Theoretical scaling of the operational density limit in tokamaks and comparison to experimental data

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Limited predictive capabilities of empirical Greenwald limit

\[ n_{GW} = \frac{I_p}{\pi a^2} \]

Maximum achievable density in real-time controlled discharges show hidden dependencies
Two mechanisms, providing similar predictions for AUG [Manz et al., NF 2023]:

- **Radiative collapse** [Gates et al., PRL 2012; Zanca et al. PRL 2017; Stroth et al., NF 2022].
- **Enhanced turbulent transport** [Rogers et al., PRL 1998; Eich et al., NF 2021; Brown et al, NME 2021; Singh et al, PPCF 2022].
MARFE onsets precedes disruption

Phenomena triggering the MARFE are key to understand density limit

Change of $q$ and $l_i$ MHD modes, Disruption

Fueling
Density increases
No global instabilities

JET, #80823
Edge pressure gradient collapse precedes MARFE onset

Density increases → Edge cooling → Collisionality increases → Pressure profile collapses at the edge → MARFE onset

Increased turbulent transport
Based on local edge parameters, AUG operational space explained in terms of transition between turbulent regimes.
Properties of boundary turbulence

- $n_{fluc} \sim n_{eq}$
- $L_{fluc} \sim L_{eq}$
- Fairly collisional magnetized plasma ($<100$ eV, $n_e \sim 10^{19}$ m$^{-3}$)
- Role of neutrals
- Sheath physics
A model to evolve boundary plasma turbulence

Collisional Plasma \rightarrow \text{Braginskii model} \rightarrow \rho_i \ll L, \omega \ll \Omega_{ci} \rightarrow \text{Drift-reduced Braginskii equations}

\frac{\partial n}{\partial t} + [\phi, n] = \hat{C}(nT_e) - n\hat{C}(\phi) - \nabla ||(nV||_e) + n_n\nu_{ion} - n\nu_{rec} + S_n

\begin{align*}
T_e, T_i, \Omega \text{ (vorticity)} & \rightarrow \text{similar equations} \\
V_{||e}, V_{||i} & \rightarrow \text{parallel momentum balance} \\
\nabla \cdot (n\nabla \perp \phi) &= \Omega - \tau \nabla ^2 \perp p_i \rightarrow \text{Poisson equation} \\
\nabla ^2 \perp \psi &= j_|| \rightarrow \text{Ampère equation}
\end{align*}
A model to evolve boundary plasma turbulence

+ coupling with kinetic neutrals

\[
\frac{\partial f_n}{\partial t} + \mathbf{v} \cdot \frac{\partial f_n}{\partial x} = -\nu_{\text{ion}} f_n - \nu_{\text{CX}} (f_n - n_n f_i / n_i) + \nu_{\text{rec}} f_i
\]

**STREAMING**
\[\nu_{\text{ion}} = n \langle v_e \sigma_{\text{ion}} \rangle\]

**IONIZATION**
\[\nu_{\text{CX}} = n \langle v_{\text{rel}} \sigma_{\text{CX}}(v_{\text{rel}}) \rangle\]

**CHARGE EXCHANGE**
\[\nu_{\text{rec}} = n \langle v_e \sigma_{\text{rec}} \rangle\]

**RECOMBINATION**

We solve in 3D geometry, taking into account turbulent transport, ionization and charge exchange processes, and losses at the vessel

Wersal & Ricci, NF 2015
Boundary conditions at the plasma-wall interface

- Set of b.c. for all quantities, generalizing Bohm-Chodura
- Checked agreement with PIC kinetic simulations
- Neutrals: reflection and re-emission with cosine distribution

Loizu et al., PoP 2012
GBS: our simulation code

- [Ricci et al., PPCF 2012]
- [Paruta et al., PoP 2018]
- [Giacomin et al., JPP 2020]
- [Giacomin et al., JCP 2022]
- [Coelho et al., NF 2022]
- [Halpern et al., JCP 2016]
Turbulent simulations to investigate edge turbulent regimes

- Retain core-edge-SOL interplay
- No separation of equilibrium and fluctuating quantities
- Validated against experimental results [Oliveira, Body et al., NF 2022]

Ricci et al., PPCF 2012, Giacomini et al., JCP 2022
Four regimes of boundary turbulence

- Reduced transport (Drift-wave instability)
- L-mode (Resistive ballooning)
- Beyond the $\beta$-limit (Ideal ballooning)
- Beyond the $n$-limit (Resistive ballooning)

[Giacomini et al., JPP 2020; PoP 2022]
L-mode turbulence driven by resistive ballooning modes

Paolo Ricci

\( \beta_{\text{limit}} \)

\( \sqrt{\frac{\nu_0}{S_p}} \)

(Density / Heating power)
SOL width: balance of perpendicular and parallel transport

\[
\frac{P_{\text{SOL}}}{aR} \sim \frac{1}{qR} \sim c_s p \\
= \langle \tilde{\rho} \tilde{v}_E \times B, r \rangle_t = \frac{1}{B} \left( \tilde{\rho} \frac{\partial \tilde{\phi}}{\partial \theta} \right)_t \sim \frac{\gamma \bar{p}}{L_p k_r^2} \sim \frac{\gamma \bar{p}}{k_\theta}
\]

Removal of driving gradient

Nonlocal linear theory,

\[
\frac{\partial \tilde{\rho}}{\partial r} \sim \frac{\partial \bar{p}}{\partial r}
\]

\[
k_r \tilde{\rho} \sim \frac{\bar{p}}{L_p} (P_{\text{SOL}}, a, R, n, ...)
\]

[Bohm’s SOL width: balance of perpendicular and parallel transport]

[Ricci et al., PRL 2008; PoP 2013; Giacomin et al, JPP 2020]
Good agreement between analytical $L_p$ scaling and simulations

\[ L_p \simeq q^{12/17} R^{7/17} P_{\text{SOL}}^{-4/17} a^{12/17} (1 + \kappa^2)^{6/17} n^{10/17} B_T^{-12/17} \]
Good agreement between analytical estimate and multimachine database

Prediction for ITER L-mode: $\lambda_q \simeq 3.5$ mm
Transition to large transport at high density
Theoretical estimate of density limit based on operational parameters

\[ L_p = L_p(P_{SOL}, a, R, n, ...) \]

Collapse of edge pressure gradient

\[ L_p \sim a \]

\[ n_{\text{lim}} = n_{\text{lim}}(P_{SOL}, a, R, ...) \]

[Giacomin et al., PRL 2022]
No need of EM effects to access the density limit: electrostatic modes become large with collisionality
Density limits depends on $I_p$ and $\alpha$, but also on $P_{\text{SOL}}$.

- **Density limit in physical units:**
  \[ n_{\text{lim}} = \alpha A^{1/6} a^{3/14} P_{\text{SOL}}^{10/21} R^{-43/42} q^{-22/21} (1 + \kappa^2)^{-1/3} B_T^{2/3} \]

  $\alpha$: Numerical coefficient rising from order of magnitude estimates and numerical factors

- **Empirical Greenwald density limit:**
  \[ n_{GW} = \frac{I_p}{\pi a^2} \]

- **Density limit in terms of the plasma current:**
  \[ n_{\text{lim}} \sim P_{\text{SOL}}^{0.48} R^{0.02} B_T^{-0.38} (1 + \kappa)^{-0.33} \left( \frac{I_p}{a^{1.88}} \right)^{1.05} \]

- **Dependence on power observed in experiments** [Bernert *et al*, PPCF 2014; Esposito *et al*, PRL 2008; Huber *et al*, JNM 2013]
Comparison with density limit in AUG, TCV and JET, in two scenarios

Standard L-mode:

Density increase $\rightarrow$ MARFE $\rightarrow$ MHD modes, Disruption

ITER-relevant H-mode:

L-H transition $\rightarrow$ Density increase $\rightarrow$ H-L transition $\rightarrow$ L mode $\rightarrow$ MARFE $\rightarrow$ MHD modes, Disruption

Data range:

$n: 2 \times 10^{19} - 1.2 \times 10^{20} \ m^{-3}, I_p: 0.1 - 2.5 \ MA, B_0: 1.4 - 3 \ T, P_{SOL}: 0.1 - 9$
Good agreement with experimental data

\[ R^2 = 0.80 \]

[Giocomin et al., PRL 2022]
Significant improvement with respect to Greenwald

\[ n_{\text{lim}} = \alpha A^{1/6} a^{3/14} P_{\text{SOL}}^{10/21} R_0^{-43/42} q^{-22/21} (1 + \kappa^2)^{-1/3} B_T^{2/3} \]

- Promising approach for real-time control in MAST-U [Berkery et al., PPCF 2023]
- Experimental campaign planned in DIII-D
- Prediction for ITER \((P_{\text{SOL}}=50 \text{ MW}, q=3, B_T=5.3 \text{ T})\): \(n_{\text{lim}} \sim 2.5 \times 10^{20} \text{ m}^{-3} > 2n_{GW}\)
Final remarks

- Density limit set by edge dynamics
- Increase of density leads to higher collisionality, larger transport, triggering MARFE and disruption
- Analytical scaling provided show $I_p$ and $a$ dependence similar to Greenwald, but also $P_{SOL}$ dependence
- Good agreement with AUG, JET and TCV discharges, as well as MAST-U
- Significantly larger safety margin than Greenwald in case of unintentional H-L transition in ITER
- Given possible role of other phenomena in setting density limit in tokamaks, further experimental investigations urgently needed.
Moving forward: multispecies simulation with detachment

A multispecies (D, D⁺, D₂, D₂⁺, e⁻) model allowed first simulations of highly-radiative (detached) scenarios

[Calado et al., PoP 2022, NF 2022, Mancini NF 2023]

Density increases, ionization front moves, heat flux to vessel reduced

Role in density limit?