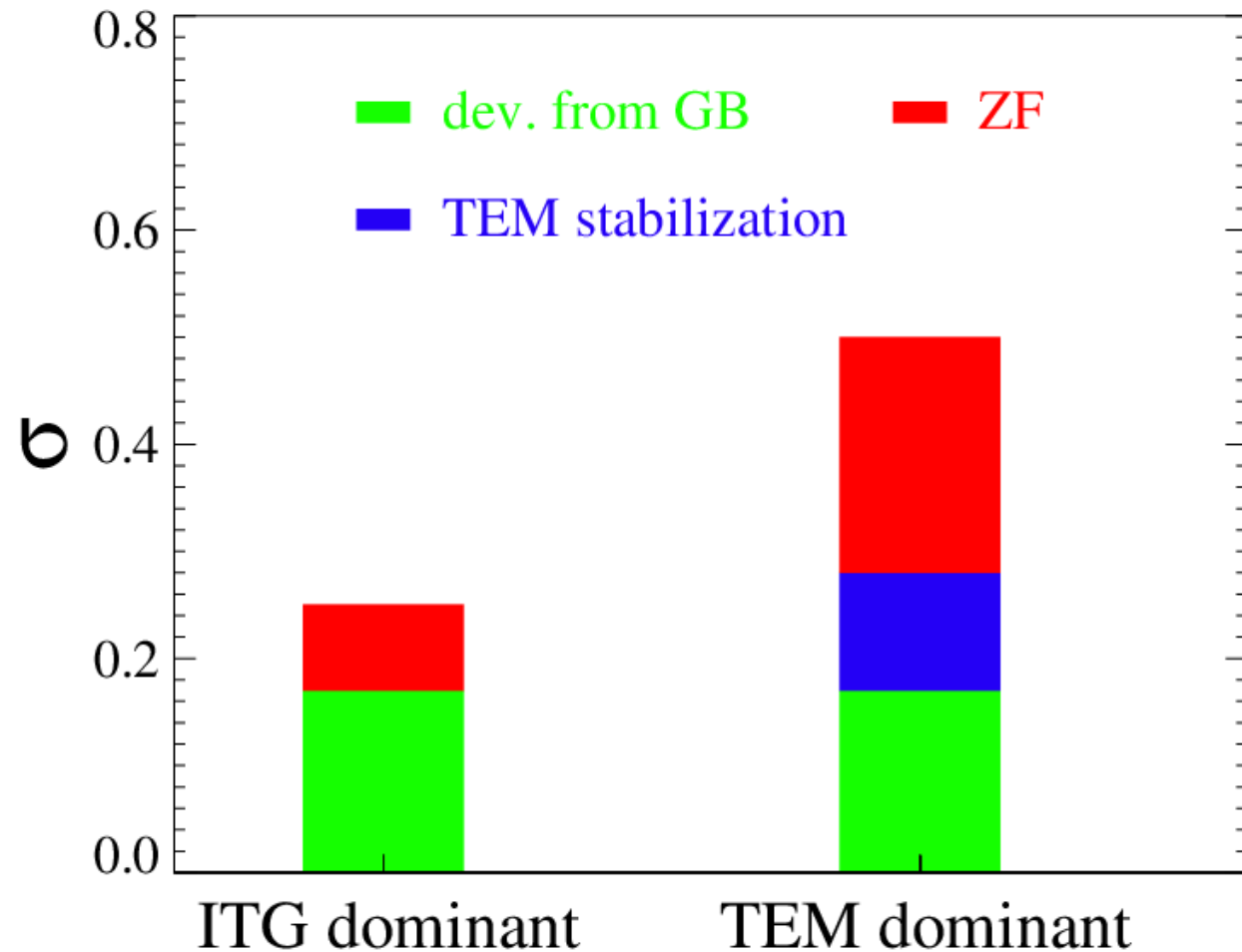
Selected cases: both with and without zonal flows

TEM dominant:  $n = 1.0, 3.0, 4.0 \times 10^{19}/m^3$

ITG dominant:  $n = 6.0, 8.0 \times 10^{19}/m^3$

$$\sigma_{TEM}^{(w\ zf)} = 0.5, \sigma_{TEM}^{(w/o\ zf)} = 0.28,$$

$$\sigma_{ITG}^{(w\ zf)} = 0.25, \sigma_{ITG}^{(w/o\ zf)} = 0.17$$

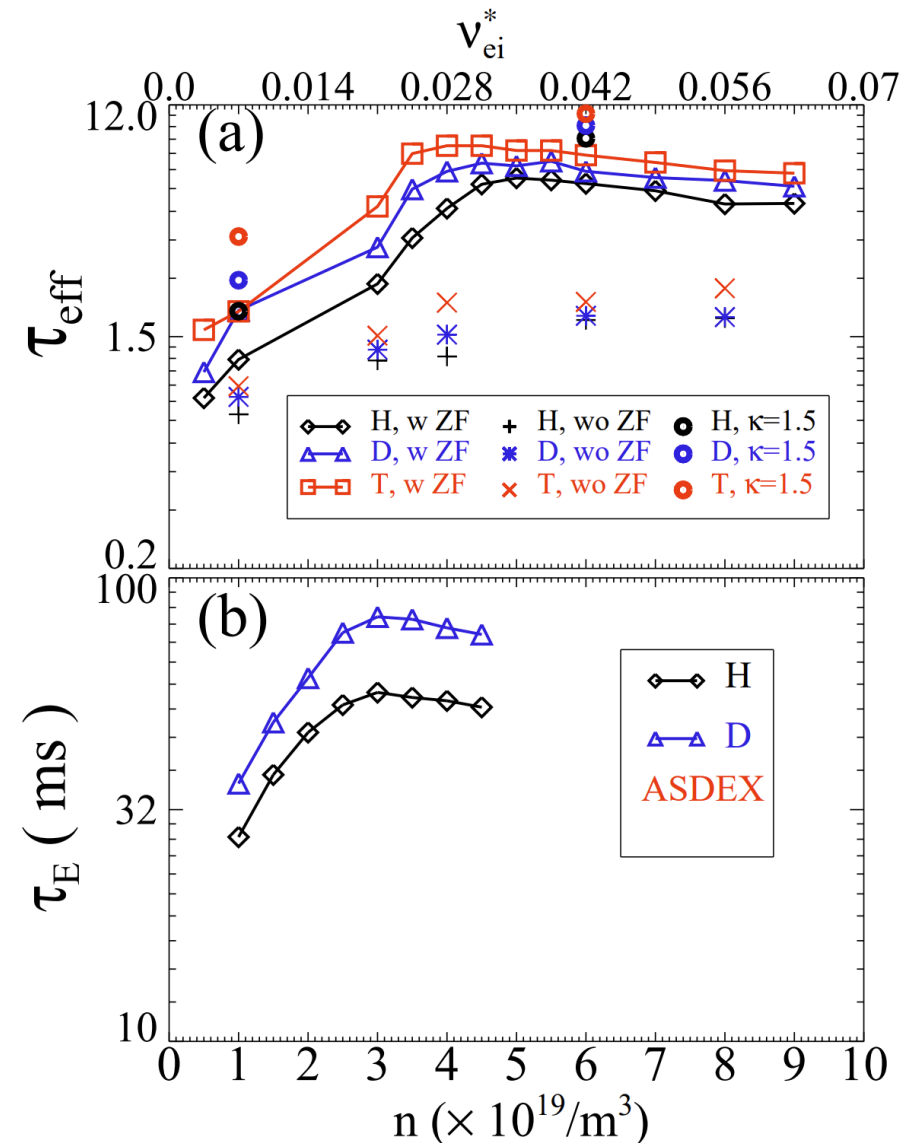


# Discussions

Commonality:

Results hold for varying magnetic configurations, a test over **plasma elongation,  $\kappa = 1.5$** .

1. Isotope effects in elongated plasma shape.
2. Elongation improves the energy confinement, consistent with our previous publications. (*Lei Qi et al., NF 2017, 2019*)



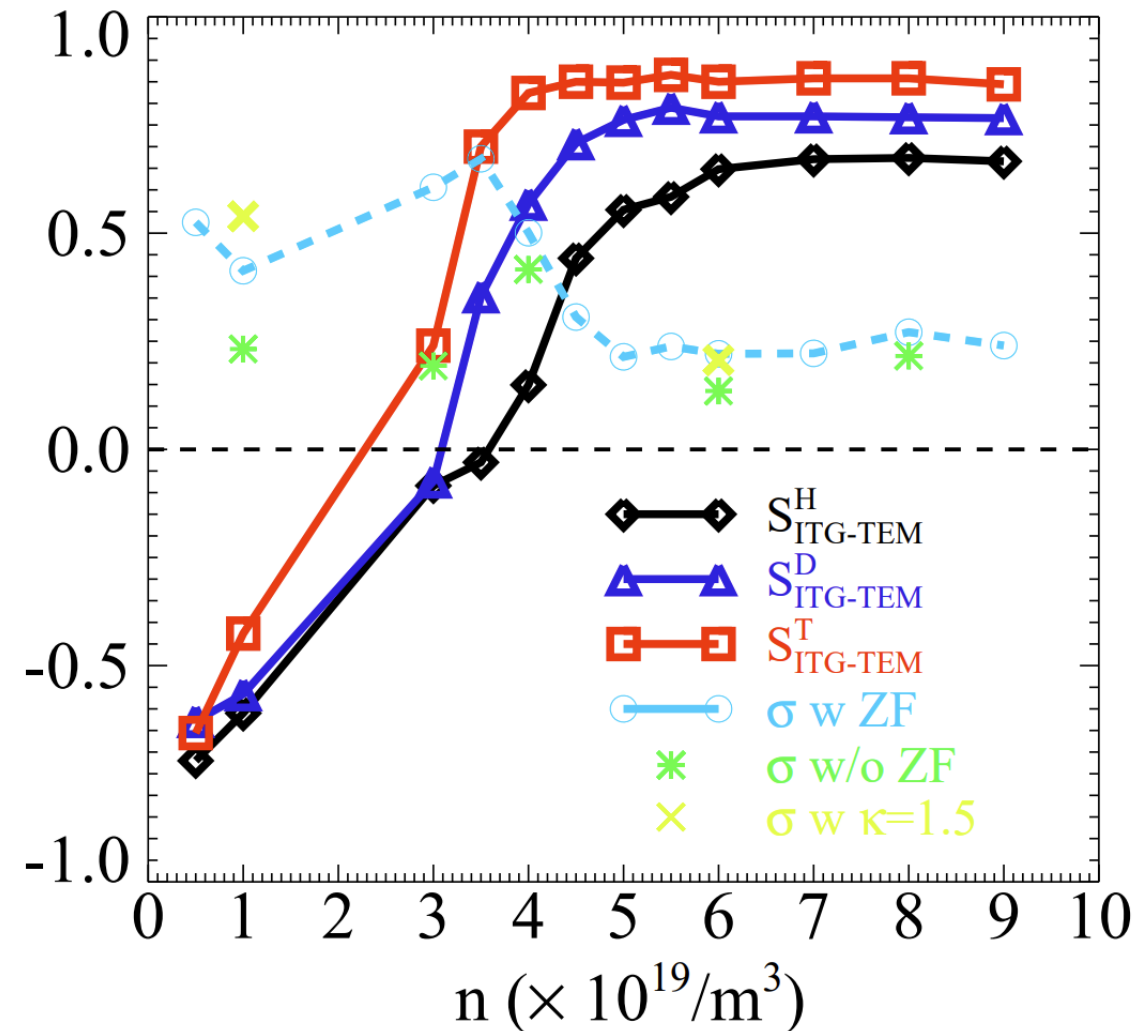


# Discussions

Commonality:

Results hold for varying magnetic configurations, a test over **plasma elongation,  $\kappa = 1.5$** .

1. Quantitative agreement holds in elongated plasma shape.



# Discussions

Commonality:

Results hold for varying profiles and parameters.

Tests were conducted for different profile and parameters:

Case1: **ITG**

$$\frac{R}{L_n} = 2, \frac{R}{L_{Ti}} = 6, \frac{R}{L_{Te}} = 2, n = 1.0 \times 10^{19}/m^3$$

$$\tau \propto M_i^{0.22} \text{ and } l_{cr} \propto M_i^{0.13}$$

Case1: **TEM**

$$\frac{R}{L_n} = 2, \frac{R}{L_{Ti}} = 2, \frac{R}{L_{Te}} = 6, n = 1.0 \times 10^{19}/m^3$$

$$\tau \propto M_i^{0.36} \text{ and } l_{cr} \propto M_i^{0.16}$$

# Discussions

Commonality:

**Plan:**

1. To conduct more simulations with varying profiles, parameters, magnetic configurations and for experiments.
2. We are particularly interested in simulating experimental results!
3. Essential role of zonal flow, and in isotope effects for a wider range of parameters?

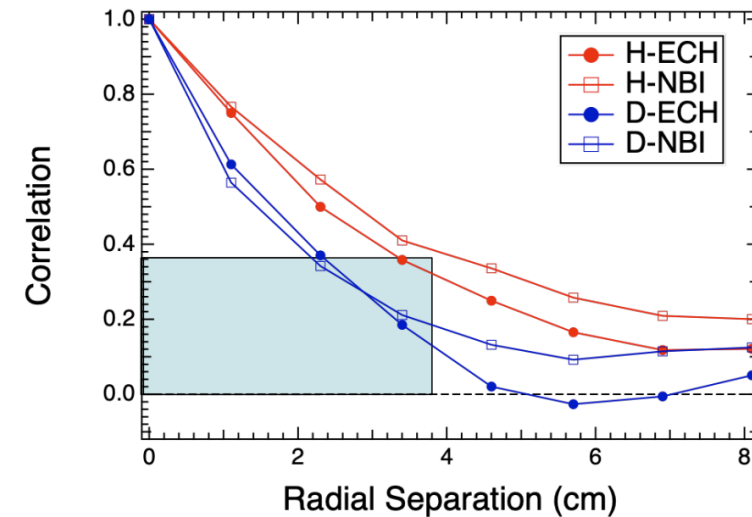
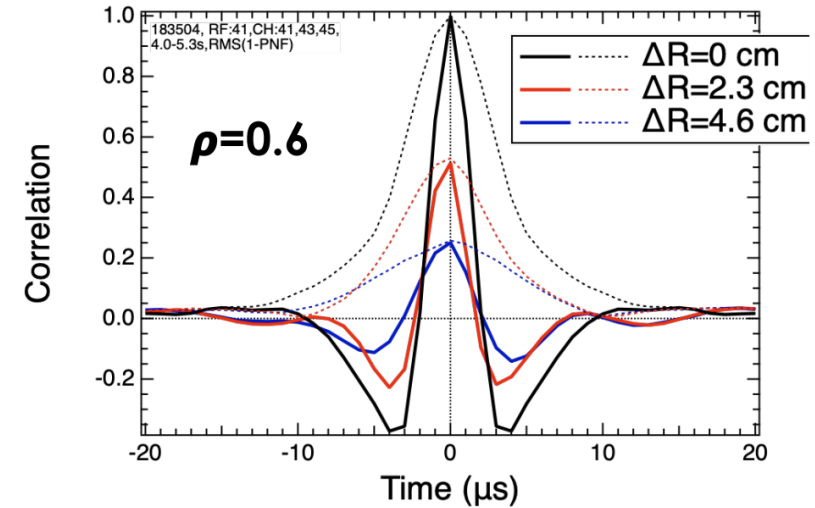
# Discussions

Isotopic dependence of radial correlation length might be the right direction to understand the isotope effects. Experimental evidence.

G. McKee, et al, IAEA-FEC2023, London, UK

## Correlation Length (cm)

	H	D
NBI+ECH	3.4	2.4
NBI	4.2	2.4



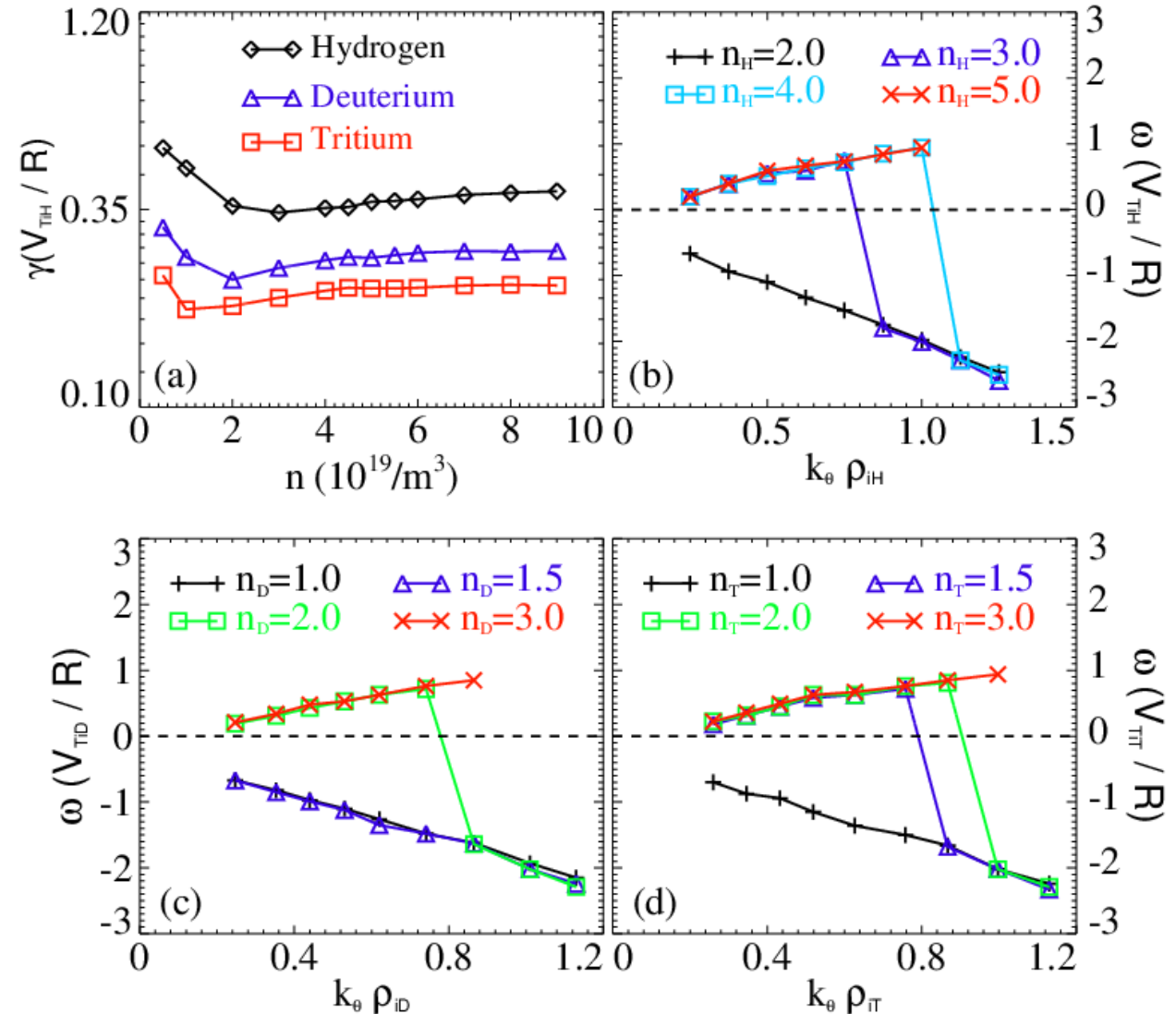


# Discussions

## Linear frequency properties:

- (a) **Growth rates** for the most linearly growing modes as a function of density for hydrogen (black), deuterium (blue) and tritium (red).
- (b) - (c) are plots of **frequency** as a function of poloidal wave number for hydrogen, deuterium and tritium plasmas, respectively.

Note that in (a), the growth rates are normalized to  $V_{TiH}/R$  with  $V_{TiH}$  being the hydrogen ion thermal velocity. In (b)-(c) frequency and wave number are normalized to hydrogen isotopes.



# Discussions

Frequency analysis:

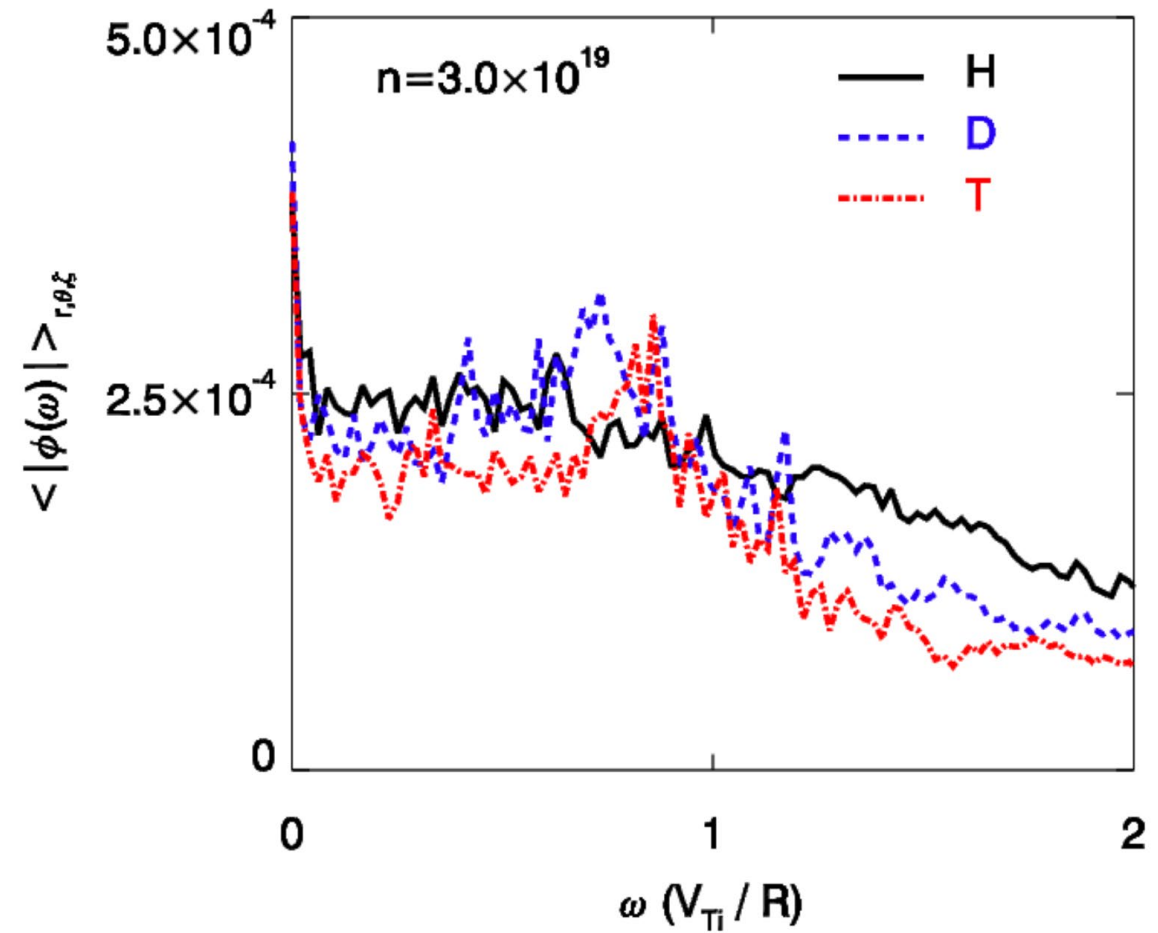
Averaged frequency spectrum over the whole volume. Frequency is normalized to isotope's mass.

Turbulence intensity averaged frequency:

$$\bar{\omega} = \sqrt{\frac{\sum \omega^2 \phi^2}{\sum \phi^2}}$$

M	$\omega(V_{Ti}/R)$
1	1.3
2	1.11
3	0.98

$\sigma = -0.25(-0.5) = -0.75$ , consistent with the correlation time analysis



# Conclusions

- **Quantitative agreement:** gKPSP gyrokinetic simulation investigating the isotopic dependence of energy confinement exhibits a robust quantitative agreement with the experimental empirical scaling law, which shows the **scaling factor  $\sigma$**  is in the range **from 0.2 to 0.5** in general.
- Mitigation of turbulence radial electric field intensity  $|\delta E_r|^2$  and associated poloidal  $\delta \mathbf{E} \times \mathbf{B}$  fluctuating velocity with the radial correlation length  $l_{cr} \propto M_i^{0.11}$  strongly **deviating from the gyro-Bohm scaling** are demonstrated to be the dominant mechanism contributing to isotope effects. This clarifies why gyro-Bohm scaling cannot predict the isotope effects

# Conclusions

- **Three main contributors** in the isotope effects are identified:
  - **Deviation from gyro-Bohm scaling:**
    - ✓ turbulence radial correlation length  $l_{cr} \propto M_i^{0.11}$
    - ✓ intrinsic isotopic dependence of linear growth rate  $\gamma \propto M_i^{-0.5}$
  - **Zonal flow:**
    - ✓ Reinforce the inverse dependence of turbulence decorrelation rate on isotope mass with  $\omega_c \propto M_i^{-0.76}$
  - **Trapped electron turbulence stabilization**
    - Additional enhancement on isotope effects: linear growth rate  $\gamma \propto M_i^{-0.8}$
    - **Collisional resonance broadening** and  $f_{dt} \propto M_i^{0.6}$
- **Future work:** Delve deeper into the role of deviation from gyro-Bohm scaling across a wider range of regimes.



**Thank you for your attention**

Ref: W. W. Lee and R. A. Santoro, Phys. Plasmas 4, 169-173 (1997)

Steep local peaks observed in the poloidal wave number and frequency spectra implies there exist limited number of wave modes in the simulation system.

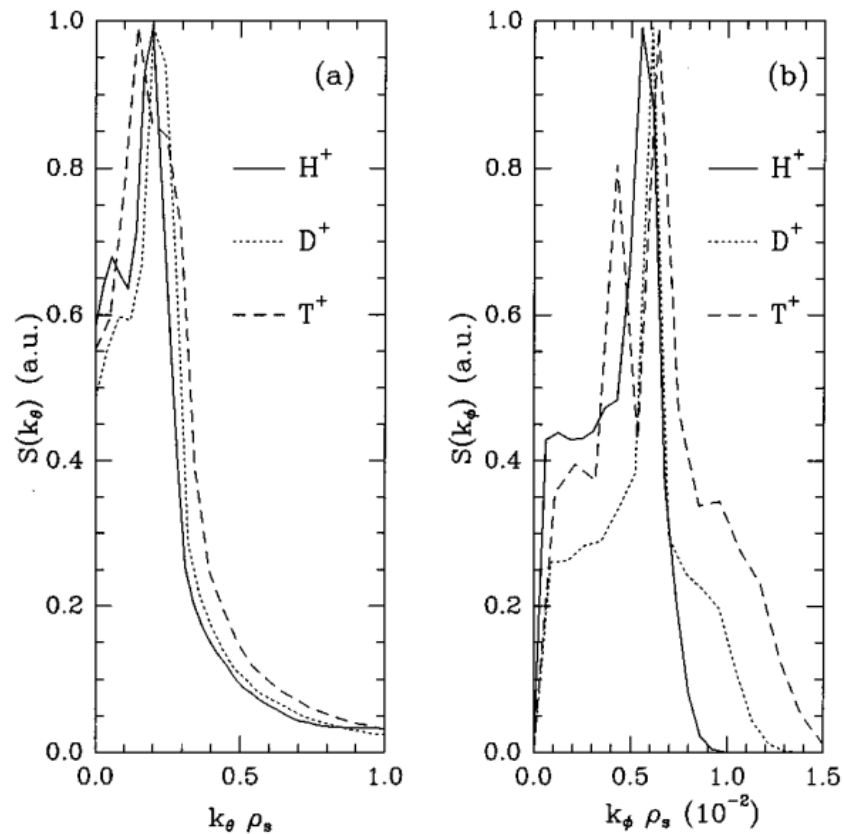


FIG. 3. The time-averaged poloidal  $k_\theta$  spectra (a) and toroidal  $k_\phi$  spectra (b) in the fully nonlinear state, where  $\rho_s$  is mass dependent.

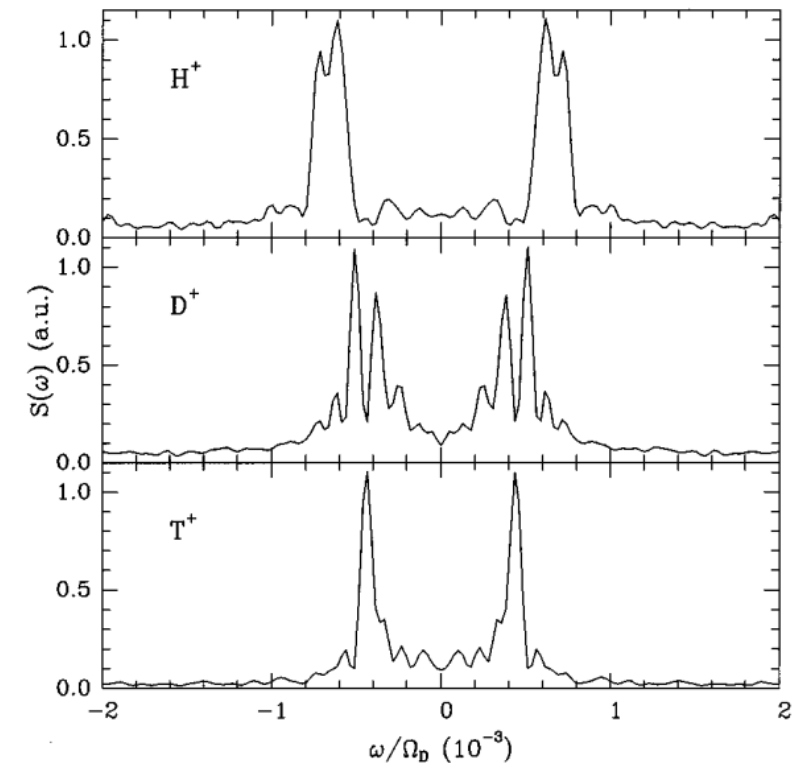


FIG. 4. Spatially averaged frequency spectra along the weak magnetic field side of the midplane for the three hydrogenic isotopes, where subscript D denotes deuterium.

# Discussions

Commonality:

Results hold for varying radial domain size  $\rho^{*-1}$ .

We have tested the radial domain size from  $\rho^{*-1} = 220$  to 440.

1. Isotopic dependence of confinement is consistently observed for different domain sizes.
2. The weak isotopic dependence of radial correlation length is consistently observed.

$l_{cr}$	H ( $\rho_{iH}$ )	D ( $\rho_{iH}$ )	T ( $\rho_{iH}$ )	scaling
$\frac{a}{\rho_{iH}} = 220$	8.64	9.55	10.2	$l_{cr} \propto M_i^{0.15}$
$\frac{a}{\rho_{iH}} = 440$	12.27	13.66	14.61	$l_{cr} \propto M_i^{0.158}$