EPFL

Reducing turbulent transport in tokamaks by combining intrinsic rotation and the low momentum diffusivity regime

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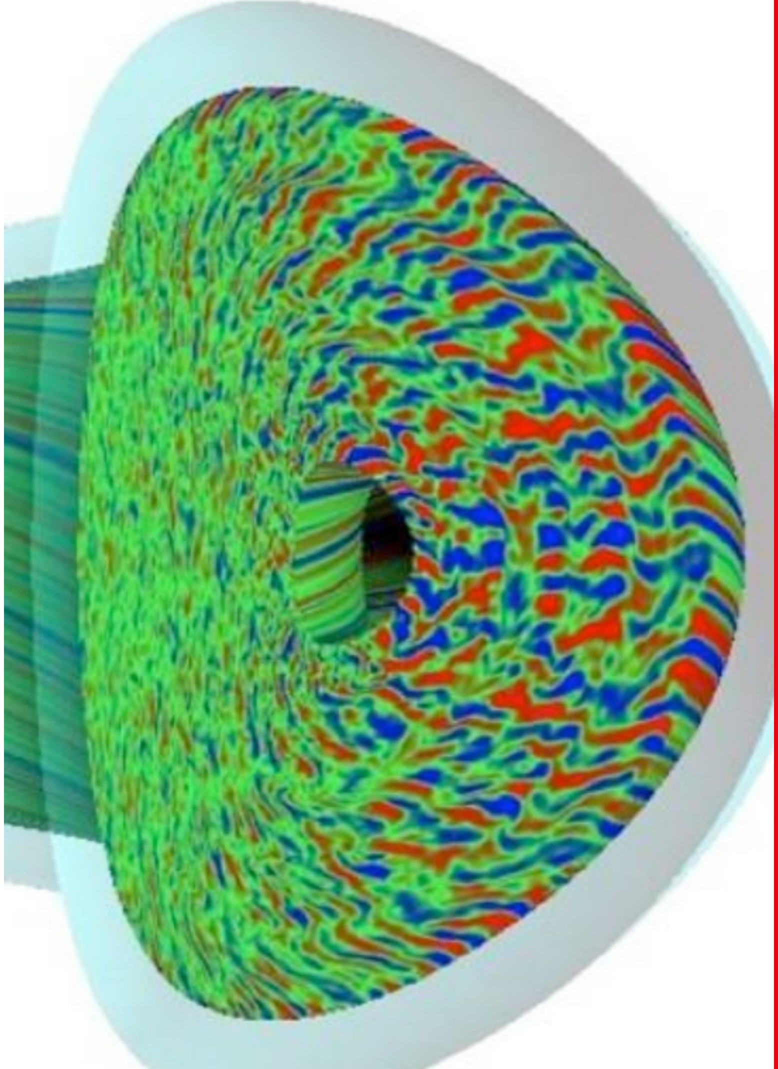
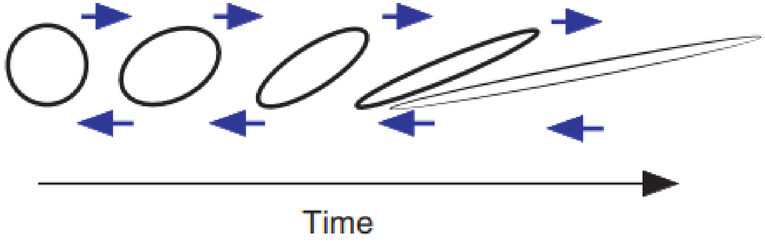
Outline

- Introduction to turbulence stabilization by flow shear
- Finding Low Momentum Diffusivity (LMD) regime using circular geometry
- Combining up-down asymmetry and the LMD regime to stabilize turbulence
- Studies of a MAST equilibrium from shot #24600
- Simulations of preliminary SMART geometry

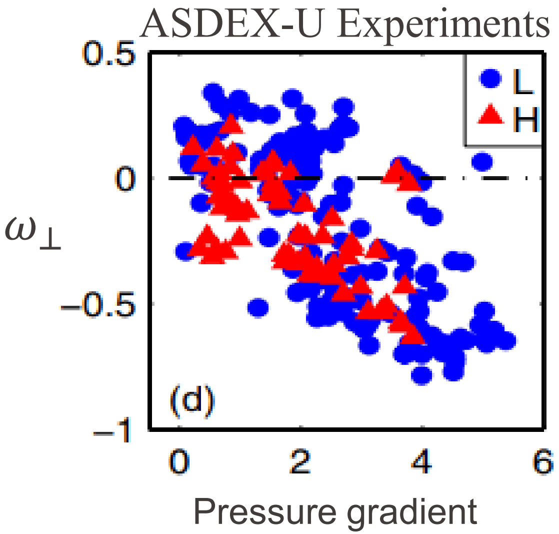


Introduction to turbulence stabilization by flow shear

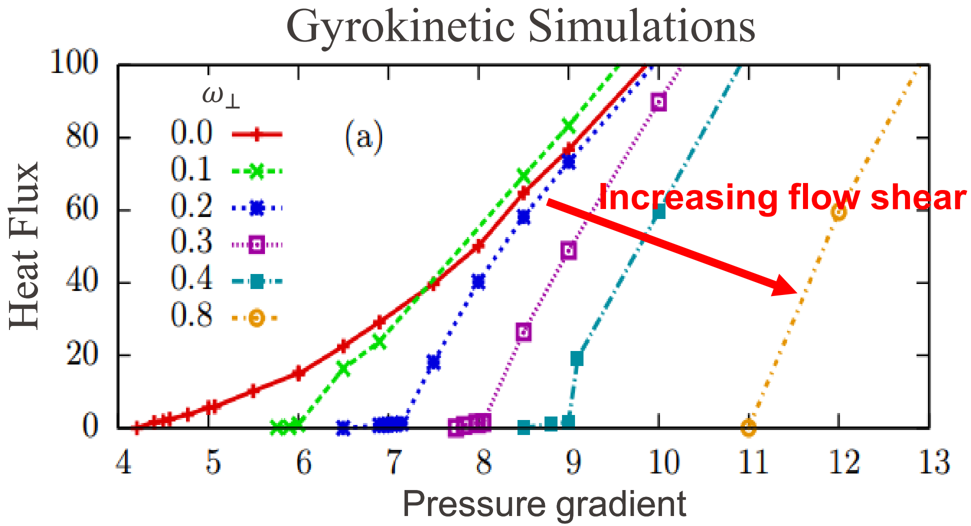
Suppression of turbulence by flow shear



Flow shear $\omega_\perp \sim \partial\Omega/\partial r$ twists turbulence eddies into smaller spatial scale, reducing their amplitude



[Angioni et al., PRL, 2011]

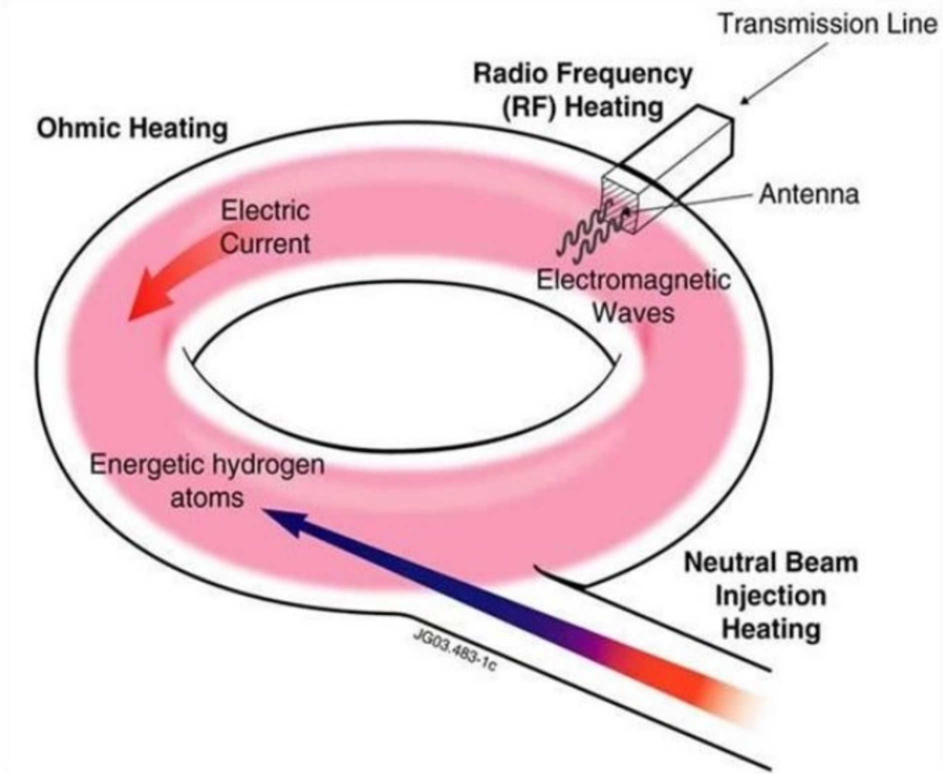


[Highcock et al., PRL, 2010]

Experiments and simulations have shown that flow shear can stabilize turbulence, improving tokamak performance

Generating flow shear using neutral beam/radio frequency waves

[Liu et al., Nuclear Fusion, 2004]



Flow shear is usually generated from external momentum sources such as NBI or RF waves.

External injections do not scale well to large devices: $\frac{\Pi_{inj}}{Q_{inj}} \sim \frac{1}{v} \sim \frac{1}{\sqrt{Q_{inj}}}$

ITER: $\omega_\perp \sim 0.02 v_A/R_0$

Alternatives?

Generate flow shear using up-down asymmetry

Typical expression for momentum flux (Taylor expansion of flow and flow shear)

$$\Pi_i = \boxed{\Pi_{i,int}} - \boxed{n_i m_i R_0^2 D_{\Pi_i} \frac{d\Omega_i}{dr}} - \boxed{\cancel{n_i m_i R_0^2 P_{\Pi_i} \Omega_i}}$$

Intrinsic momentum flux   Turbulent diffusion   Pinch term

In steady state $\Pi_i = 0$, so assuming pinch term is small, we have

$$\Pi_{i,int} = \boxed{D_{\Pi_i}} \frac{d\Omega_i}{dr} n_i m_i R_0^2$$

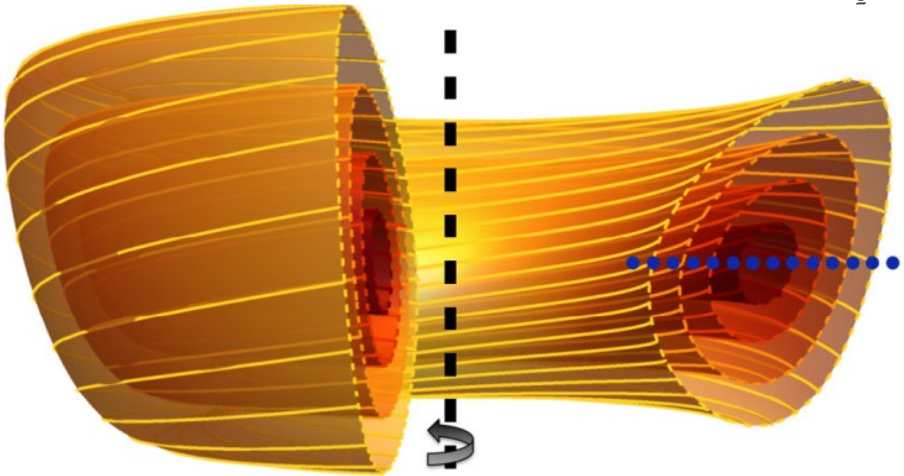
$$Q_i = -D_{Q_i} \frac{dT_i}{dr}$$

Define Prandtl number: $\mathbf{Pr}_i = \frac{D_{\Pi_i}}{D_{Q_i}}$

Prandtl number estimates the rotation relative to turbulence amplitude

- Many people assumed $\mathbf{Pr}_i \approx 1$

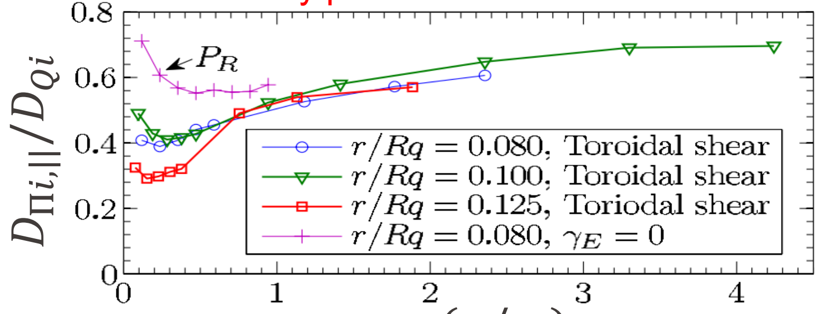
To lowest order in gyrokinetics, $\Pi_{i,int} = 0$ unless magnetic equilibrium is up-down asymmetric



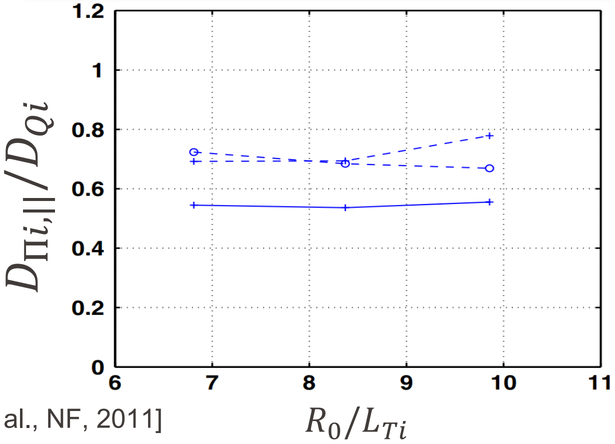
[Parra et al., POP, 2011]
[Ball et al., Nuclear Fusion, 2018]

Low Momentum Diffusivity (LMD) has been observed in recent simulations and experiments

Only parallel momentum flux



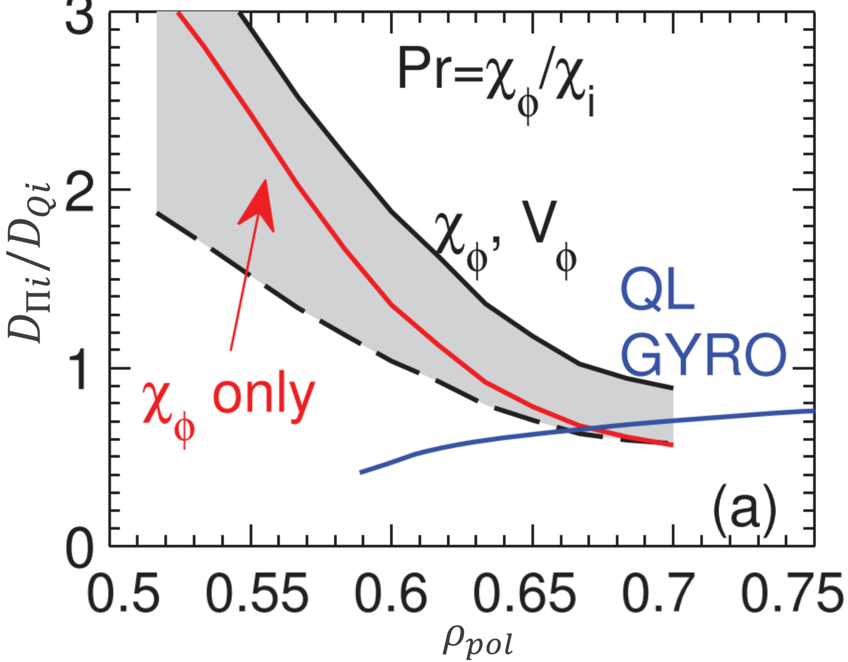
[Casson et al., POP, 2009]



[Camenen et al., NF, 2011]

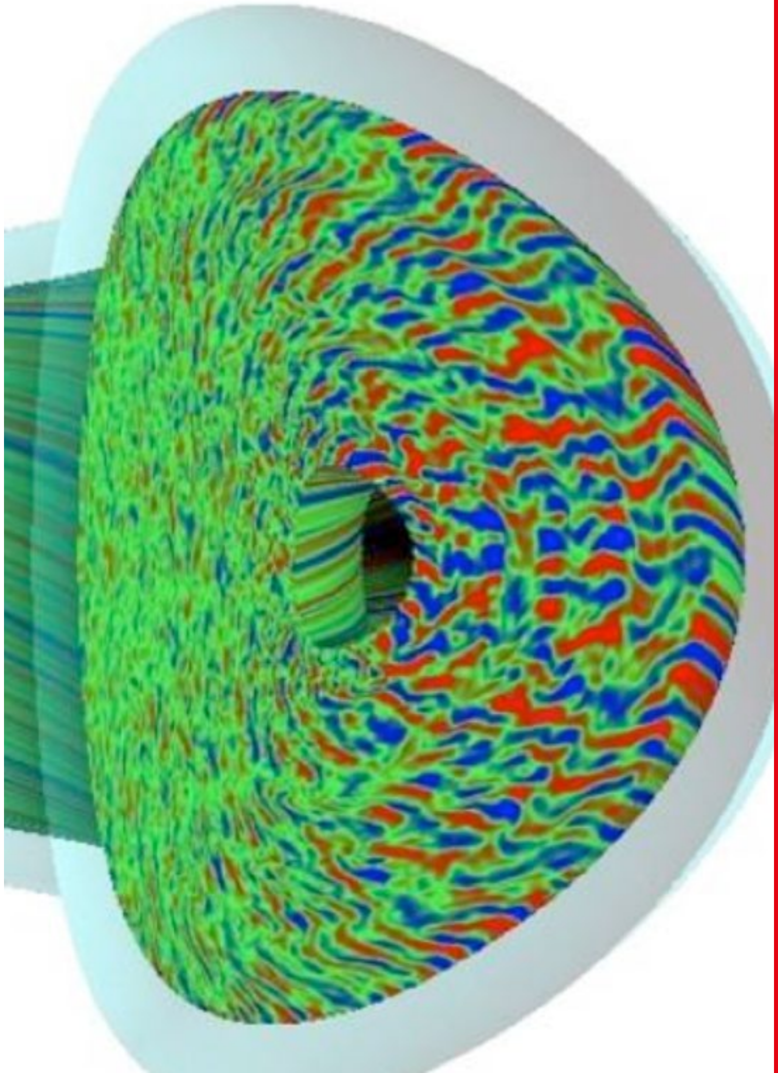
Often simplified calculation of toroidal angular momentum flux

[Guttenfelder et al., NF, 2017]



Experimental data is often hard to interpret

A thorough parameter scan searching for the optimized parameter regime with lowest Prandtl number is required



Finding low momentum diffusivity regime using circular geometry [1,2]

[1] Haomin Sun, Justin Ball, Stephan Brunner et al.,2024, https://doi.org/10.48550/arXiv.2410.10555

[2] Haomin Sun, Justin Ball, Stephan Brunner et al.,2024, https://doi.org/10.48550/arXiv.2408.12331

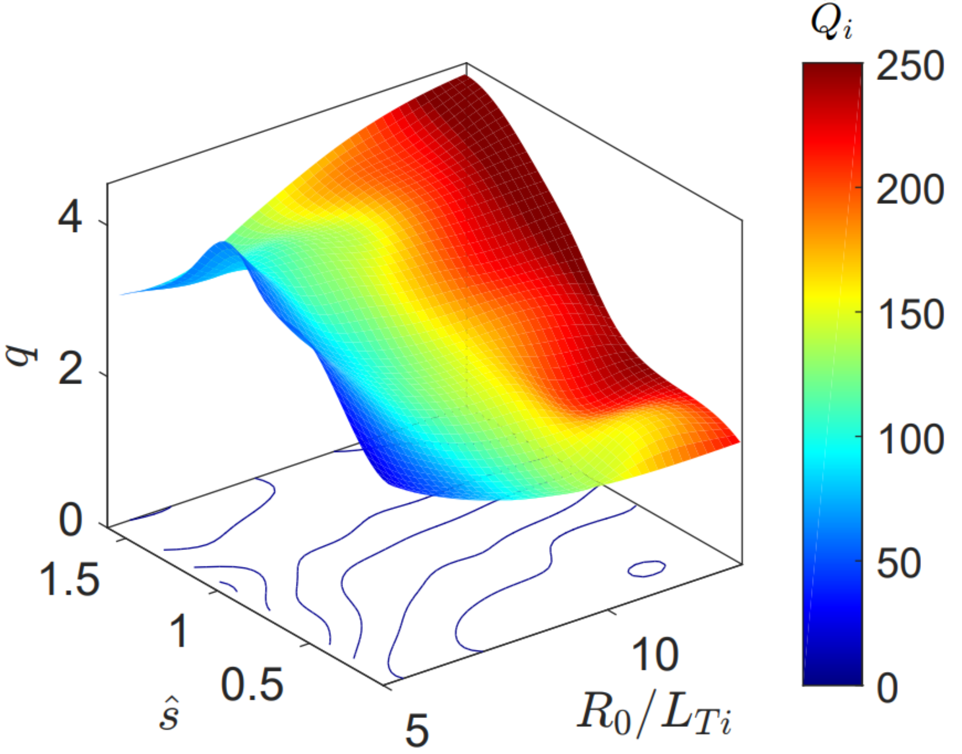
Low Momentum Diffusivity (LMD) manifold

$\mathrm{Pr}_i = 0.5$ **manifold for circular geometry and** $\epsilon = 0.36$

$$\mathrm{Pr}_i = \frac{\hat{\Pi}_{i,tor}}{\hat{Q}_i}\frac{R_0}{L_{Ti}}\frac{\epsilon}{q\omega_\perp}\frac{c_s}{R_0}$$

Flow shear in toroidal direction

Toroidal angular momentum flux



$$\Pi_{i,tor} = m_i \int d^3v R\, \boldsymbol{v}\cdot \boldsymbol{e_\phi} f\, \boldsymbol{v_D}$$

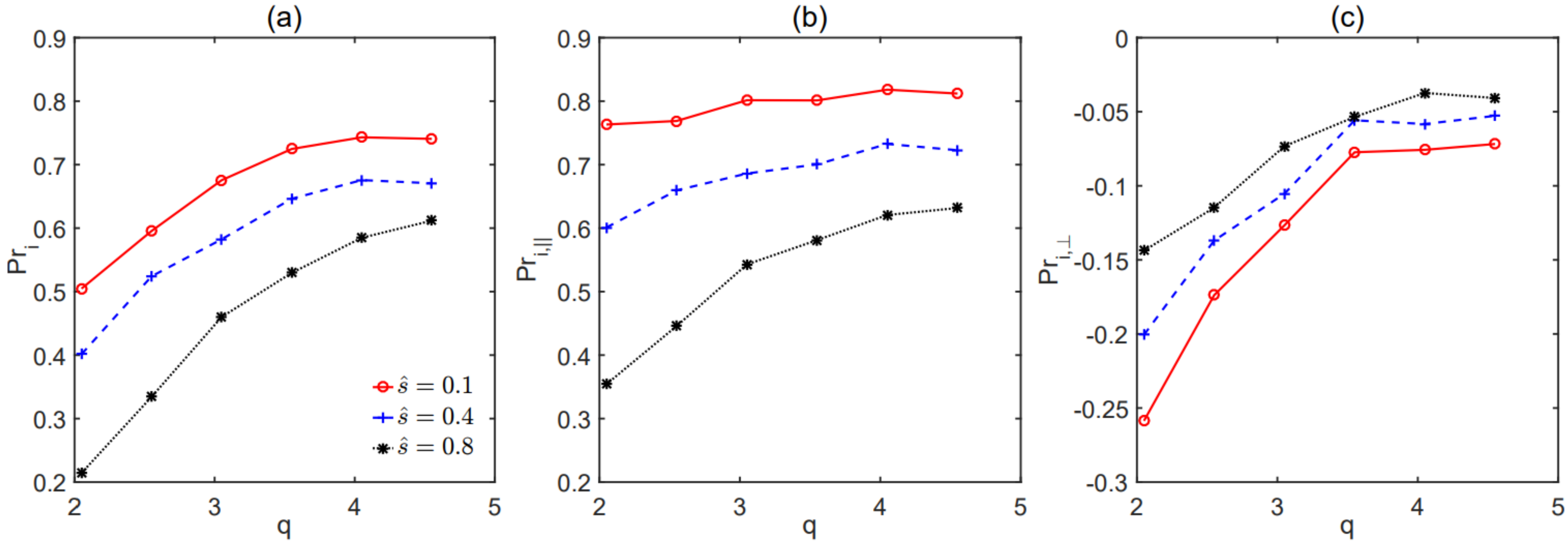
$$= m_i \int d^3v R\, (v_{||} + v_\perp) f\, \boldsymbol{v_D}$$

$$= \Pi^{||}_{i,tor} + \Pi^{\perp}_{i,tor}$$

576 nonlinear GENE simulations with adiabatic electrons.

It is important to consider $\Pi_{\mathrm{i,tor}}^{\perp}$ in Prandtl number calculation at tight aspect ratio

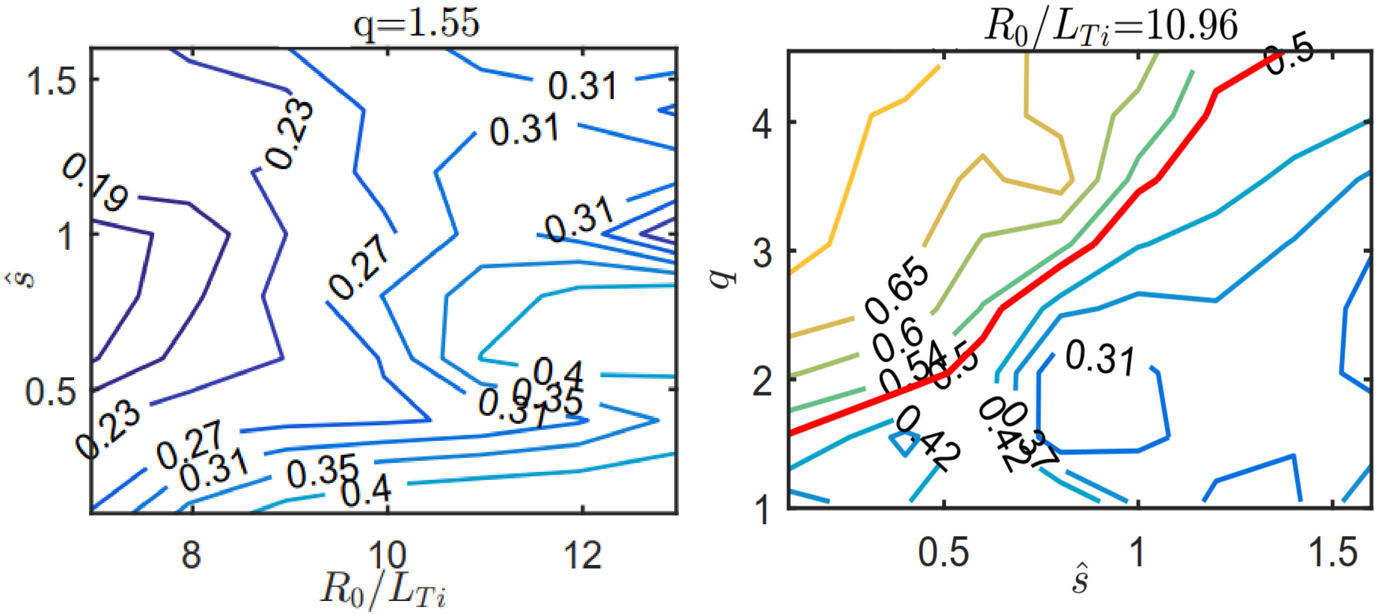
$\frac{R_0}{L_{Ti}} = 6.96, \epsilon = 0.36$, **circular geometry, adiabatic electrons**



$\Pi_{\mathrm{i,tor}}^{\perp}$ becomes important especially in the LMD regime

Low Momentum Diffusivity (LMD) regime
tight aspect ratio, low q, normal to high $\hat{s}$

Contours of Pr_i for circular geometry, $\epsilon = 0.36$

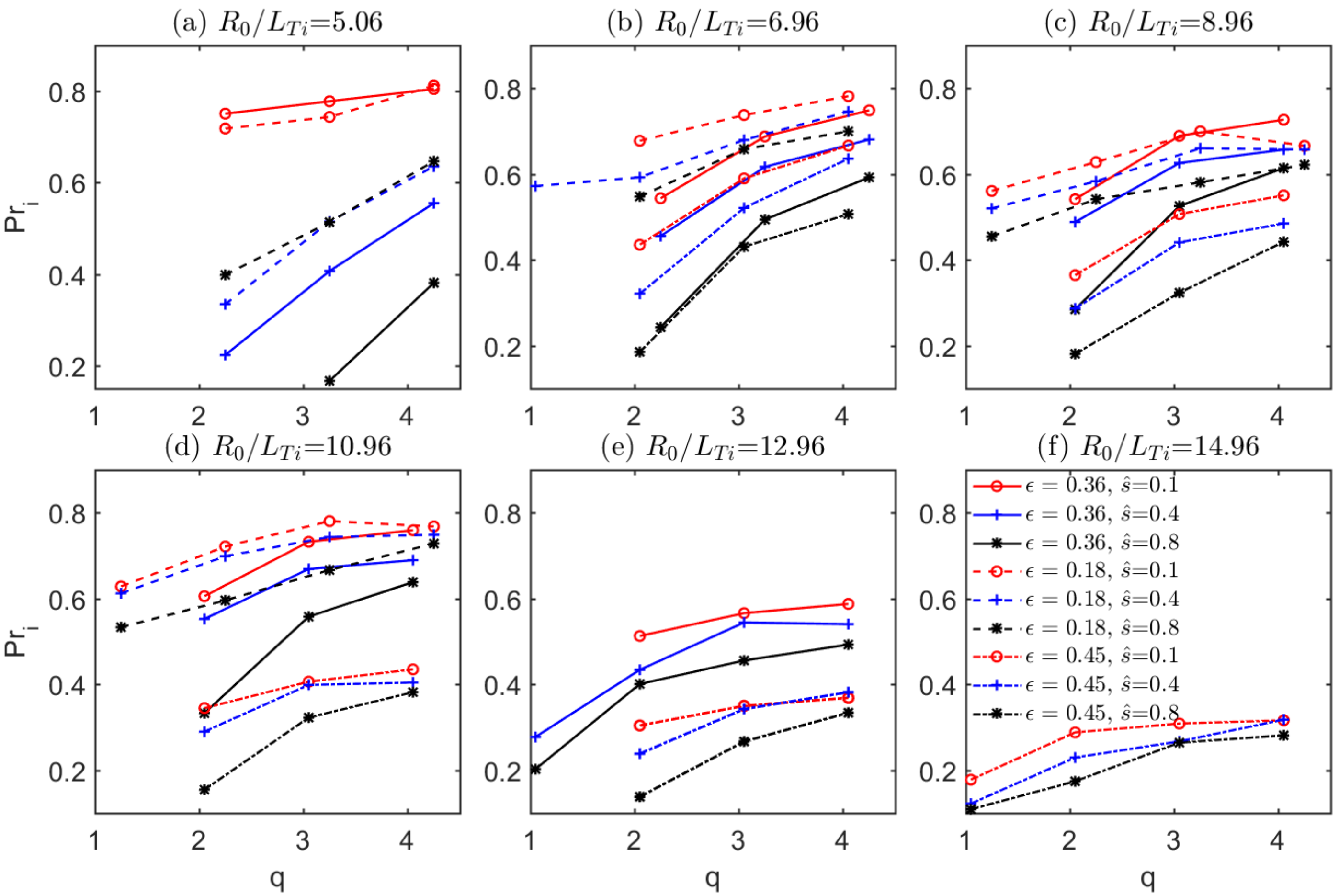


A more comprehensive study inspired by previous work

[McMillan & Dominski, Journal of Plasma Physics, 2019]

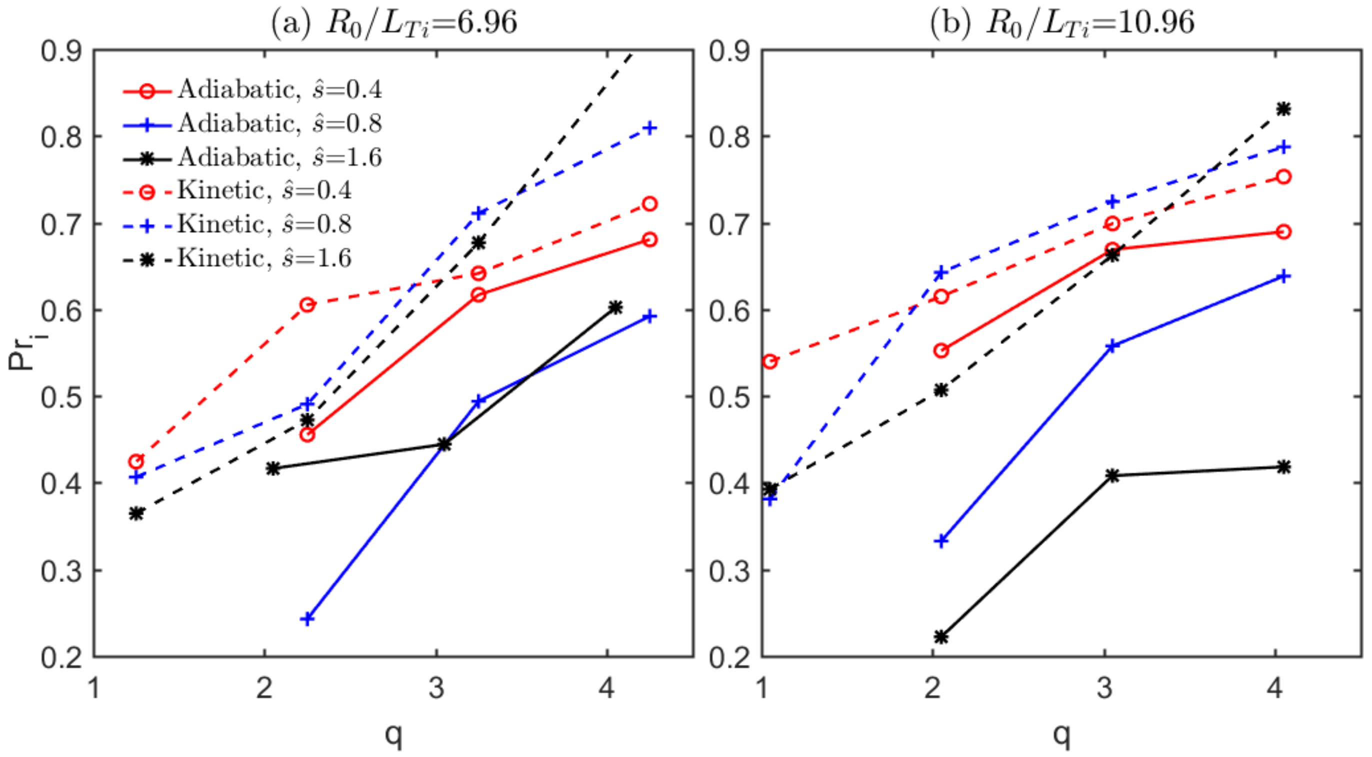
Effects of other parameters (ϵ, kinetic electrons, TEM turbulence)?

Tight aspect ratio reduces Prandtl number



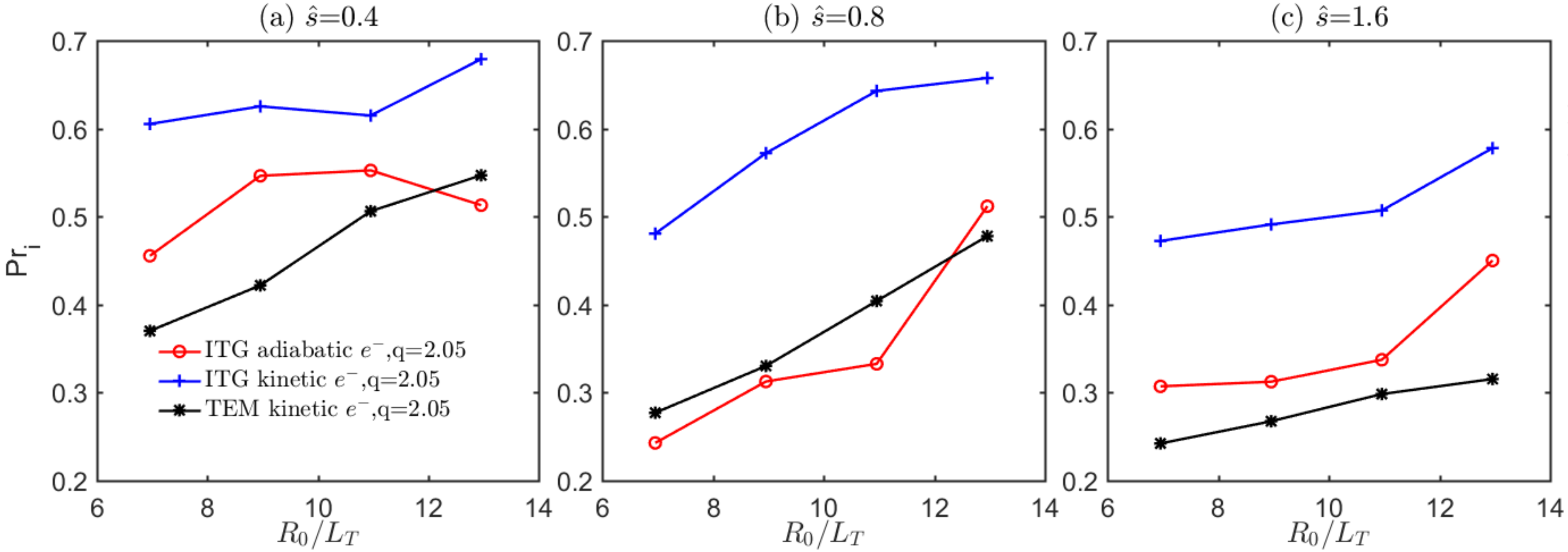
Tight aspect ratio and high magnetic shear reduce Prandtl number

Kinetic electrons increase Prandtl number, but do not affect basic trend



One can still get very small Prandtl number with kinetic electrons.
With kinetic electrons, simulations are further away from marginal stability

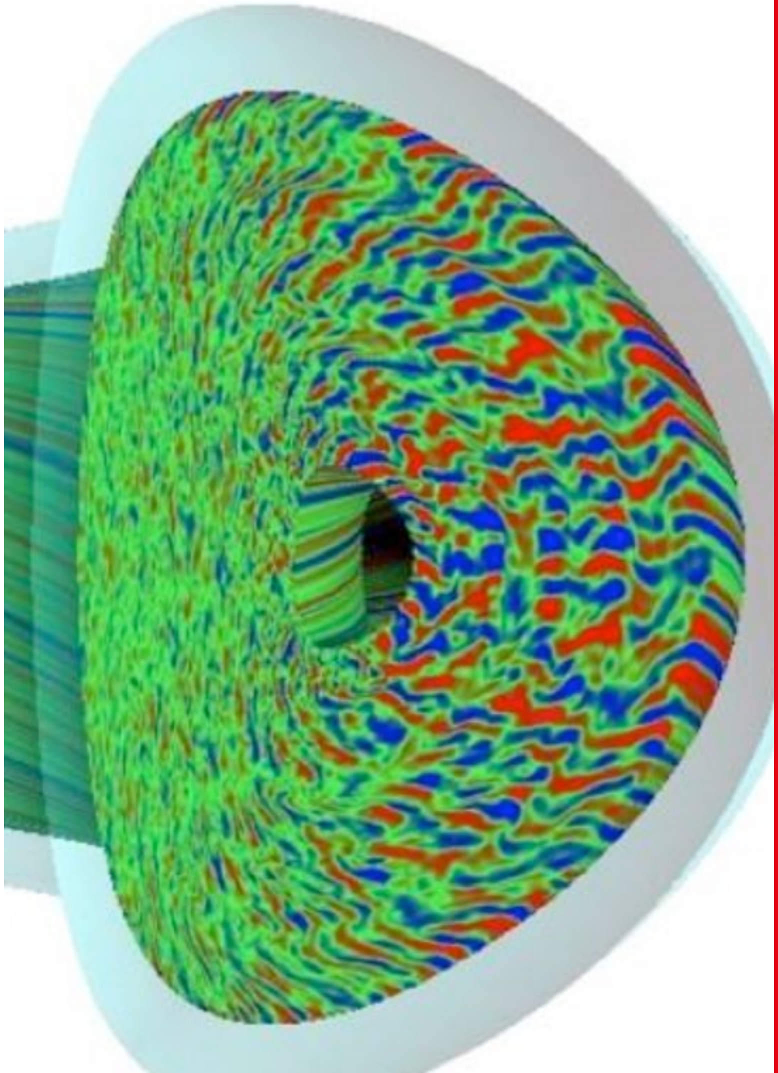
TEM turbulence does not change the Prandtl number significantly



Considered TEM mode with kinetic electrons with $R_0/L_{Ti} = 0$ $\mathbf{Pr}_i = \frac{\widehat{\Pi}_i}{\widehat{Q}_e}\frac{R}{L_{Te}}\frac{\epsilon}{q\omega_\perp}\frac{c_s}{R}$

TEM mode does not change Prandtl number significantly.

- ETG does not contribute to momentum flux because $m_e \ll m_i$

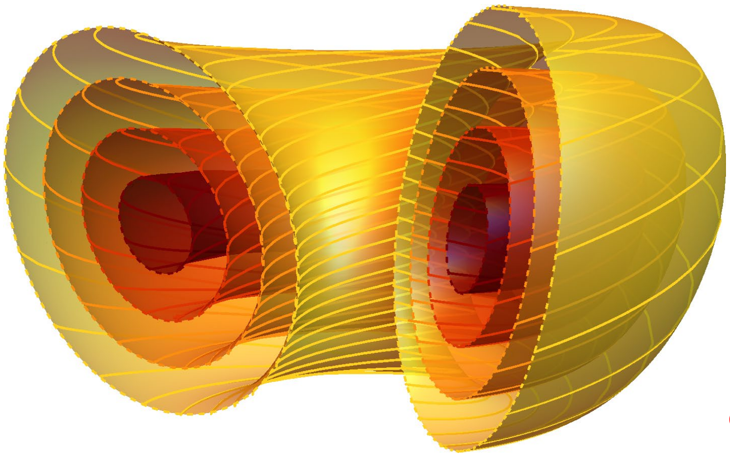


Combining up-down asymmetry and LMD regime to stabilize turbulence [1,2]

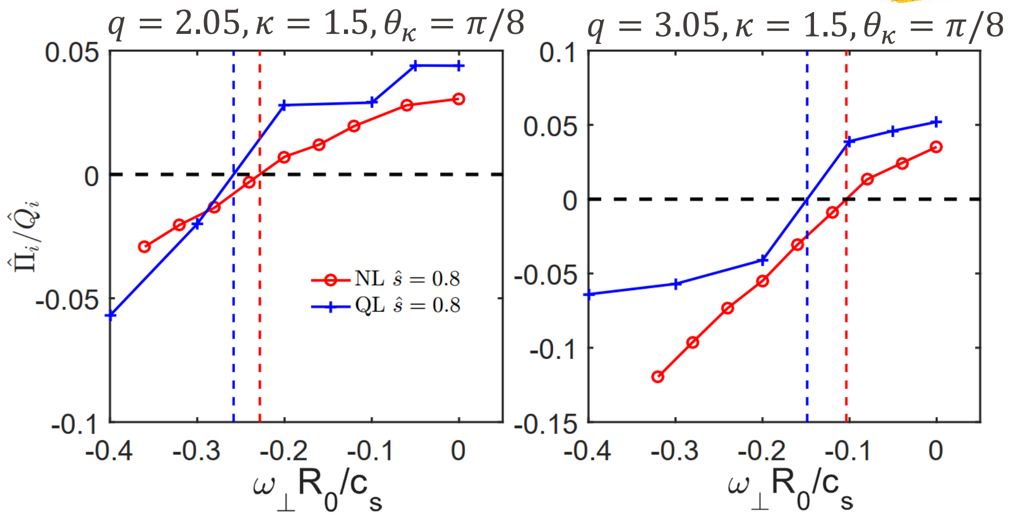
[1] Haomin Sun, Justin Ball, Stephan Brunner et al.,2024, https://doi.org/10.48550/arXiv.2410.10555

[2] Haomin Sun, Justin Ball, Stephan Brunner et al.,2024, https://doi.org/10.48550/arXiv.2408.12331

Using quasilinear (QL) model to estimate flow shear at equilibrium



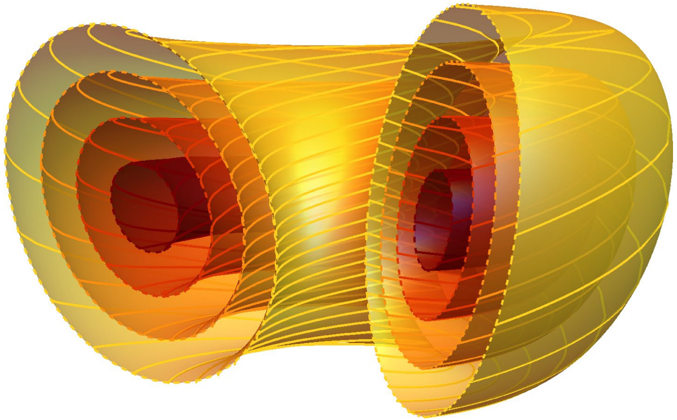
QL model: [Sun et al., NF, 2024]



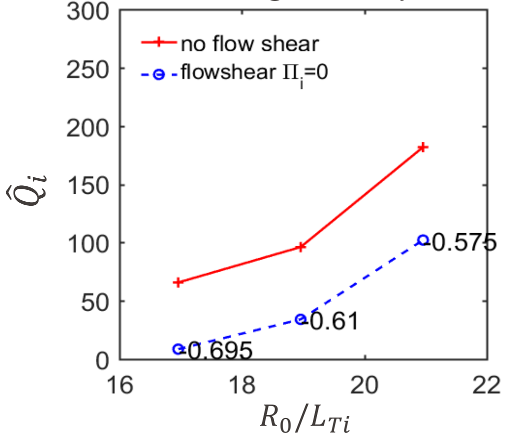
Scan flow shear to find the $\omega_\perp$ value at which $\widehat{\Pi}_i = 0$

Using our QL model to predict the steady-state flow shear, which shows good agreements

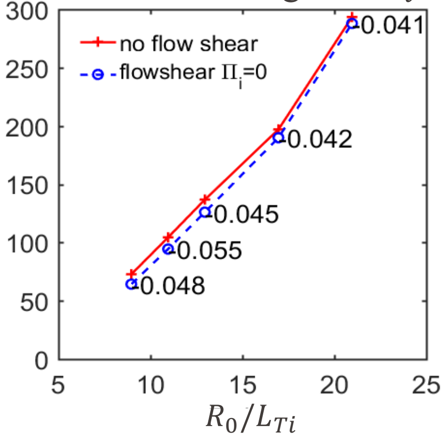
LMD regime with up-down asymmetry drives strong flow shear



In the LMD regime, $Pr_i \approx 0.3$



Out of the LMD regime $Pr_i \approx 1$



$$\boldsymbol{\Pi_i = \Pi_{i,intrinsic} - D_{\Pi_i}\frac{d\Omega_i}{dr}}$$

In the LMD: strong flow shear, significant suppression of turbulence especially near marginal stability

Out of the LMD: weak flow shear, little effect on heat transport

Effect of pinch term on intrinsic rotation

$$\Pi_i = \Pi_{i,int} - n_i m_i R_0^2 D_{\Pi_i} \frac{d\Omega_i}{dx} - n_i m_i R_0^2 P_{\Pi_i} \Omega_i = 0$$

We therefore have

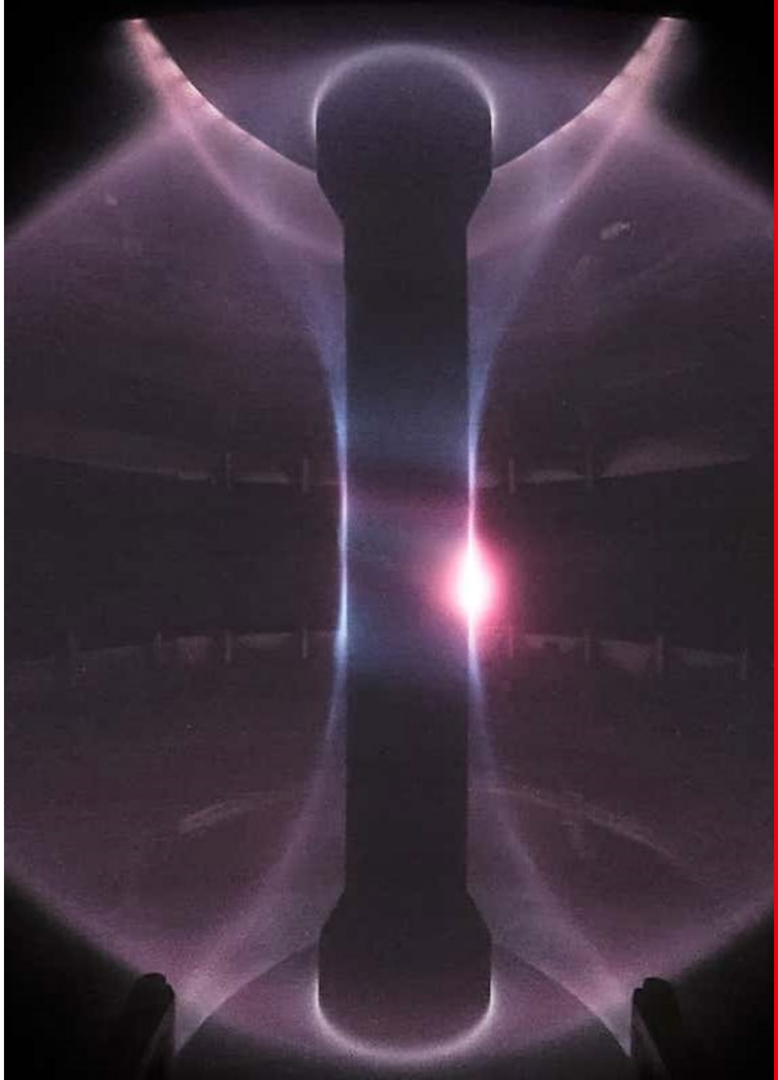
$$\Pi_{i,int} = n_i m_i R_0^2 D_{\Pi_i} \frac{d\Omega_i}{dx} + n_i m_i R_0^2 P_{\Pi_i} \Omega_i$$

Important to note that both D_{Π_i} and P_{Π_i} are positive [Peeters et al., PRL, 2007]

$$\Rightarrow \begin{cases} \Omega_i(x) = -e^{\int_x^a \frac{P_{\Pi i}}{D_{\Pi i}} dx'} \int_x^a \frac{\Pi_{i,int}}{n_i m_i R_0^2 D_{\Pi i}} e^{-\int_{x''}^a \frac{P_{\Pi i}}{D_{\Pi i}} dx'} dx'' + \Omega_{edge} e^{\int_x^a \frac{P_{\Pi i}}{D_{\Pi i}} dx'} \\ \frac{d\Omega_i}{dx}(x) = \frac{P_{\Pi i}}{D_{\Pi i}} e^{\int_x^a \frac{P_{\Pi i}}{D_{\Pi i}} dx'} \int_x^a \frac{\Pi_{i,int}}{n_i m_i R_0^2 D_{\Pi i}} e^{-\int_{x''}^a \frac{P_{\Pi i}}{D_{\Pi i}} dx'} dx'' + \frac{\boxed{\Pi_{i,int}}}{n_i m_i R_0^2 D_{\Pi i}} - \Omega_{edge} \frac{P_{\Pi i}}{D_{\Pi i}} e^{\int_x^a \frac{P_{\Pi i}}{D_{\Pi i}} dx'} \end{cases}$$

Flip up-down symmetry changes its sign

Considering pinch term will only make the intrinsic rotation stronger



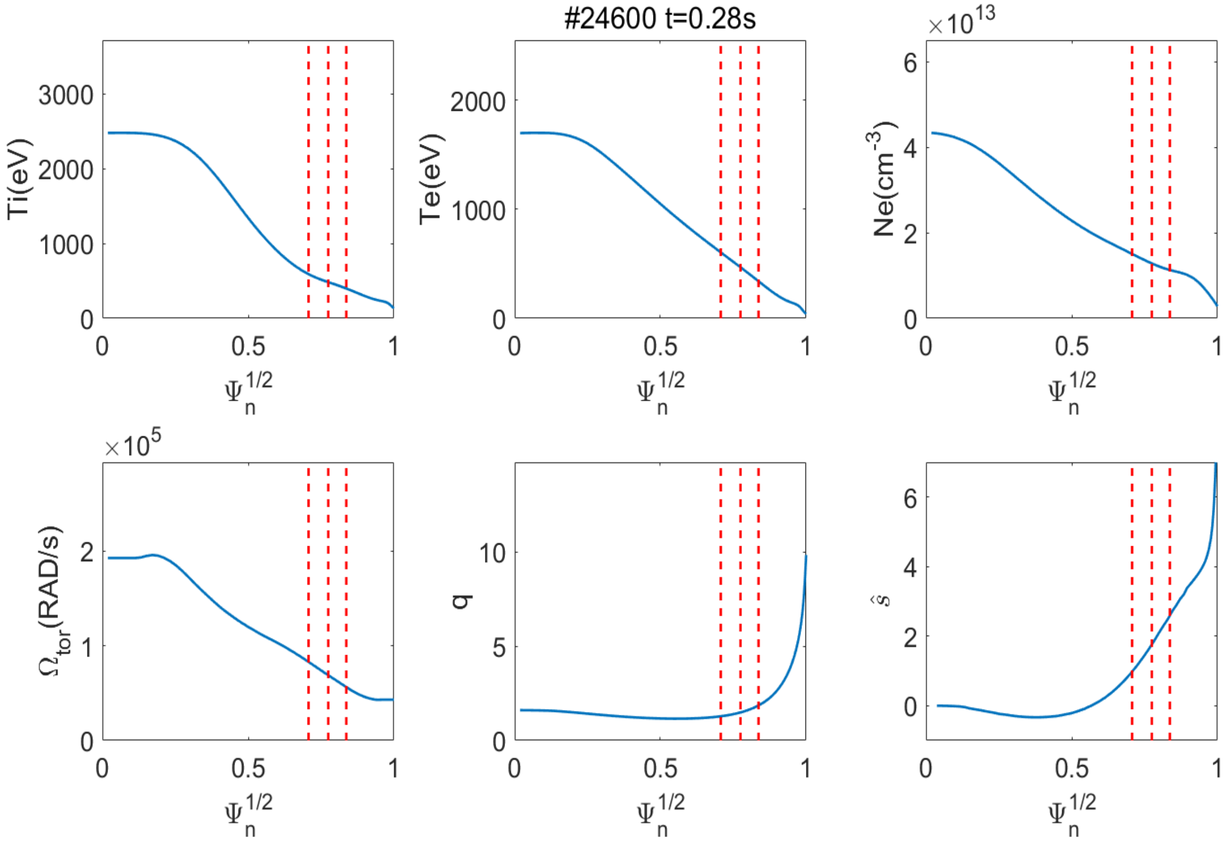
Test if MAST #24600 is in the LMD regime [1,2]

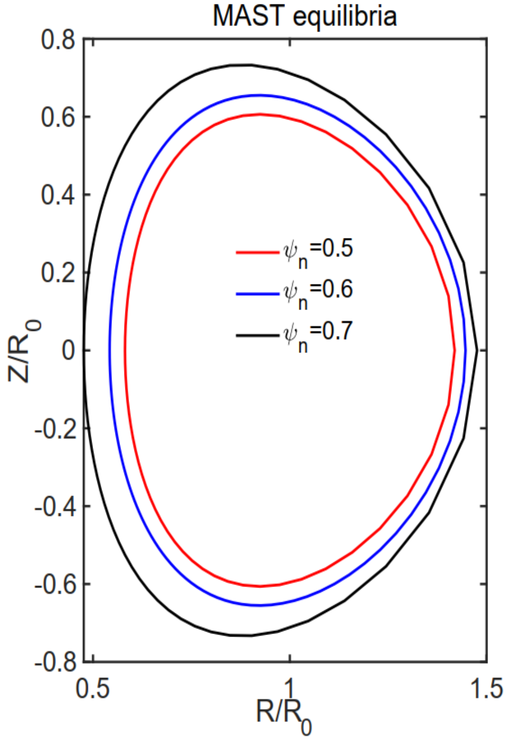
[1] Haomin Sun, Justin Ball, Stephan Brunner et al.,2024, https://doi.org/10.48550/arXiv.2410.10555

[2] Haomin Sun, Justin Ball, Stephan Brunner et al.,2024, https://doi.org/10.48550/arXiv.2408.12331

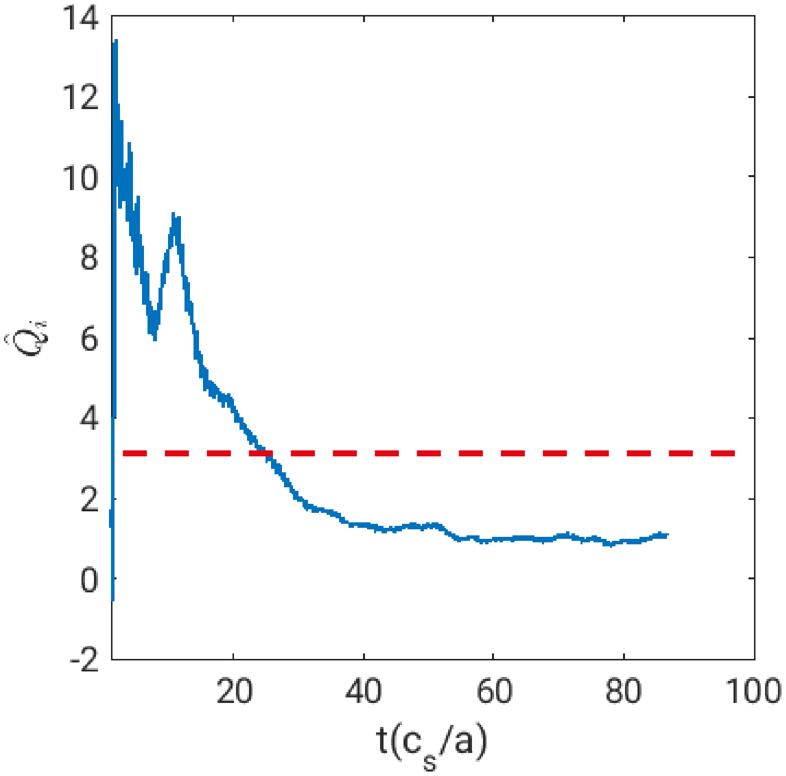
Simulate MAST #24600 at t=0.28s

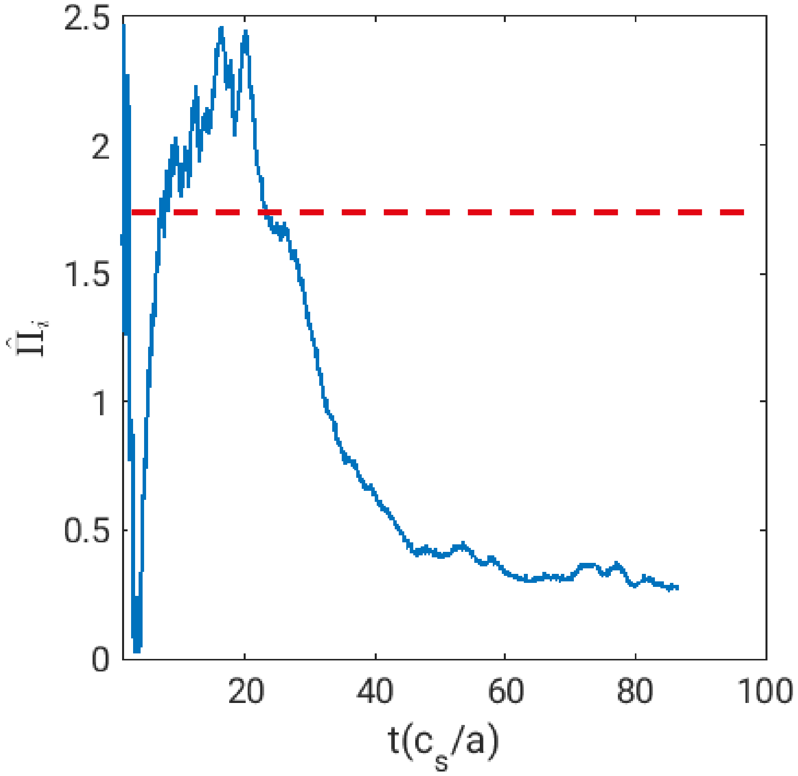
Chose #24600 at t=0.28s as it has a large radial range with low q, and is in a quasi-steady state, nearly free of MHD instabilities





Benchmark with experiment using measured flow shear at $\psi_n = 0.5$

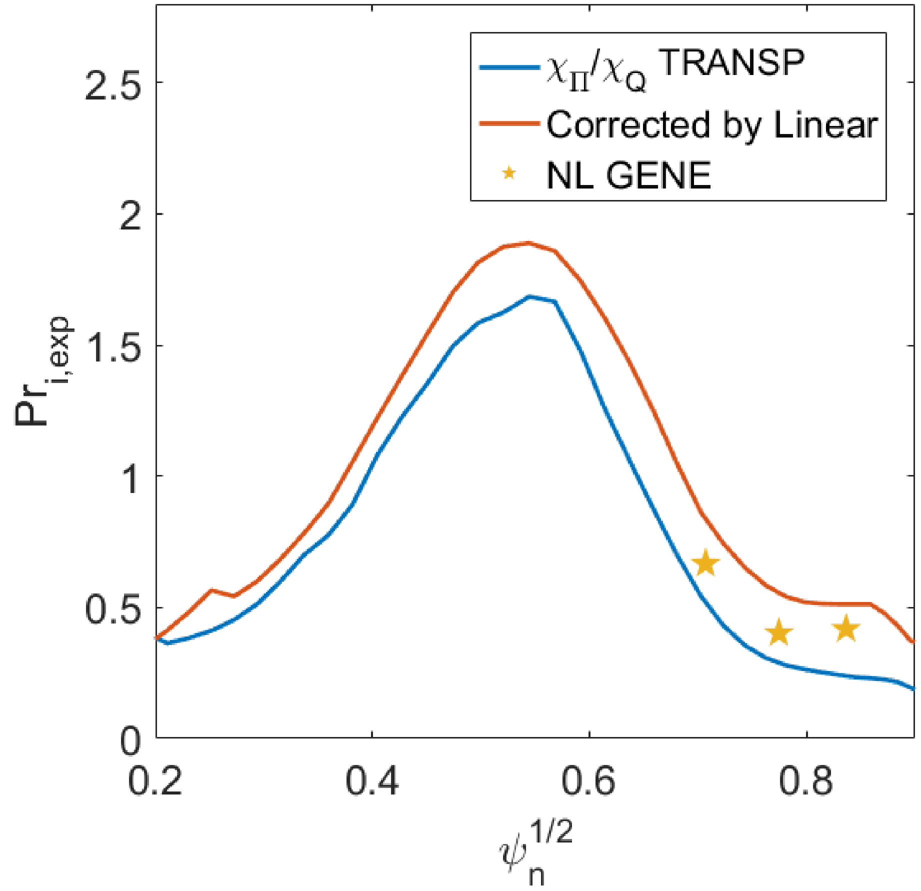




Exp value: $\hat{Q}_i = \frac{Q_i}{Q_{gB}} = 3.06$

Exp value: $\hat{\Pi}_i = \frac{\Pi_i}{\Pi_{gB}} = 1.74$

Prandtl number comparison



[Peeters et al., 2007, PRL]

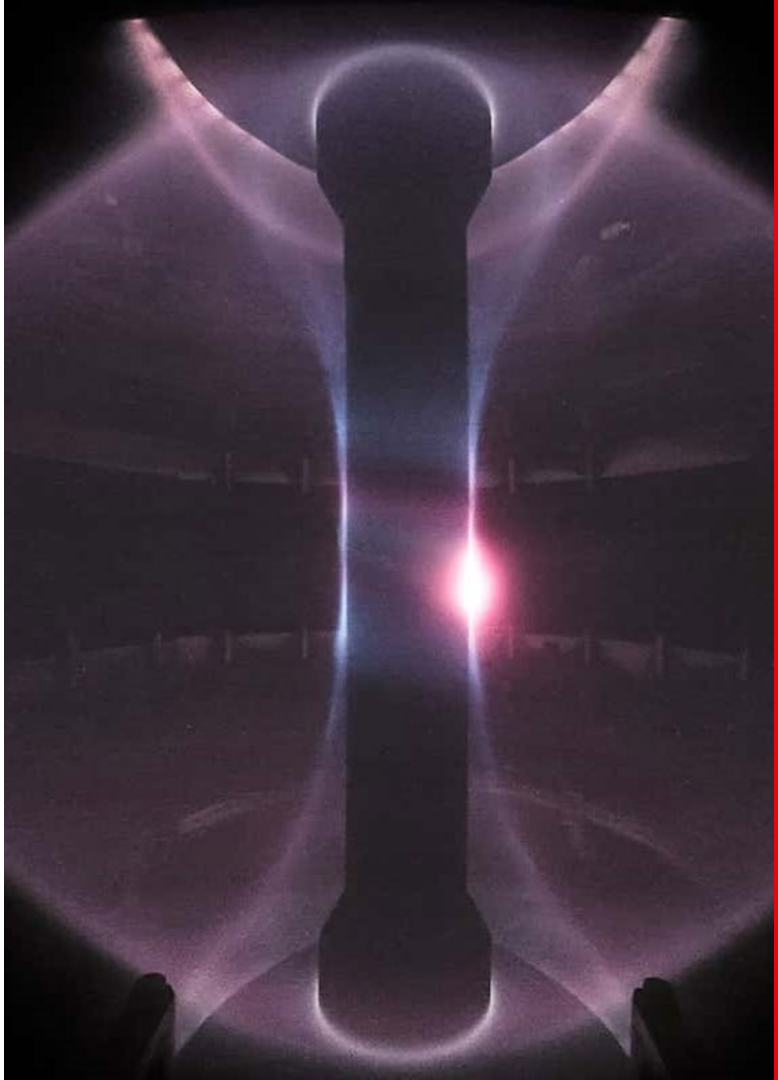
$$\boldsymbol{\Pi_i = \chi_\Pi u' + V_{pinch} u}$$

TRANSP calculates

$$\chi_{\Pi,eff} = \chi_{\Pi,real}\left(1 + \frac{RV_{pinch}}{\chi_{\Pi,\mathrm{real}}}\frac{1}{R/L_u}\right)$$

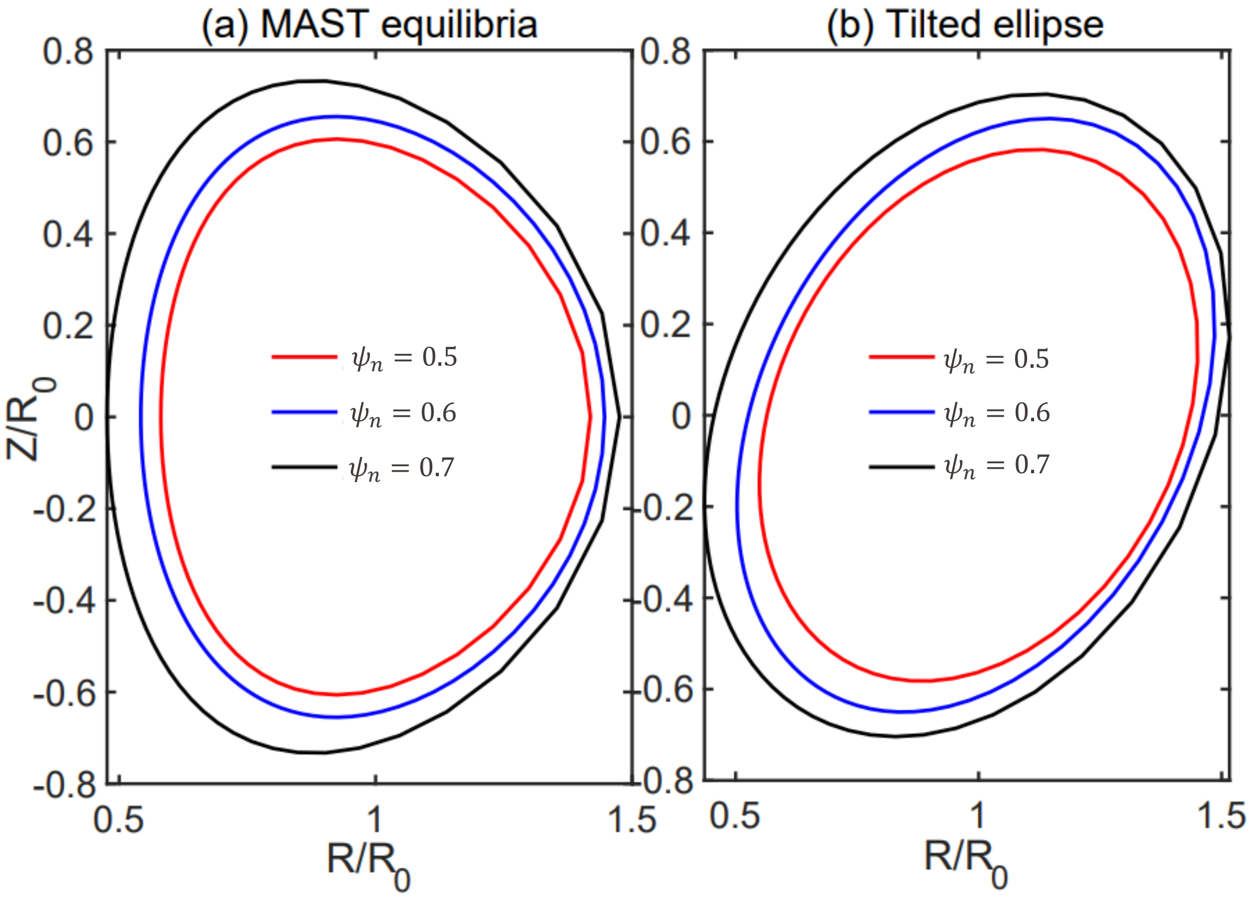
Linearly estimated pinch term using a given $k_y \rho_i = 0.3$, and then corrected the experimental Prandtl number

A low Prandtl number can be obtained on MAST.



Tilt the MAST geometry

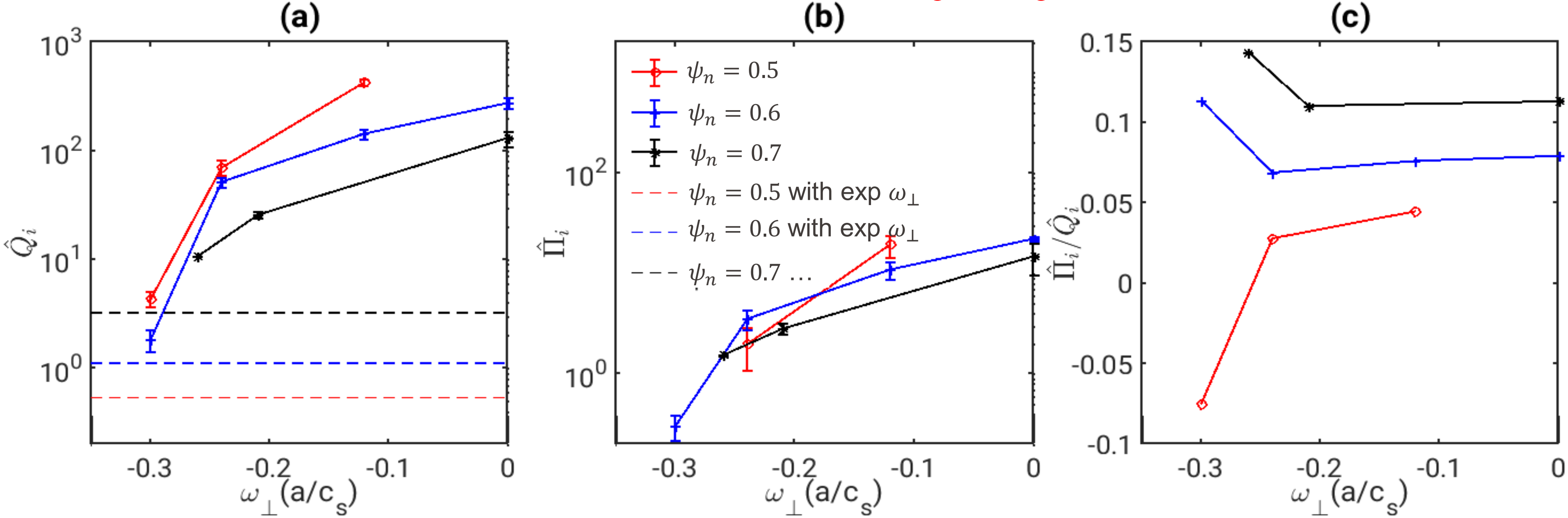
Artificially tilt MAST geometry to study intrinsic flow shear



$\theta_\kappa = \pi/8$ No triangularity

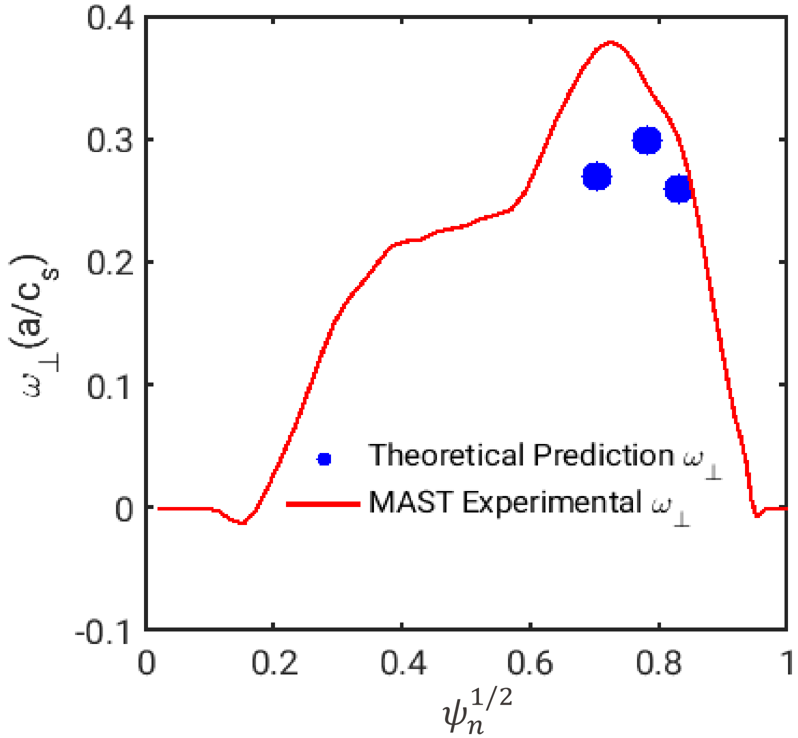
Predicting flow shear generated by up-down asymmetry

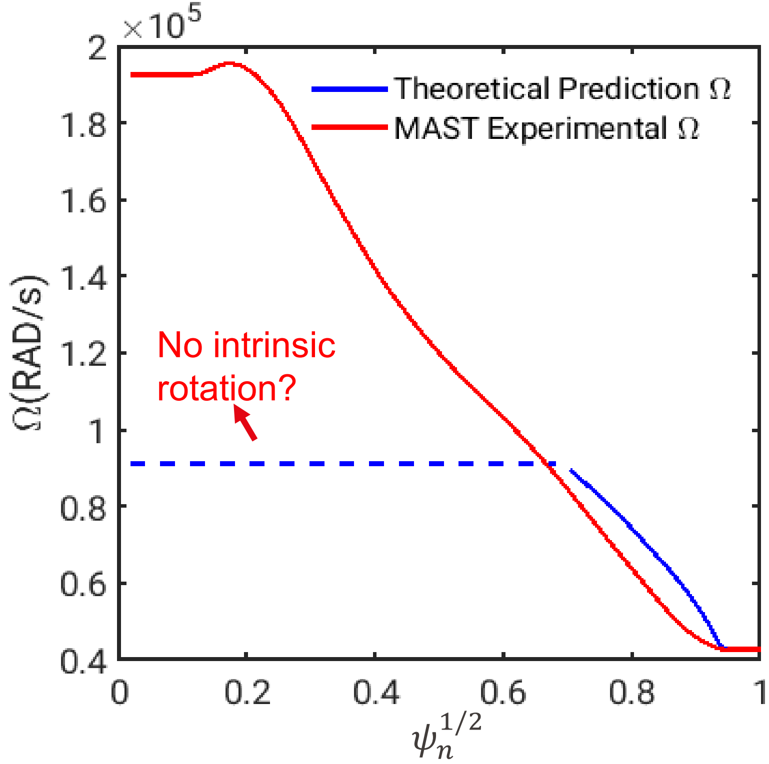
Errorbars come from rolling average



Radial Location ψ_n	Theoretical Prediction $\omega_\perp$ (c_s/a, by up-down asymmetry)	Experimental $\omega_\perp$ (c_s/a, by NBI)
0.5	−0.27	−0.37
0.6	−0.30	−0.34
0.7	−0.26	−0.30

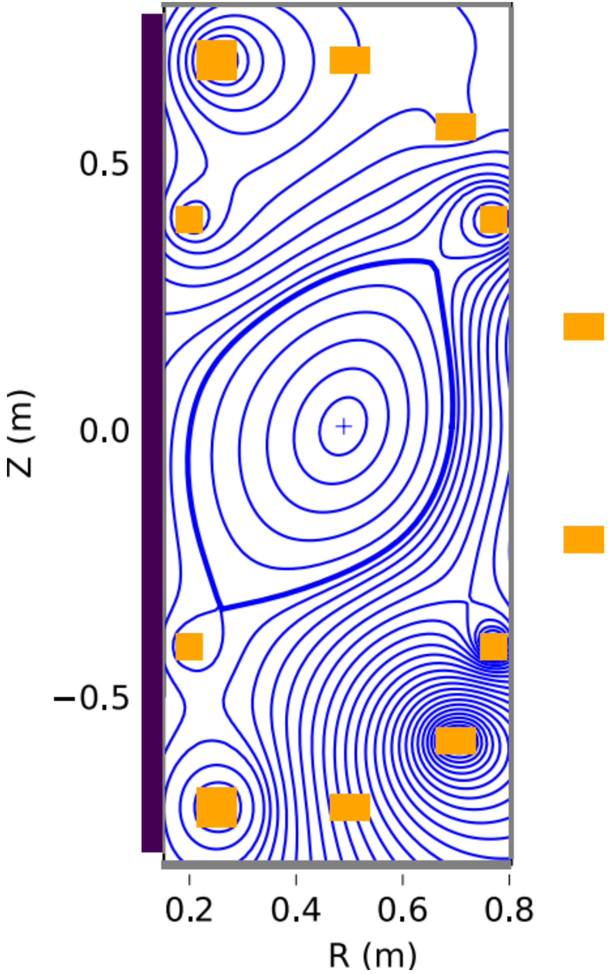
Reconstruct rotation profile





At least 1/3 of the experimental rotation can be generated

For larger devices, red curves are lower but blue dots are expected to remain the same



SMART preliminary geometry simulation [1,2]

[1] Haomin Sun, Justin Ball, Stephan Brunner et al.,2024, https://doi.org/10.48550/arXiv.2410.10555

[2] Haomin Sun, Justin Ball, Stephan Brunner et al.,2024, https://doi.org/10.48550/arXiv.2408.12331

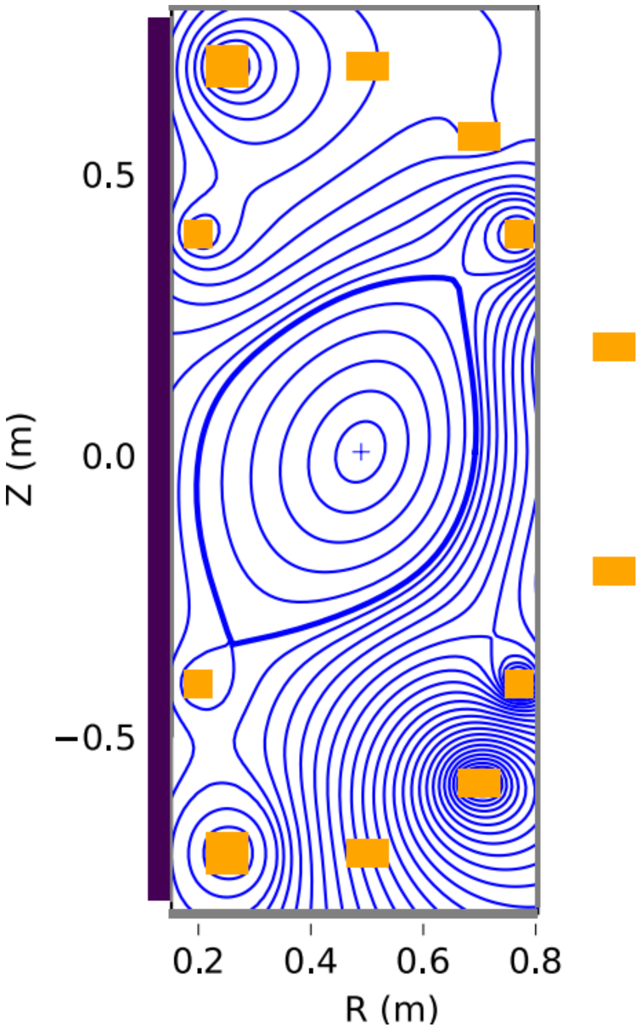
Parameters

$\kappa \approx 1.39$

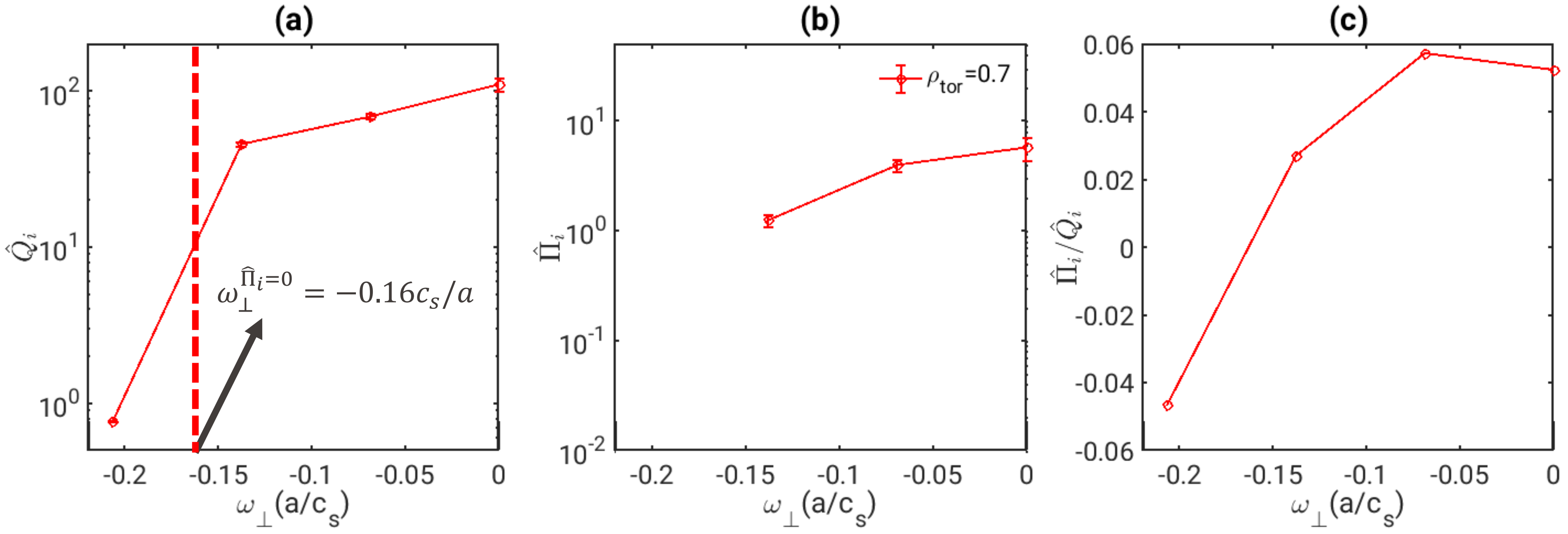
$\theta_\kappa \approx 0.51 = 29^o$

At $\rho_{tor} = 0.7$, we have $\epsilon = 0.393, q = 1.33, \hat{s} = 1.25$ (LMD regime)

Use miller general geometry for GENE simulations



Summary of SMART simulations



Flow shear created: SMART: $0.16c_s/a$ MAST (hypothetical): $0.26c_s/a$ TCV: $0.03c_s/a$

Conclusions

https://arxiv.org/abs/2408.12331 https://arxiv.org/abs/2410.10555

- Outlined a new approach to drive strong flow shear in large spherical tokamaks
- Prandtl number can be much smaller than 1, termed the Low Momentum Diffusivity (LMD) regime
  - Enabled by tight aspect ratio, low q, high $\hat{s}$, and low $\frac{R_0}{L_{Ti}}$
- Combining the LMD regime with up-down asymmetry creates intrinsic flow shear that significantly reduces the heat flux
- Simulations of MAST and SMART show they can exhibit LMD
- Hypothetical tilted MAST showed flow shear stabilization
- Studied a tilted geometry that may be achievable on SMART, which also demonstrated flow shear stabilization

Useful for STEP design?

Full expression of EM toroidal angular momentum flux

Haomin Sun

$$\Pi_s = -\left\langle\left\langle\left\langle\int d^3v\, f_s m_s R\left(\vec{v}\cdot\hat{e}_\xi\right)\left(\vec{v}\cdot\nabla\psi\right)\right\rangle_\psi\right\rangle_{\Delta\psi}\right\rangle_{\Delta t}$$

$$\begin{aligned}\Pi_{\zeta s} = \frac{4\pi^2 i}{V'}\Bigg\langle\sum_{k_\psi,k_\alpha} k_\alpha \oint d\theta JB \int dw_{||} d\mu\, h_s\left(-k_\psi,-k_\alpha\right) \\ \times\Bigg\{\phi\left(k_\psi,k_\alpha\right)\left[\left(\frac{I}{B}w_{||}+R^2\Omega_\zeta\right)J_0\left(k_\perp\rho_s\right)+\frac{i}{\Omega_s}\frac{k^\psi}{B}\frac{\mu B}{m_s}\frac{2J_1\left(k_\perp\rho_s\right)}{k_\perp\rho_s}\right] \\ - A_{||}\left(k_\psi,k_\alpha\right)\left[\left(\frac{I}{B}w_{||}+R^2\Omega_\zeta\right)w_{||}J_0\left(k_\perp\rho_s\right)+\left(i\frac{w_{||}}{\Omega_s}\frac{k^\psi}{B}+\frac{I}{B}\right)\frac{\mu B}{m_s}\frac{2J_1\left(k_\perp\rho_s\right)}{k_\perp\rho_s}\right] \\ + B_{||}\left(k_\psi,k_\alpha\right)\frac{1}{\Omega_s}\left[\left(\frac{I}{B}w_{||}+R^2\Omega_\zeta\right)\frac{\mu B}{m_s}\frac{2J_1\left(k_\perp\rho_s\right)}{k_\perp\rho_s}+\frac{i}{2\Omega_s}\frac{k^\psi}{B}\frac{\mu^2B^2}{m_s^2}G\left(k_\perp\rho_s\right)\right]\Bigg\}\Bigg\rangle_{\Delta t}\end{aligned}$$

$$\begin{aligned}\Pi_{\zeta B} = \frac{2\pi i}{\mu_0 V'}\Bigg\langle\sum_{k_\psi,k_\alpha} k_\alpha \oint d\theta J A_{||}\left(k_\psi,k_\alpha\right) \\ \times\left[-ik^\psi A_{||}\left(-k_\psi,-k_\alpha\right)+IB_{||}\left(-k_\psi,-k_\alpha\right)\right]\Bigg\rangle_{\Delta t}\end{aligned}$$

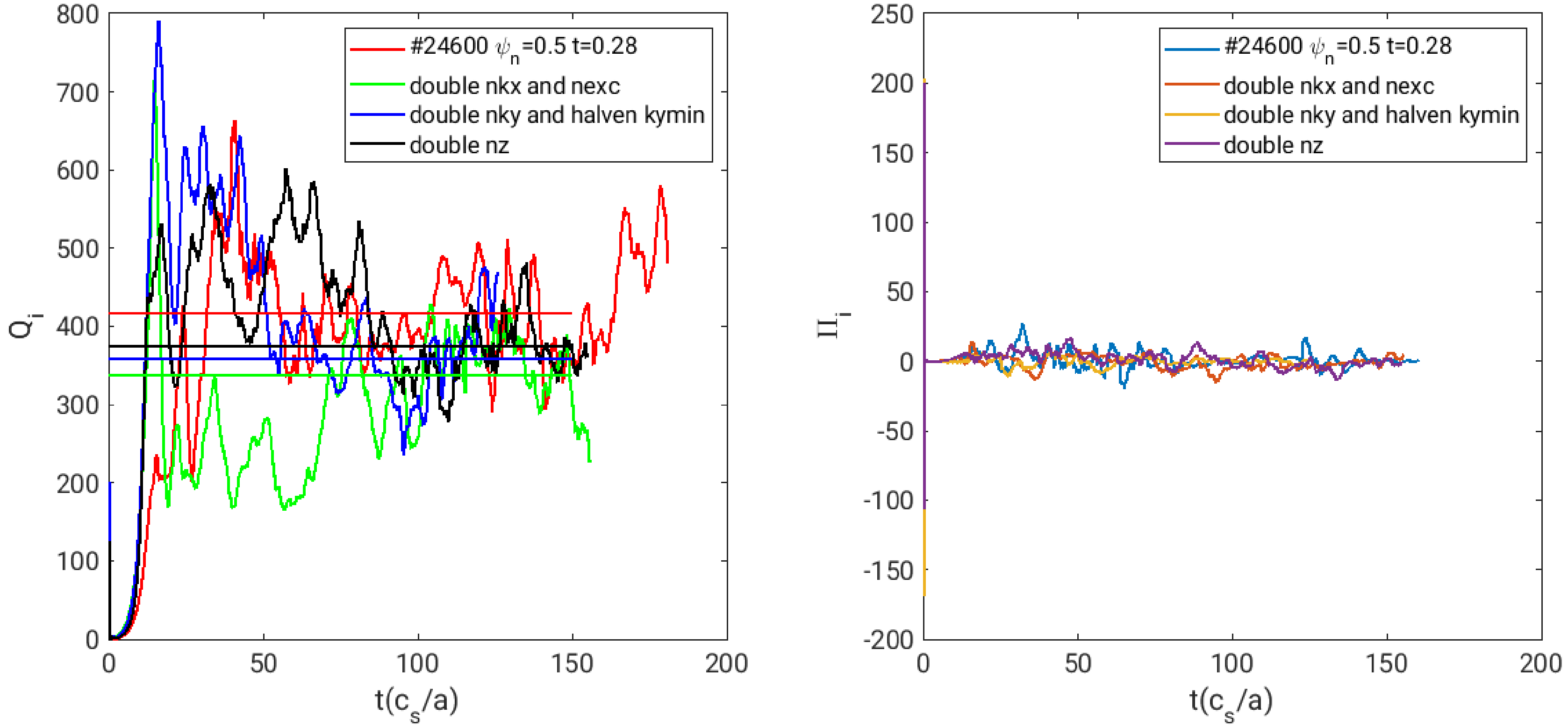
$$h_s = H_s - \frac{Z_s e F_{Ms}}{T_s}\left(\bar{\phi}-\langle\bar{\phi}\rangle_\varphi\right)+\mu\frac{F_{Ms}}{T_s}\langle\bar{B}_{||}\rangle_\varphi$$ Pull back operation

H_s: distribution in guiding center coordinate

Note: Not really self-consistently written, because the φ dependence of h_s and other parts must be integrated together

[Parra et al., 2011; Ball PhD thesis 2016; Sugama & Horton 1998]

$\psi_n = 0.5$ Nonlinear Simulations, realistic geometry, no flow shear



- Converge grid: $192 \times 64 \times 64 \times 32 \times 16$