Parity-Broken Fluids: Theory and Applications

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Outline

• Motivation
• Hydrodynamics
• Odd (gyro) viscosity
• Ferrofluids
• Hele-Shaw Cell
• Microscopic Origins
• Superfluids

Goal:  Parity-breaking (odd viscosity) is common to many systems and leads to a general type of observable phenomena
Our Research Group

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Also: Sudheesh Srivastava, Chunyin Chan, Zeshui Song

Research Interests:
• Hydrodynamics of parity broken flows
• Active matter and odd viscosity
• Boundary dynamics of fractional quantum Hall fluids
• Kuramoto oscillators and machine learning

Nonlinear shallow water dynamics with odd viscosity
Gustavo M. Monteiro and Sriram Ganeshan
Phys. Rev. Fluids 6, L092401 – Published 7 September 2021

Hele-Shaw flow for parity odd three-dimensional fluids
Dylan Reynolds, Gustavo M. Monteiro, and Sriram Ganeshan
Phys. Rev. Fluids 7, 114201 – Published 16 November 2022

Kardar-Parisi-Zhang Universality at the Edge of Laughlin States.
Monteiro, G. M., Reynolds, D., Glorioso, P., & Ganeshan, S.
preprint arXiv:2305.16511 May 2023
Parity Breaking “spinny particles”

Ubiquitous across all scales...

Some possible methods of study:

Odd ideal gasses – collisions of chiral particles, kinetic theory [5]


Odd fluids - odd viscosity [7]

Parity Breaking in Plasmas

- If Larmor radius is smaller than plasma length scale → Gyro Viscosity

- Split into two terms, perpendicular to $B$-field and parallel to $B$-field [8]
  \[
  \eta_{\alpha \perp} = \frac{v_{\alpha}^2}{2\omega_{\alpha}} \quad \eta_{\alpha \parallel} = \frac{v_{\alpha}^2}{\omega_{\alpha}} \quad \omega_{\alpha} = \frac{e_{\alpha}B}{m_{\alpha}}
  \]

- Contributes to gyro viscous stress in Braginskii Eq. ($B$-field along $z$ direction)

\[
\frac{1}{n_{\alpha}m_{\alpha}}(\nabla \cdot \pi)_{\perp} = -\left(\eta_{\alpha \perp} \nabla_{\perp}^2 + \eta_{\alpha \parallel} \frac{\partial^2}{\partial z^2}\right) v_{\alpha} \times z - \eta_{\alpha \parallel} (\nabla \times z) \frac{\partial}{\partial z} v_{\alpha z}
\]

\[
\frac{1}{n_{\alpha}m_{\alpha}}(\nabla \cdot \pi)_{z} = -\eta_{\alpha \parallel} \frac{\partial}{\partial z} z \cdot (\nabla \times v_{\alpha}),
\]

- Dissipationless, forces always perpendicular to flow and appears in real part of dispersion

Note: Units of $\eta_{\alpha \perp}$ are $L^2/T$

Gyro viscosity is like specific angular momentum

\[
e vB = \frac{mv^2}{R} \quad Rv = \frac{mv^2}{eB}
\]

Active matter

• Collection of agents that convert environment energy into directed mechanical motion
  
  *bacteria, cells, liquid crystals, Janus particles, active colloids, spinny particles...*

• Fluid description $\rightarrow$ properties of active matter captured by viscosity coefficients

• Studied extensively in 2D, less so in 3D

• Markovich & Lubensky 2021 [9]:
  
  - Coarse-grain collection of complex molecules
  
  - Introduce intrinsic angular momentum density $\vec{\ell}$
  
  - Gives parity-broken viscosity in both 2D and 3D

Possible microscopic descriptions

3D odd viscosity can arise from relaxation of $\vec{\ell}$:

$$\dot{g}_i(r) = \sum_{\alpha\mu} p_{i}^{\alpha\mu} \delta (r - r^{\alpha\mu})$$

which gives “coarse grained” momentum

$$g(r) = g^c + \frac{1}{2} \nabla \times \ell$$

If $\vec{\ell}$ is relaxed, then momentum conservation gives:

$$\dot{g}_i + \nabla_j (v_j g_i) = -\nabla_i \tilde{P} + \eta \nabla^2 v_i + \frac{1}{2} \ell \cdot \nabla \omega_i$$

with $\tilde{P} \equiv P + \ell \cdot \omega$

Note:
- Same form as ferrofluid (later)
- 3D odd viscosity is stemming from angular momentum; common to most 3D cases, including gyro viscosity

Hydrodynamics

Study of global conserved quantities $Q$:
\[
\frac{dQ}{dt} = 0
\]

Gives rise to local conservation laws for density $q$:
\[
\partial_t q + \partial_i \Pi_i^q = 0
\]

Flux $\Pi_i^q (q, v_i, ...)$ given by constitutive relation

We focus on mass and momentum:
\[
\partial_t \rho + \partial_i g_i = 0 \quad \quad g_i = \rho v_i
\]
\[
\partial_t g_i + \partial_j \Pi_{ij} = 0 \quad \quad \Pi_{ij} = \rho v_i v_j - T_{ij}
\]

Ignore any thermal effects, so energy conservation comes automatically

\[
Q = \int d\vec{r} \ q(\vec{r}) : \\
\text{mass} \\
\text{momentum} \\
\text{energy} \\
\text{entropy} \\
\text{angular momentum} \\
... \\
\rho = \text{mass density} \\
v_i = \text{flow velocity} \\
g_i = \text{momentum density} \\
\Pi_{ij} = \text{momentum flux}
Hydrodynamics

Governing equations typically written:

$$\partial_t \rho + \partial_i (\rho v_i) = 0$$

$$\rho D_t v_i = \partial_j T_{ij} + f_i$$

with material derivative $D_t = \partial_t + v_j \partial_j$

In first order hydro $T_{ij}$ is first order in gradients of velocity:

$$T_{ij} = -\delta_{ij} P + \eta_{ijkl} \partial_k v_l + \cdots$$

Viscosity tensor $\eta_{ijkl}$ is in general a function of $\rho$ and $\mathbf{r}$

Simplifications:

- If compressible:

  $$P = P_0 + c^2 (\rho - \rho_0) + \cdots$$

- If incompressible:

  $$\partial_i v_i = 0$$
Viscosity Tensor in 2D

- In 2D compressible fluid a general viscosity tensor $\eta_{ijkl}$ has 16 independent components.

- For isotropic 2D fluids symmetry leaves only 6 unique transport coefficients [10]

$$\eta_{ijkl} = \eta(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}) + \zeta\delta_{ij}\delta_{kl} + \Gamma_e \epsilon_{ij}\epsilon_{kl} + \eta_H(\epsilon_{ik}\delta_{jl} + \epsilon_{jl}\delta_{ik}) + \zeta_H \delta_{ij}\epsilon_{kl} + \Gamma_H \epsilon_{ij}\delta_{kl}$$

  
  \begin{tabular}{c|c|c|c|c} 
    Shear & Bulk & Rotational & Odd/Hall & Odd pressure & Odd torque \\
  \end{tabular}

- In the incompressible limit, and no internal torque ($T_{ij}$ symmetric under $i \leftrightarrow j$)

$$\eta_{ijkl} = \eta(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk}) + \eta_H(\epsilon_{ik}\delta_{jl} + \epsilon_{jl}\delta_{ik}) + \zeta_H \delta_{ij}\epsilon_{kl}$$

$$\partial_i v_l = 0, \quad \delta_{kl} \rightarrow 0$$

- To be written as a Hamiltonian system, only $\eta_H$ remains (with constant $\rho$)

$$T_{ij} \sim \eta_{ijkl} \partial_k v_l$$

Basic Results in 2D

Odd viscosity has no effect on incompressible bulk:

\[ D_t v_i = -\partial_i P + \partial_j \sigma_{ij}, \quad \sigma_{ij} = \nu_e (\partial_i v_j + \partial_j v_i) + \nu_o (\partial_i v_j^* + \partial_j^* v_i) \]

\[ D_t v_i = -\partial_i \tilde{P} + \nu_e \nabla^2 v_i, \quad \tilde{P} = P - \nu_o \omega \]

Thus, if boundary conditions depend only on flow:

- Flow does not depend on \( \nu_o \)
- Net force on closed contour is independent of \( \nu_o \)
- Net torque depends on change in area

\[ \tau_o = 2 \nu_o \frac{dA}{dt} \]

Odd viscosity does not produce any drag or lift forces in the incompressible case

Results in 2D

• For a compressible ocean (free surface), effects of odd viscosity or confined to compressible boundary layer [12]

• In the nonlinear regime, odd viscosity modifies the KdV equation and soliton solutions [13]

\[ \eta_x \pm \frac{\eta_t}{\sqrt{gh}} + \frac{3}{2h} \eta \frac{\eta_x}{\sqrt{gh}} + h^2 \left( \frac{1}{6} \pm \frac{\nu_o}{\sqrt{gh^3}} \right) \eta_{xxx} = 0 \]

• If particle number conservation is violated, lift is possible [14]

\[ \partial_t \rho + \partial_k (\rho v_k) = -\frac{1}{\kappa} (\rho - \rho_0) \]

• Current project: Odd compressible Oseen equation

\[ \tilde{v} = U_0 \hat{x} + \epsilon \tilde{u} \]

Viscosity Tensor in 3D

- In 3D, a general viscosity tensor $\eta_{ijkl}$ has 81 independent components (compressible case)

- For rotational invariance in a single plane ($xy$), 19 independent coefficients [15]

- Importantly, there exist 2 ‘odd viscosity’ coefficients

- Underlying microscopic theory determines the precise relation between coefficients

Results in 3D

• Bulk flow is modified (Stokeslet) [15]

\[ \gamma = 0 \quad \gamma = 1 \quad \gamma = \infty \]

\[ \mathbf{F} = -\hat{z} F_z \delta^3(\mathbf{x}) \]

\[ \gamma = \frac{\eta_2}{\mu} , \text{ with } \eta_1^0 = -2\eta_2^0 \]

• Novel “odd” mechanical waves are stable [9]

• Still active area of research; flows without “−2” constraint, and for non-isotropic not understood


3D Odd Viscosity in Plasmas

Role of gyro viscosity depends on relevant scaling (ordering) of the problem

\[ \delta = \frac{R}{L} = \frac{\text{ion Larmor Radius}}{\text{length scale}} \]

• Hall MHD Ordering (large accelerations): gyro viscosity enters with pressure at 2\(^{\text{nd}}\) order
• MHD Ordering (subsonic flows): gyro viscosity enters alone at 2\(^{\text{nd}}\) order
• Drift Ordering: gyro viscosity enters at 2\(^{\text{nd}}\) order WITH acceleration and viscosity

\[ \nabla p + \mathbf{J} \times \mathbf{B} = \delta^2 \left( n \frac{dV_i}{dt} + \nabla \cdot \Pi_i^{gy} + b \cdot \nabla \cdot \Pi_i^{nc} - n\mu_A \nabla^2 V_i \right) \]

**gyroviscous cancellation:**

\[ n \left( \frac{\partial V_i}{\partial t} + V_i \cdot \nabla V_i \right) + \nabla \cdot \Pi_i^{gy}(V_i) \approx \nabla \chi - b n V_i \cdot \nabla V_i \]

\[ \chi = -p_i b \cdot (\nabla \times V_i) \]

\[ V_e = V_{i,e} + V_{E} + V_{*e} \]

[16] Ordered Fluid Equations D. D. Schnack
Magnetic Colloids to Ferrofluids

Magnetic colloids [17]:

- These relatively large (~μm) particles show odd viscosity experimentally.
- For small particles like Ferrofluids (~nm) these effects ‘washout’.

Seem to possess all essential ingredients for parity breaking.

Is there a version of Ferrofluids that breaks parity?

Ferrofluids in a nutshell

Colloidal suspension with 3 components

• 5% Magnetized particles (magnetite, hematite, ~10 nm)
• 10% Surfactant (prevents clumping via Van der Waals)
• 85% Carrier fluid

*Regular fluid if no external B field*: Brownian Motion randomizes magnetic moments

*Large scale magnetization in the presence of external B field*


$\text{arXiv:2301.07096}$
Ferrohydrodynamics

Degrees of Freedom

Fluid flow velocity: \( \vec{v} = (v_x, v_y, v_z) \)

Particle rotation rate: \( \vec{\Omega} = (\Omega_x, \Omega_y, \Omega_z) \)

Magnetization: \( \vec{M} = (M_x, M_y, M_z) \)

Pressure: \( P = P(x, y, z) \)

Fixed Quantities

Applied magnetic field: \( B = (B_x, B_y, B_z) \)

Density: \( \rho = \text{const.} \)

Moment of inertia per mass: \( I = \text{const.} \) (small!)

arXiv:2301.07096
Recently:

- Markovich & Lubensky couple $\ell = I\Omega$ to $\omega = \nabla \times \mathbf{v}$
  - leads to 3D odd viscosity in active matter systems [9]
- Relevant for magnetic colloids & large complex molecules
- Ferrofluid particles are too small; these effects don’t manifest on hydrodynamic scale [17]

Idea:

**Couple magnetization $\mathbf{M}$ to $\omega$**
Ferrofluid Hamiltonian

\[ H = \int d^3r \left[ \frac{1}{2} v_i v_i + \frac{1}{2I} \ell_i \ell_i + \frac{1}{2} \ell_i \omega_i + \frac{\gamma}{2} \omega_i M_i - M_i B_i \right] \]

- \( \rho = 1 \) for simplicity
- \( \ell = I \Omega \) is angular momentum density (per particle)
- \( \gamma \) = coupling constant
- Markovich & Lubensky

New coupling

\[ R = \int d^3r \left[ \frac{1}{2} \nu (\partial_i v_j + \partial_j v_i)^2 + \frac{1}{2} \Gamma (2 \Omega_i - \epsilon_{ijk} \partial_j v_k)^2 + \frac{1}{2\tau} (M_i - M_i^0)^2 \right] \]

Use appropriate Poisson Brackets and dissipation function

\[ \rightarrow \text{derive ferrohydro equations} \]

Note: Don’t need Hamiltonian framework, but convenient for parity breaking (dissipationless)
**Governing Equations**

Skip technical details…

**Key Physical Steps:**

- Fix uniform $\vec{B}$ field in the $z$ direction
- Small particle size ($I$ goes to zero)
  
  $\rightarrow$ particles rotate before magnetization re-adjusts
  
  (contrast to larger magnetic colloids)

\[
D_t v_i + \partial_i P = \nu \nabla^2 v_i + \frac{\gamma}{4} M^0 \partial_z \omega_i
\]

Unique to 3D flows! No term in standard ferrohydrodynamics

*arXiv:2301.07096*
Ferrofluids in Hele-Shaw Cells

New coupling (Modified Darcy’s Law):

\[
0 = \partial_x V_x + \partial_y V_y
\]

\[
-\frac{h^2}{12} \partial_x P = \mu V_x - \frac{\gamma}{4} M^0 V_y
\]

\[
-\frac{h^2}{12} \partial_y P = \mu V_y + \frac{\gamma}{4} M^0 V_x
\]

“Table-top” experiment to probe parity-odd fluid


arXiv:2301.07096
Darcy’s Law

Incompressible 3D fluid with shear viscosity:

$$\partial_i v_i = 0$$

$$\rho (\partial_t v_i + v_j \partial_j v_i) = \partial_j T_{ij} + f_i$$

$$T_{ij} = -P \delta_{ij} + \eta (\partial_i v_j + \partial_j v_i),$$

Confine fluid between plates with separation $h$, assume $xy$ variables vary over large scales compared to $h$:

$$v_x = 6z(1-z)V_x$$
$$v_y = 6z(1-z)V_y$$
$$v_z = 0$$
$$P = P(x, y)$$

Phys. Rev. Fluids 7, 114201 (2022)
Modified Darcy’s Law

Incompressible 3D fluid with general (uniform) viscosity tensor:

\[ \partial_i v_i = 0 \]

\[ \rho (\partial_t v_i + v_j \partial_j v_i) = \partial_j T_{ij} + f_i \]

\[ T_{ij} = -P \delta_{ij} + \eta_{ijkl} \partial_k v_l, \]

Same scaling analysis. Remarkably, only four coefficients remain (out of 64 in principle):

\[ V_i = -\frac{h^2}{12} (\eta^{-1})_{ij} \partial_j P \]

\[ \eta = \begin{pmatrix} \eta_{xzzx} & \eta_{xzyy} \\ \eta_{yzxx} & \eta_{yyyy} \end{pmatrix} \]

Phys. Rev. Fluids 7, 114201 (2022)
Modified Darcy’s Law

\[ V_i = -\frac{h^2}{12}(\eta^{-1})_{ij} \partial_j P \]
\[ \eta = \begin{pmatrix} \eta_{xzzx} & \eta_{xzzy} \\ \eta_{yzzx} & \eta_{yzzz} \end{pmatrix} \]

Analogous to:

- Flow through anisotropic porous media (symmetric unless parity is broken)
- Ohm’s law with general conductivity (Hall Effect, \( \vec{E} \sim \vec{\nabla}P \))

We examine special case of cylindrical symmetry

\[ \eta = \eta_{xzzx} = \eta_{yzzy} \quad (\text{shear viscosity}) \]
\[ \eta_* = \eta_{xzzy} = -\eta_{yzzx} \quad (3d \text{ odd viscosity}) \]

Coefficient \( \eta_* \) is one of the 2 parity odd viscosities associated with 3D cylindrical symmetry

*Longitudinal... Not 2D Hall viscosity!*

*Phys. Rev. Fluids 7, 114201 (2022)*
Exp 1: Channel Flow

- Misalignment between flow and pressure gradient
  \[ \theta = \arctan \left( \frac{\eta_*}{\eta} \right) \]
- Transverse force on channel walls
  \[ F = 2\eta_*V_0(RLh) \]
- Hall Analogy:
  Transverse electric field \( E_x \) is required to maintain vertical current
Exp 2: Expanding Bubble

- Laplacian Growth, well studied
- Polubarinova-Galin equation:
  \[ \frac{\partial P}{\partial t} - \frac{\eta}{\eta^2 + \eta_*^2} \left| \nabla P \right|^2 \bigg|_{\partial \Omega(t)} = 0 \]
- Modification to cusp formation time
- Circulation at infinity gives measure of parity odd-odd terms

\[ \Gamma \sim \frac{\eta_*}{\eta^2 + \eta_*^2} \frac{dA}{dt} \]

Exp 3: Saffman/Taylor Instability

• Perform linear stability analysis on free surface of the form:

\[ H(x, t) = \epsilon \text{Re} \left( e^{ikx + \Omega t} \right) \]

• Interface boundary conditions
  - Kinematic: \( V_n \) continuous & matched with interface
  - Forces: \( P \) continuous

• Constraint on asymptotic flow angle

\[ \theta = \arctan \left( \frac{\eta_2^* - \eta_1^*}{\eta_1} \right) \]

• Modified stability condition

\[
(p^{(1)} - p^{(2)})g + V_0 (\eta^{(1)} - \eta^{(2)}) + \frac{V_0 (\eta_2^* - \eta_1^*)^2}{\eta_1 + \eta_2} < 0
\]

• Parity-odd terms introduce more stable regions in the parameter space
Applications

Facilitates the measurement of parity-odd viscosity coefficients in active matter systems, especially useful for complex biological systems.

[22] Circularly confined microswimmers exhibit multiple global patterns, Alan Cheng Hou Tsang and Eva Kanso, Phys. Rev. E 91, 043008 – Published 13 April 2015

Experimental Setup

No coupling → no circulation (yes or no result)

\[ \gamma = 0 \]
\[ \Gamma = 0 \]

\[ \gamma \neq 0 \]
\[ \Gamma = -\frac{q\gamma M^0}{4\mu} \]

(a) without parity breaking

(b) with parity breaking
Quantum Superfluids

- Fluid behavior of many-body quantum system
- Map complex wavefunction $\psi$ to hydrodynamic variables
- Constraint on vorticity $\bar{\omega} = \vec{\nabla} \times \vec{v}$
- Vortex solutions $\rightarrow$ density fluctuations important (compressible)

Typical Prescription:

- $\psi = \sqrt{\rho} e^{i\theta}$
- $v_i = \frac{\hbar}{m} \partial_i \theta$

Fluid-like equations:

$$\partial_t \rho + \partial_i (\rho v_i) = 0$$

$$\rho D_t v_i = \partial_j T_{ij} + f_i$$

$$T_{ij} = -\delta_{ij} P + \eta_{ijk} \partial_k v_l + \cdots$$

“Quantum Pressure”

$$P = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

- Helium-4
- Superconductivity
- Ultra cold atomic gases
- Bose-Einstein condensates
- Fractional quantum Hall
- Polaritons
Fractional Quantum Hall Effect

- Collective behavior of 2D electron system in strong magnetic field
- Laughlin states admit fluid description

Start with Chern-Simons-Ginzburg-Landau action

\[
S_{\text{CSGL}} = \int d^3x \left[ i\hbar \Phi^\dagger D_t \Phi - \frac{\hbar^2}{2m} |D_i \Phi|^2 - V(|\Phi|^2) - \frac{\hbar \nu}{4\pi} \epsilon^\mu\lambda\kappa a_\mu \partial_\lambda a_\kappa \right]
\]

where $\Phi$ is bosonic matter field, $A_\mu$ is external electromagnetic field, $a_\mu$ is statistical Chern-Simons field

Define velocity:

\[
v_i = \frac{\hbar}{m} \left( \partial_i \theta + a_i + \frac{e}{\hbar} A_i + \frac{\epsilon_{ij} \dot{n}_j}{2n} \right)
\]

arXiv:2305.16511
Fluid Equations

Charge and Momentum conservation:

\[
\frac{\partial t}{\partial t} n + \frac{\partial}{\partial x_i} (n v_i) = 0
\]
\[
\frac{\partial t}{\partial t} v_i + v_j \frac{\partial}{\partial x_j} v_i + \frac{eB}{m} \epsilon_{ij} v_j - \frac{1}{mn} \frac{\partial}{\partial x_j} T_{ij} = 0
\]

Hall Constraint:

\[
n - \frac{\nu eB}{2\pi \hbar} + \frac{\nu}{4\pi} \frac{\partial}{\partial x_i} \left( \frac{\partial n}{n} \right) + \frac{\nu m}{2\pi \hbar} \epsilon_{ij} \frac{\partial}{\partial x_j} v_j = 0
\]

Stress Tensor:

\[
T_{ij} = \left( V - n V' - \frac{\pi \hbar^2 n^2}{\nu m} \right) \delta_{ij} - \frac{\hbar n}{2} \left( \epsilon_{ik} \partial_k v_j + \epsilon_{jk} \partial_i v_k \right)
\]

2D Odd Viscosity!

\[
\nu_o \sim \frac{\hbar}{2}
\]

arXiv:2305.16511
Polaritons

• Exciton-Polaritons created when light modes (photons) hybridize with quantum well excitons. [24]

• Resulting quasiparticles are bosonic

• If a condensate is formed, described by wavefunction $\Psi$ in 2D
  → admits a fluid description

• System is inherently out of equilibrium, so no action principle

Goals: Find fluid description of 2D polariton system

Drag and lift forces on impurities or obstacles?

[25] Michael D. Fraser
Wavefunction Description

Start with generalized Gross-Pitaevskii Equation (nonlinear Schrodinger) given by

\[ i\hbar \partial_t \Psi = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) + \alpha |\Psi|^2 + \alpha_R n_R \right] \Psi + \frac{i\hbar}{2} (Rn_R - \gamma) \Psi, \]

where

- \( m \) = effective polariton mass
- \( V(\vec{r}) \) = obstacle potential
- \( \alpha \) = polariton-polariton interaction coupling
- \( \alpha_R \) = polariton-exciton interaction coupling
- \( R \) = stimulated scattering rate from reservoir to polariton mode
- \( \gamma \) = polariton decay rate
- \( P(\vec{r}) \) = pumping intensity
- \( \gamma_X \) = exciton decay rate.

For this work, assume pumping intensity is uniform and constant.
**Fluid Description**

\[
\Psi(t, \vec{r}) = \sqrt{\rho(t, \vec{r})} e^{i\theta(t, \vec{r})}
\]

\[
v_i = \frac{\hbar}{m} \left( \partial_i \theta + \frac{1}{2\rho} \epsilon_{ij} \partial_j \rho \right)
\]

gGPE splits into real and imaginary pieces which are fluid type equations:

\[
\partial_t \rho + \partial_i (\rho v_i) = \frac{R}{\gamma_X} \rho P_0 \left( 1 - \frac{R}{\gamma_X} \rho \right) - \gamma \rho
\]

\[
\rho D_t v_i = \partial_j T_{ij} + \rho f_i
\]

where the stress tensor and force are:

\[
T_{ij} = -\frac{\alpha}{2m} \left( 1 - \frac{\alpha_R}{\alpha} \frac{RP_0}{\gamma_X^2} \right) \delta_{ij} \rho^2 - \frac{\hbar R^2 P_0}{4m \gamma_X^2} \epsilon_{ij} \rho^2 - \frac{\hbar}{2m} \rho \left( \partial_i v_j^* + \partial_j^* v_i \right)
\]

\[
f_i = -\frac{1}{m} \partial_i V
\]
Concluding Thoughts

• Parity breaking appears in many physical systems (active fluids, plasmas, superfluids)

• Odd viscosity, in 2D and 3D encapsulates these properties in a fluid description

• Observable effects and common between systems (edge dynamics in 2D, bulk spiral in 3D)

• 3D effects, in general, are not well understood

Thanks for your time…Questions?