Simulating electromagnetic instabilities in tokamaks with gyrokinetic particle-in-cell code GTS *

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Abstract

- Recently, the numerical scheme presented in [1] enabled explicit gyrokinetic simulations of low-frequency electromagnetic instabilities in tokamaks at experimentally relevant values of plasma beta.

- This scheme resolved the long-standing ”cancelation problem” that previously hindered gyrokinetic particle-in-cell code simulations of electromagnetic phenomena with inherently small parallel electric fields.

- Moreover, the scheme did not employ approximations that eliminate critical tearing-type instabilities.

- Here, we report on the implementation of this numerical scheme in the global gyrokinetic particle-in-cell code GTS.

- Additionally, we present a comprehensive set of verification simulations of numerous electromagnetic instabilities: kinetic ballooning mode (KBM), the internal kink mode, the tearing mode, the micro-tearing mode (MTM) and toroidal alfven eigenmode (TAE) destabilized by energetic ions, which are all instrumental in understanding tokamak physics.

- We also showcase the preliminary nonlinear simulations of the kinetic ballooning instability and (2,1) island formation due to the tearing mode instability.

[1] A. Mishchenko, M. Cole, R. Kleiber, A. Konies, Phys. Plasmas 21 (2014) 052113.

Gyrokinetic Equations solved in PIC codes

- Particle drift equations for $(\mathbf{R}, v_\parallel, \mu)$ are

$$\dot{\mathbf{R}} = \frac{1}{B_\parallel^*}\left[v_\parallel \mathbf{B}^* + \frac{c}{q}\mathbf{b}\times\nabla H\right], \quad m\dot{v}_\parallel = -\frac{1}{B_\parallel^*}\mathbf{B}^*\cdot\nabla H - \frac{q}{c}\frac{\partial \bar{A}_\parallel}{\partial t}, \quad \dot{\mu} = 0, \quad (1)$$

where $\mathbf{B}^* = \nabla\times\mathbf{A}^*$, $B_\parallel^* = \mathbf{b}\cdot\mathbf{B}^*$, $\mathbf{A}^* = \mathbf{A}_0 + (A_\parallel + \frac{mc}{q}v_\parallel)\mathbf{b}$ and $H = mv_\parallel^2/2 + q\phi$.

- In δf simulations the particle weight is defined as $w = \delta f/f$ with $\delta f = f - f_0$. and Vlasov equation with collision operator $C[w]$ reads as

$$\frac{dw}{dt} = -(1-w)\frac{d}{dt}\ln f_0 + C[w]. \quad (2)$$

- The gyrokinetic Poisson for ϕ is given by

$$-\nabla_\perp\cdot\frac{n_0 c}{B\Omega_i}\nabla_\perp\phi = \delta\bar{n}_i - \delta n_e, \quad (3)$$

and Ampere's law for electromagnetic potential $A_\parallel$

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c}\delta j = -\frac{4\pi}{c}(\delta j_e + \delta\bar{j}_i). \quad (4)$$

Illustration of Cancellation Problem

- To remove time derivative $\frac{\partial A_\parallel}{\partial t}$ in Eq.(1) introduce $p_\parallel = v_\parallel + \frac{q}{mc} A_\parallel$. which modifies Ampere's law

$$k_\perp^2 A_\parallel + \frac{1}{\lambda_e^2} A_\parallel = -\frac{4\pi}{c} \delta j_p.$$

- The skin-term will be incompletely canceled numerically by adiabatic contribution from δj_p

$$k_\perp^2 \left(1 + \frac{\epsilon}{\lambda_e^2 k_\perp^2}\right) A_\parallel = S_1 A_h.$$

- Here $A_h = -i(c/\omega)E_\parallel$. From Poisson equation $\phi = \frac{\omega}{k_\parallel} S_2 A_h / (\rho_i k_\perp)^2$, and

$$A_\parallel = A_h + \frac{k_\parallel}{\omega}\phi = A_h + \frac{S_2}{\rho_i^2 k_\perp^2} A_h \approx \frac{S_2}{\rho_i^2 k_\perp^2} A_h$$

- The dispersion relation for shear-Alfven waves then has error term $\epsilon/(\lambda_e k_\perp)^2$

$$\omega^2 = v_A^2 k_\parallel^2 \left[1 + \epsilon \frac{1}{\lambda_e^2 k_\perp^2}\right].$$

- To get correct alfven wave dispersion for $k_\perp \sim 1/a$ we need

$$\epsilon \sim \frac{1}{\sqrt{N}} \ll (\lambda_e/a)^2.$$

Another way of removing time derivative from equation of motion

- Switching to a new velocity variable v_h defined as

$$v_h = v_\parallel + \frac{q}{mc} A_h, \tag{5}$$

where field quantities A_h and A_s are defined as

$$A_h = -c \int_{t_0}^{t} E_\parallel dt, \quad A_s = -c \int_{t_0}^{t} \mathbf{b}_0 \cdot \nabla \phi dt + A_\parallel(t_0), \tag{6}$$

with $A_\parallel(t) = A_h(t) + A_s(t)$, the guiding center equations for v_h become

$$m\dot{v}_h - \tfrac{q}{c}\dot{\mathbf{R}} \cdot \nabla A_h = -\tfrac{1}{B_\parallel^*}\mathbf{B}^* \cdot \nabla H + q\mathbf{b} \cdot \nabla \phi. \tag{7}$$

- Time derivative is eliminated and the r.h.s contains only magnetic drift and mirror terms.

- Because $A_h/A_\parallel \sim \rho_i^2 k_\perp^2$ the error is reduced by factor $\rho_i^2 k_\perp^2$

$$\omega^2 = v_A^2 k_\parallel^2 \left[1 + \epsilon \frac{\rho_i^2}{\lambda_e^2} \right].$$

- To get correct alfven wave frequency we need

$$\epsilon \sim \frac{1}{\sqrt{N}} \ll (\lambda_e/\rho_i)^2 = \frac{1}{\beta m_i/m_e}.$$

Pullback scheme to reduce error

- To reduce error further do periodic shifts of t_0 in definition of A_h.

- After each shift during the simulation at some time moment t_1, A_s is reset to be
$$A_s^{new}(t_1) = A_s^{old}(t_1) + A_h^{old}(t_1) = A_\parallel(t_1) \tag{8}$$
and A_h is reset to zero $A_h^{new} = 0$ by setting $t_0 = t_1$ in Eq. (6).

- The parallel particle velocity $v_\parallel(t_1)$ at time t_1 is different from velocity $v_h^{old}(t_1)$ at the same time
$$\begin{aligned} v_\parallel(t_1) &= v_h^{old}(t_1) - \frac{q}{mc}A_h^{old}(t_1) \qquad (9)\\ &= v_h^{new}(t_1) - \frac{q}{mc}A_h^{new}(t_1) = v_h^{new}(t_1). \qquad (10)\end{aligned}$$

- The value of distribution function carried by the particle is not changed $f^{new}(t_1) = f^{old}(t_1)$ while the value of f_0 at the particle location in the phase space is changed as
$$f_0(v_h^{new}) = f_0(v_\parallel) = f_0(v_h^{old} - \frac{q}{mc}A_h^{old}). \tag{11}$$

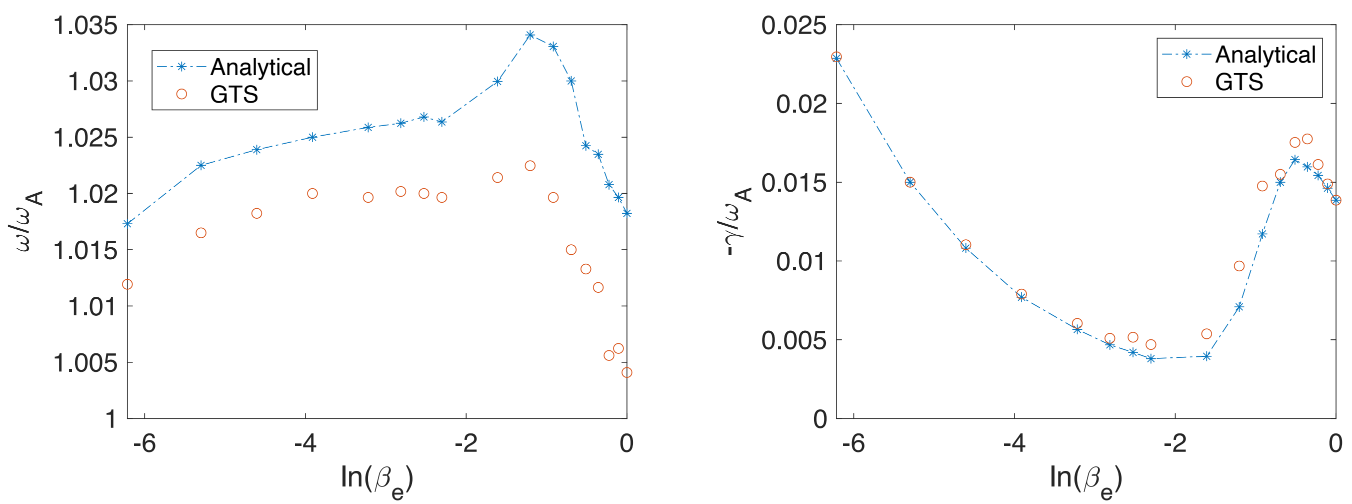
- The weights are changed according to $w = 1 - f_0/f$

Simulation of (2,2) Alfven mode

- Almost cylindrical plasma with $R/a = 100$, $a/\rho_i = 25$, $q = 2$ and $T_i = T_e$.
- Initial mode profile is an eigenfunction of $-\nabla^2$ with a smallest eigenvalue $k_\perp^2$ for a given $m = 2$.
- Solve analytical dispersion relation for ω

$$k_\perp^2 \rho_i^2 + \left(1 - \frac{\omega^2}{\omega_A^2}\right) [(1 + X_e) + (1 + X_i)] = 0 \tag{12}$$

- Here $\omega_A = v_A k_\parallel$, $X_{e,i} = \zeta_{e,i} Z(\zeta_{e,i})$ and $\zeta_{e,i} = \omega/(v_{e,i}^{th} k_\parallel)$.



Removing errors when gradients are present

- Linearized Vlasov equation can be written symbolically as

$$(\omega - kv - \omega_d)w = \omega_d\phi - \omega^*(\phi - vA) + \left(\phi - \frac{\omega}{k}A\right)kv.$$

- Here $\frac{q}{T}\phi \to \phi$ and $\frac{q}{cT}A_\parallel \to A$. Writing $A = \frac{k}{\omega}\phi + A_h$ it can be simplified to

$$(\omega - kv - \omega_d)\left(w + \frac{\omega^*}{\omega}\phi\right) = \left(1 - \frac{\omega^*}{\omega}\right)(\omega_d\phi - \omega v A_h).$$

- R.H.S contains only small terms in MHD limit.

- Large parts $\frac{\omega^*}{\omega}\phi$ contribute to Poisson equation only a small term $\delta n/n_0 = k_\perp^2\rho_i^2\frac{\omega^*}{\omega}\phi$ due to cancellations of contribution from electrons and ions.

- If not not canceled correctly, large error term is introduced $\delta n^{err}/n_0 \sim \epsilon\frac{\omega^*}{\omega}\phi$ and

$$\omega^2 = \omega_{DA}^2\left[1 + \epsilon\left(\frac{\omega^*}{\omega_{DA}}\right)\frac{1}{\rho_i^2 k_\perp^2}\right].$$

MHD equations in gyrokinetic codes

- By taking time derivative of gyro-kinetic Poisson equation, using linearized drift-kinetic Vlasov equations and Ampere's low to eliminate parallel current and setting $E_{\|} = -\partial_{\|}\phi - (1/c)\partial_t A_{\|} = 0$ we get

$$\nabla_{\perp} \cdot \frac{\omega^2}{v_A^2} \nabla_{\perp}\phi + \partial_{\|}\nabla_{\perp}^2 \partial_{\|}\phi - \frac{\mathbf{b} \times \nabla \partial_{\|}\phi}{B} \cdot \nabla \left(\frac{4\pi}{c} j_{\|0} \right) = -\frac{4\pi\omega}{c^2} \sum_s q_s n_s < \omega_{ds} w >=$$

$$\frac{4\pi}{c^2} \sum_s \frac{q_s^2 n_s < \omega_{ds}\omega_s^* >}{T_s}\phi - i\frac{4\pi^2}{c^2} \sum_s \frac{q_s^2 n_s}{T_s} < \omega_{ds}\delta(\omega - k_{\|}v)(\omega - \omega_s^*)\omega_{ds} > \phi.$$

- Here $<>$ means averaging over Maxwellian velocity distribution function.

- First two terms represent dispersion equation for shear-alfven wave.

- Third term provides free energy for tearing and kink modes.

- First term on r.h.s. is a curvature drive for ballooning instability and kink mode in toroidal geometry.

- The last terms is responsible for destabilization of TEA mode by energetic particles.

"Invariant Embedding" Method Cylindrical Eigenmode code

- For set of linear equations for $\mathbf{u} = (u_1, u_2)$

$$\frac{d^2}{dr^2}\mathbf{u} = \hat{A}(r,\omega)\mathbf{u} + \hat{B}(r,\omega)\frac{d}{dr}\mathbf{u}.$$

- Matrices $\hat{A}(r,\omega)$ and $\hat{B}(r,\omega)$ are found by solving Vlasov equation.

- Introduce matrix $\hat{R}(r,\omega)$ as

$$\mathbf{u} = \hat{R}(r,\omega)\frac{d}{dr}\mathbf{u}.$$

- Equation for $\hat{R}(r,\omega)$ is the matrix Riccatti equation

$$\frac{d}{dr}\hat{R}(r,\omega) = 1 - \hat{R}(r,\omega)\hat{B}(r,\omega) - \hat{R}(r,\omega)\hat{A}(r,\omega)\hat{R}(r,\omega).$$

- The boundary conditions $\mathbf{u}(a) = 0$ and $\mathbf{u}(b) = 0$ set boundary conditions for $\hat{R}(r,\omega)$

$$\hat{R}(a,\omega) = 0, \text{ and } \quad \hat{R}(b,\omega) = 0.$$

- Integrate Ricatti equation on the interval $a < r < r_0$ to obtain $\hat{R}^{-}(r_0,\omega)$ and on the interval $r_0 < r < b$ to obtain $\hat{R}^{+}(r_0,\omega)$.

"Invariant Embedding Method" Cylindrical Eigenmode code, cont'd

- The eigehnvalue equation is

$$\det[\hat{R}^{+}(r_0,\omega) - \hat{R}^{-}(r_0,\omega)] = 0.$$

- The method only searches in 2-d complex-ω plane, instead of 4-d $[\omega, u_2'(a)/u_1'(a)]$ space of original system.

- After ω is found, eigenfunctions are found by solving first order equations for $\mathbf{v} = d\mathbf{u}/dr$

$$\frac{d}{dr}\mathbf{v} = \hat{A}(r,\omega)\hat{R}(r,\omega)\mathbf{v} + \hat{B}(r,\omega)\mathbf{v}.$$

- With boundary conditions at rational surface $r = r_0$ for $\mathbf{v} = (v_1, v_2) = (u_1'(r_0), u_2'(r_0)$ are

$$v_1(r_0) = 1, \quad \text{and} \quad (R_{12}^{+} - R_{12}^{-})v_2(r_0) + (R_{11}^{+} - R_{11}^{-}) = 0.$$

- Next, use definition of $\hat{R}$ to find $\mathbf{u}$: $\mathbf{u} = \hat{R}(r,\omega)\mathbf{v}$.

Drift-Tearing Mode Properties

- Ampere law inside the current layer

$$-i\omega = \frac{1}{j_\parallel}\frac{\partial j_\parallel}{\partial t} = \frac{\partial_r^2 E_\parallel}{4\pi j_\parallel / c^2} = \frac{c^2}{4\pi\delta}\left[\frac{\partial_r E_\parallel|_{-\delta/2}^{\delta/2}}{E_\parallel}\right]\left(\frac{E_\parallel}{j}\right) = \left[\frac{\Delta'\delta_e^2}{\delta}\right]\left(\frac{-eE_\parallel/m_e}{v_\parallel^e}\right).$$

- where $\Delta' = \partial_r E_\parallel|_{-\delta/2}^{\delta/2}/E_\parallel = \partial_r A|_{-\delta/2}^{\delta/2}/A$ is obtained from equilibrium equation outside the layer

$$\nabla_\perp^2 A = \frac{4\pi j_0'(r)}{cB}\frac{k_\theta}{k_\parallel}A.$$

- Using electrons moment equation

$$(\nu - i\omega)v_\parallel^e = \left(1 - \frac{\omega^*}{\omega}\right)\frac{-eE_\parallel}{m_e}$$

- We get relation between the width of the current layer and mode frequency.

$$\frac{\delta}{\Delta'\delta_e^2} = \frac{\omega + i\nu}{\omega - \omega^*}.$$

Drift-Tearing Mode Properties

- The parallel electron current with frequency ω can only be driven inside the narrow region of width δ such that (for $\delta < \rho_i$)

$$\omega(\omega + i\nu) \approx [v_{te} k_\parallel(\delta)]^2 = [v_{te} k_\parallel' \delta]^2$$

- Combining, we obtain the dispersion relation for drift-tearing mode

$$(\omega - \omega^*)^2 = -\gamma_c^2 \left(1 + \frac{i\nu}{\omega}\right)$$

  where $\gamma_c \equiv v_{te} k_\parallel' \Delta' \delta_e^2$

- For $\nu = 0$ and $\omega^* = 0$, $\omega = i\gamma_c, \quad \delta = \delta_c \equiv \Delta' \delta_e^2$.

- For $\nu = 0$ and $\omega^* >> \gamma_c$, $\omega = \omega^* + i\gamma_c, \quad \delta = \delta_c \left(\frac{\omega^*}{\gamma_c}\right)$.

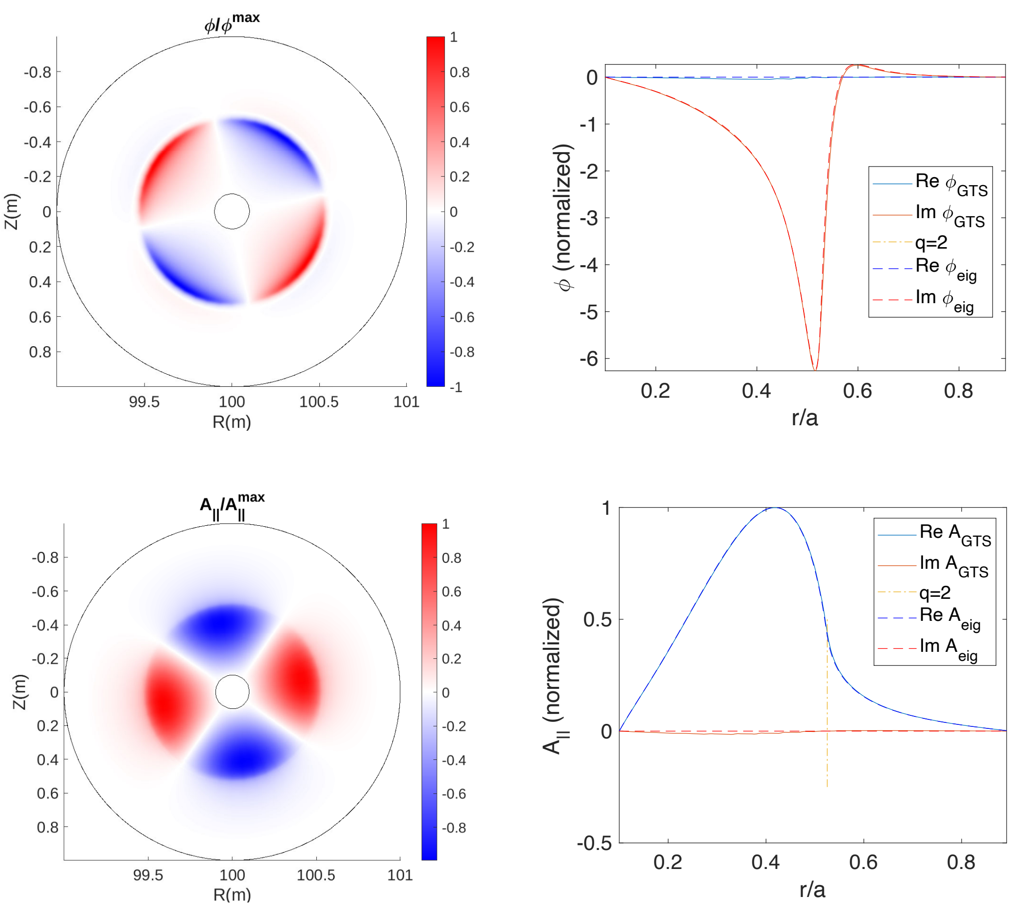
- For $\nu >> \gamma_c$ and $\omega^* = 0$, $\omega = i\gamma_c \left(\frac{\nu}{\gamma_c}\right)^{1/3}, \quad \delta = \delta_c \left(\frac{\nu}{\gamma_c}\right)^{2/3}$.

- For $\omega^* \neq 0$ and $\nu >> \omega^*$, $\omega = \omega^* + i\gamma_c \left(\frac{\nu}{\omega^*}\right)^{1/2}, \quad \delta = \delta_c \frac{(\nu\omega^*)^{1/2}}{\gamma_c}$.

Simulation of (2,1) Tearing mode

- $a/\rho_i = 50$, $R/a = 100$, $q(r) = 1.725/(1 - 0.5r^2/a^2)$, $\beta_e = 0.001$, $R/L = 0$, $\nu = 0$.

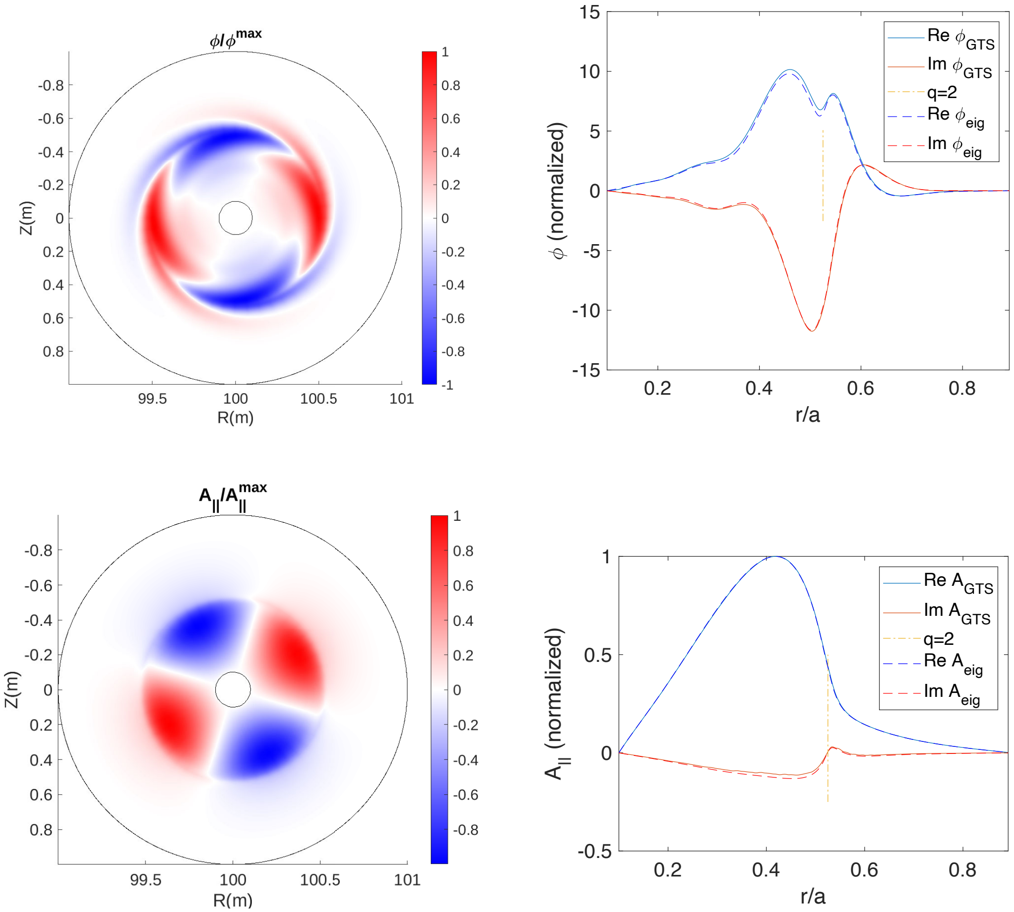
- $\omega_{GTS} = 0.113c_s/R$ while $\omega_{eig} = 0.115c_s/R$.



Simulation of (2,1) Drift-tearing mode, $R/L_n = 5$

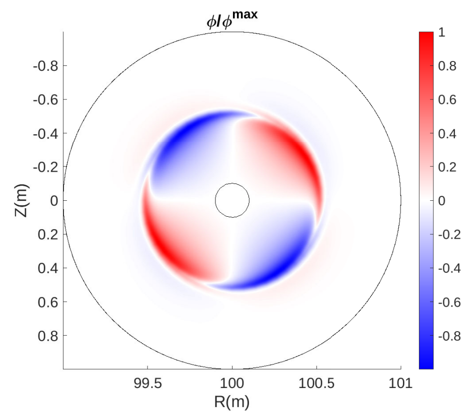
- $a/\rho_i = 50,\ R/a = 100,\ q(r) = 1.725/(1 - 0.5r^2/a^2),\ \beta_e = 0.001,\ R/L_T = 0, R/L_n = 5\ ,\ \nu = 0.$

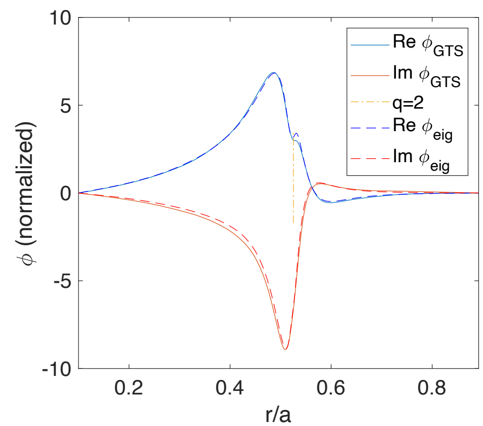
- $\omega_{GTS} = (-0.37 + i0.13)c_s/R$ while $\omega_{eig} = (-0.36 + i0.13)c_s/R$.

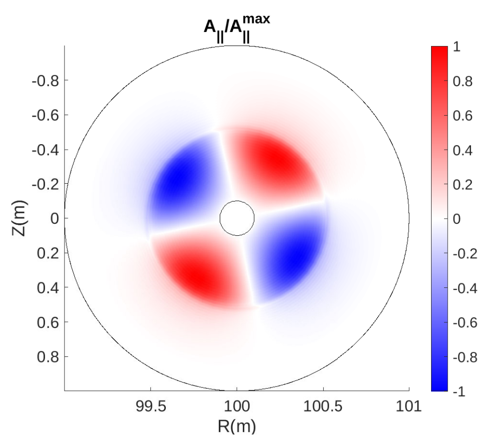


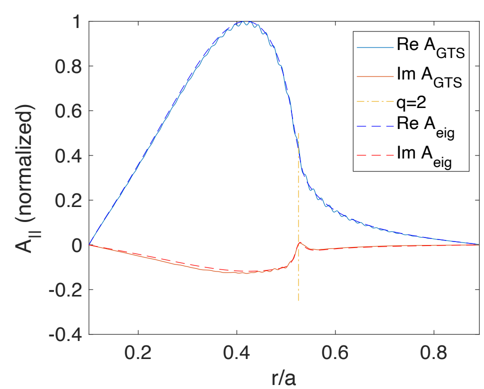
Simulation of (2,1) Drift-tearing mode, $R/L_{Te} = 5$

- $a/\rho_i = 50$, $R/a = 100$, $q(r) = 1.725/(1 - 0.5r^2/a^2)$, $\beta_e = 0.001$, $R/L_{Te} = 5$, $R/L_n = 0$, $\nu = 0$.
- $\omega_{GTS} = (-0.24 + i0.10)c_s/R$ while $\omega_{eig} = (-0.23 + i0.10)c_s/R$.









Micro-tearing Mode

- $\Delta' > 0$ is satisfied only for small mode numbers (e.g., m = 2 or 3).

- For higher mode numbers, a tearing mode can also be destabilized with a much larger growth rate $\gamma \gg \gamma_c$ due to the dependence of the electron collision frequency on velocity

$$\left\langle \left(\frac{v}{v_{te}}\right) \frac{[\omega - \omega^*(v)]}{[1 + i\nu(v)/\omega]^{1/2}} \right\rangle_v = i\gamma_c$$

  or

$$\left(1 - \frac{\omega_n^*}{\omega}\right) f_1\left(\frac{\bar{\nu}}{\omega}\right) - \frac{\omega_{Te}^*}{2\omega} f_2\left(\frac{\bar{\nu}}{\omega}\right) = i\frac{\gamma_c}{\omega} \approx 0$$

  where $\bar{\nu}$ is characteristic value of collision frequency.

- If $\omega_{Te}^* = 0$, $\omega \approx \omega_n^*$ and the mode is stable (if $\gamma_c/\omega_n^* \ll 1$ is neglected)

- If $\omega_{Te}^* \neq 0$ the mode is unstable. For example for $\omega_n^* = 0$

$$\frac{\omega}{\omega_{Te}^*} = F\left(\frac{\bar{\nu}}{\omega_{Te}^*}\right).$$

Implementation of collisions in GTS and eigenmode code

- $C[\delta f_e]$ is the electron pitch-angle scattering collision operator

$$C[\delta f_e] = \frac{\nu(v)}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial}{\partial\xi}\delta f_e,$$

where $\xi = v_\parallel/v$ has been implemented in the code using Langeven equation

$$v'_\parallel = v_\parallel[1-\nu(v)dt(1+\alpha^2)/2] - v_\perp\alpha\sqrt{\nu(v)dt},$$

where α takes randomly on values ± 1 and

$$\nu(v) = \bar{\nu}\left(\frac{v_{te}}{v}\right)^3$$

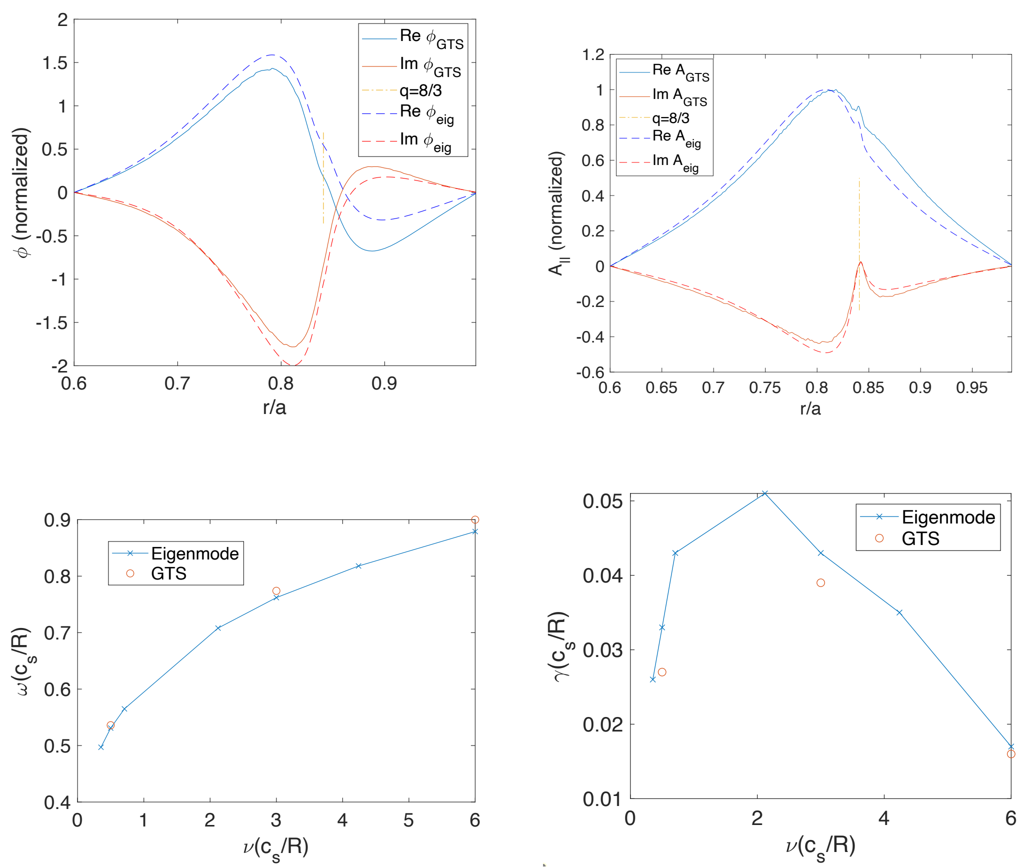
- Matrices $\widehat{A}(r,\omega)$ and $\widehat{B}(r,\omega)$ are found by solving collisional Vlasov equation and solution for electron current and density can be expressed exactly using "continued" fractions, i.e.,

$$A_{12} = \int v^n dv F_{e0}(v)[a_0 + \cfrac{b_0}{a_1 + \cfrac{b_1}{a_2 + \cfrac{b_2}{a_3+\cdots}}}],$$

- with a_n and b_n being functions of $k_\parallel v/\omega$, ω^*/ω, and $\nu(v)/\omega$ and index n, and converge very fast for $\bar{\nu}/\omega > 0.1$.

Simulation of (8,3) Micro-tearing mode

- $a/\rho_i = 25$, $R/a = 100$, $q(r) = 1.725/(1 - 0.5r^2/a^2)$, $\beta_e = 0.06$, $T_i = T_e$, $R/L_n = 0$, $R/L_{Te} = 2$, $R/L_{Ti} = 0$, $\nu = \bar{\nu}/(v/v_{th})^3$



[1] "Electron temperature gradient driven microtearing mode", N. T. Gladd et. al., Physics of Fluids **23**, 1182 (1980).

(1,1) Kink mode

- In tokamaks with $R/a >> 1$ and circular flux-surfaces and no Shafranov shift MHD equations for $\xi = \phi/r$ with $\xi = \sum_m \exp(im\theta)\xi_m$ take form

$$\frac{d}{dr} r^3 \left(n^2_{(1,1)\|} + \frac{\gamma^2}{\omega_A^2} \right) \frac{d}{dr} \xi_{m=1} = R \frac{d}{dr} \beta' r^2 \xi_{m=2}$$

$$\frac{d}{dr} r^3 \left(n^2_{(1,2)\|} + \frac{\gamma^2}{\omega_A^2} \right) \frac{d}{dr} \xi_{m=2} - 3 \left(n^2_{(1,2)\|} + \frac{\gamma^2}{\omega_A^2} \right) \xi_{m=2} = -R \frac{r^3}{2} \frac{d}{dr} \frac{\beta'}{r} \xi_{m=1}$$

  here $n_{(n,m)\|} = n - m/q(r)$

- Terms on the r.h.s come from pressure-curvature coupling.

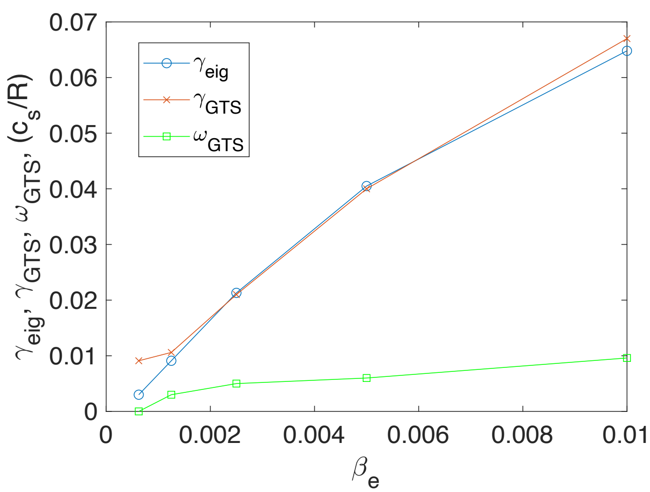
- The strength of the coupling scales as $\beta \frac{R}{L_p}$.

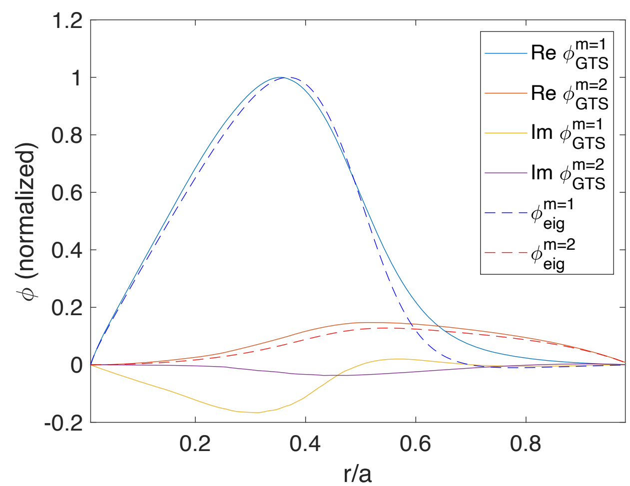
- For $\beta R/L_p \ll 1$, $\xi_{m=1} \approx const$ all the way to rational surface $q = 1$ where coupling becomes significant, since $n_{(1,1)\|} \approx 0$.

- Coupling removes the singularity near rational surface and localizes the mode inside $q = 1$ surface.

Simulation of (1,1) Kink mode

- $a = 1m,\ R/a = 10,\ B = 1T,\ a/\rho_i = 180,\ q(r) = 0.8(1 + r^2/a^2)$.
- $n(r) = n_0 \exp\left[-0.6 tanh\left(\frac{r/a - 0.5}{0.2}\right)\right]$.





[1] "Pullback scheme implementation in ORB5", A. Mishchenko et. al., Computer Phys. Comm., **238**, 194-202 (2019).

Toroidal Alfvenic Eigenmode

- MHD equations for TAE in the limit $R/a \gg 1$ and $\beta R/a \ll 1$ are

  $$\nabla_\perp \cdot \frac{\omega^2}{\omega_A^2} \nabla_\perp \phi - n_\parallel \nabla_\perp^2 n_\parallel \phi = 0,$$

  where $n_\parallel = n - m/q(r)$.

- Near a surface where $n_{m\parallel} = -n_{m+1\parallel}$ the modes with m and $m+1$ are resonantly coupled due to a weak dependence of $\omega_A \sim B$ on poloidal angle θ.

- This coupling removes the degeneracy at the crossing point producing a frequency gap.

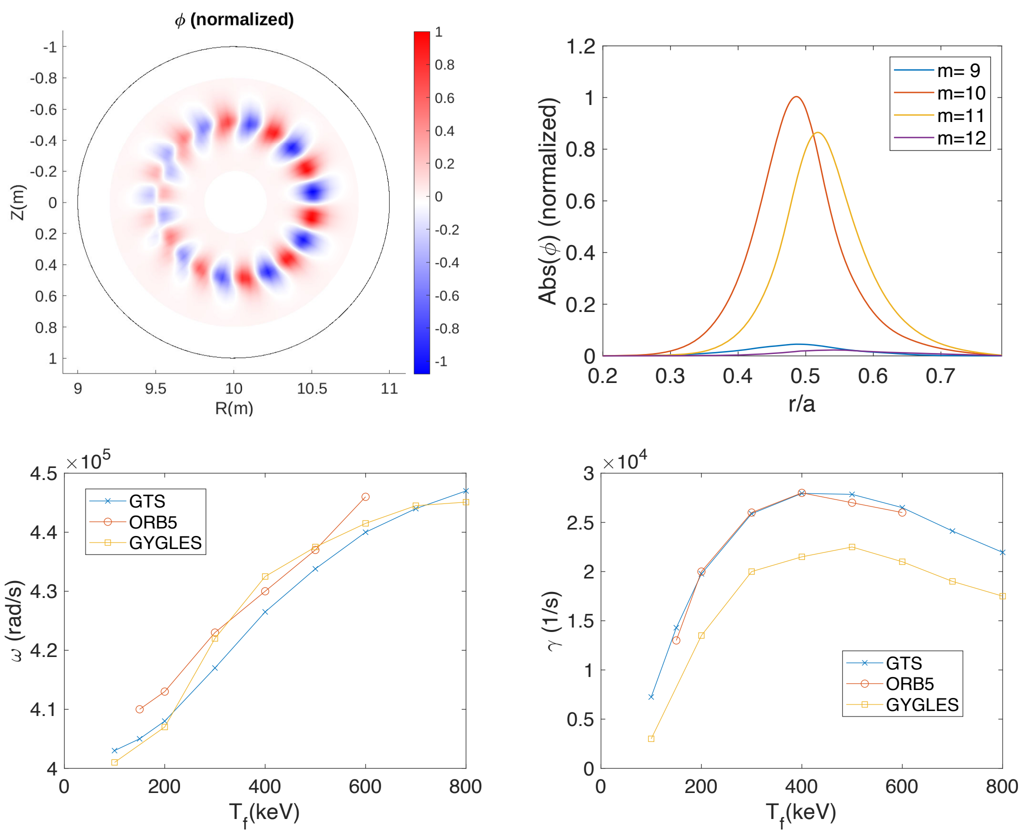
- It also produces an effective potential well for the mode which confines it near the crossing point producing a global TAE mode.

- If energetic particles are present near phase velocity of the wave, the wave can become unstable.

- The mode is damped by electrons and background ions and by interaction with alfvenic continuum.

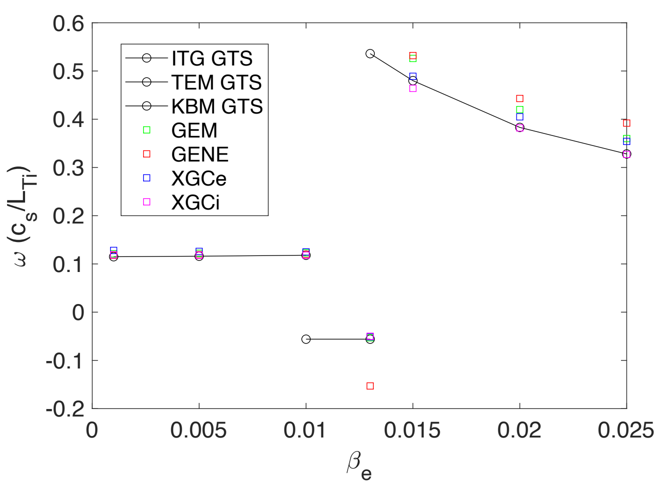
Simulation of n=6 TAE mode

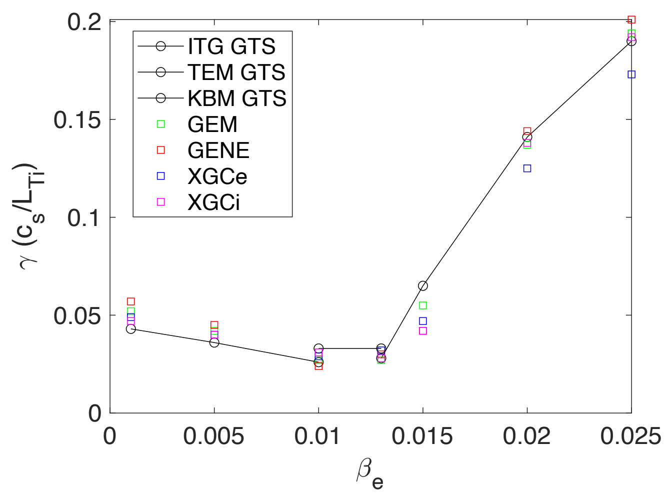
- Hydrogen plasma with $a = 1m, R = 10m$, $B = 3T$, $q(r) = 1.71 + 0.16(r/a)^2$, $n = 2 \cdot 10^{19} m^{-3}$, $T_i = T_e = 1keV$
- Hot species: deuterium with Maxwellian velocity distribution with $100keV \leq T_h \leq 800keV$
- $n_h(r) = n_{0h} \exp\left[-\frac{2}{3} tanh\left(\frac{r/a - 0.5}{0.2}\right)\right]$. , $n_{0h} = 0.75 \cdot 10^{17} m^{-3}$



Simulation of ITG-TEM-KBM transition

- Cyclone base case (CBC) hydrogen plasma with $a/\rho_i = 50$, $R/L_n = 2.22$, $R/L_{Ti} = R/L_{Te} = 6.92$.

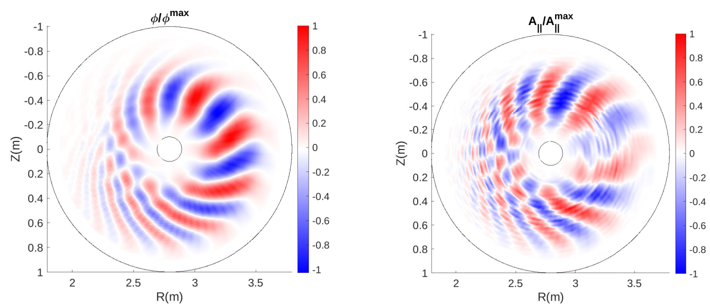


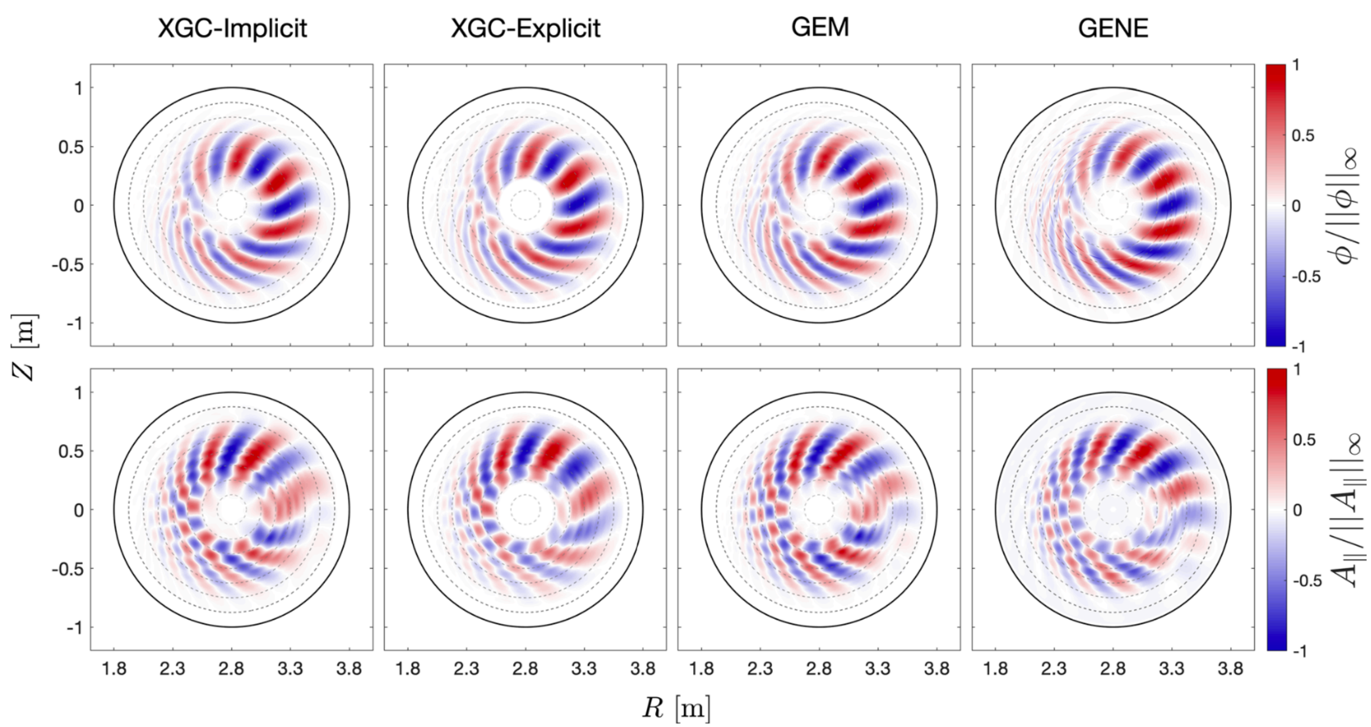


[1] "Verification of a fully implicit particle-in-cell method for the $v_{\parallel}$-formalism of electromagnetic gyrokinetics in the XGC code", B. J. Sturdevant, Physics of Plasmas **28**, 072505 (2021).

ITG mode, $\beta_e = 0.001$

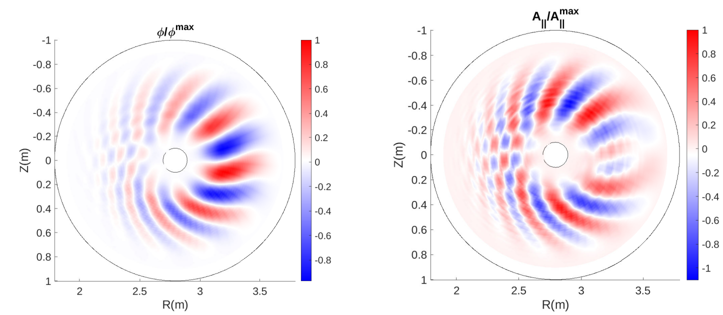
- Comparison of GTS results with results of XGC, GEM and GENE.

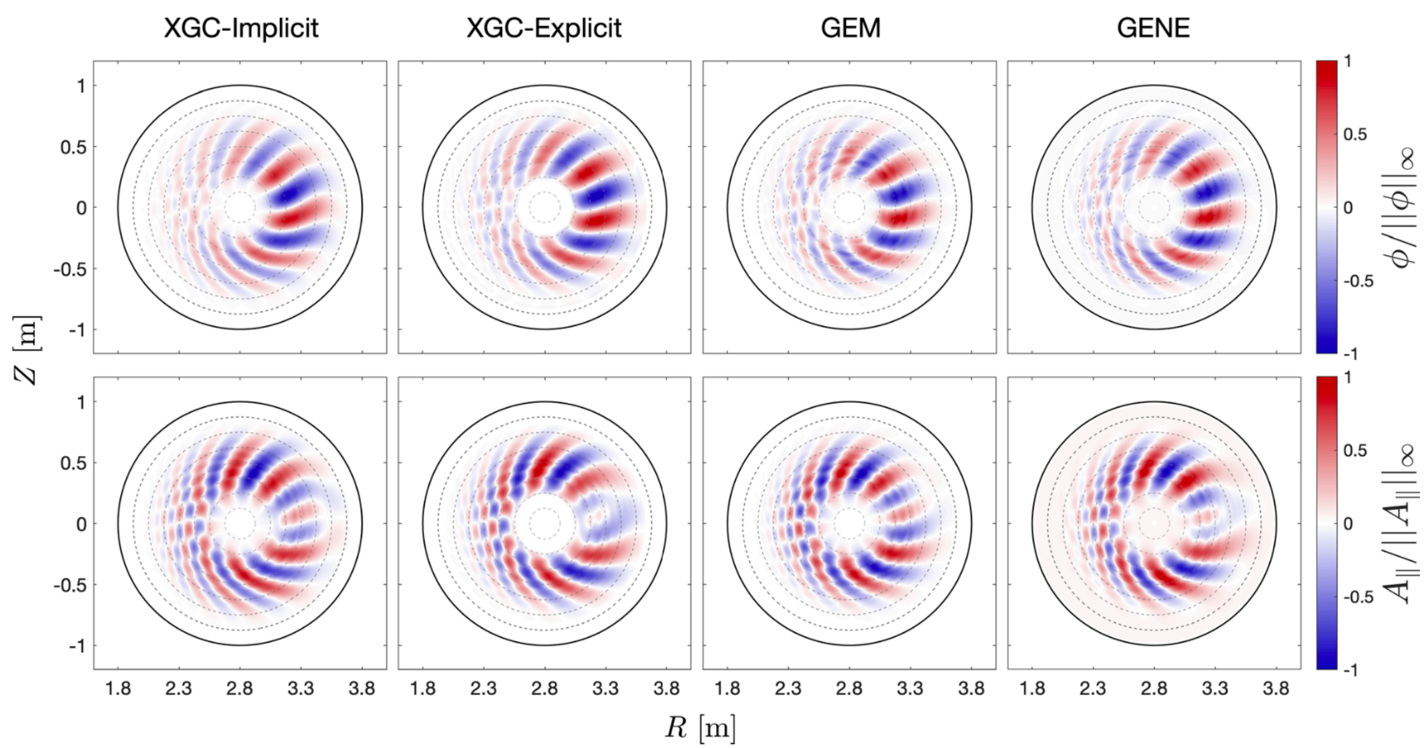




KBM mode, $\beta_e = 0.025$

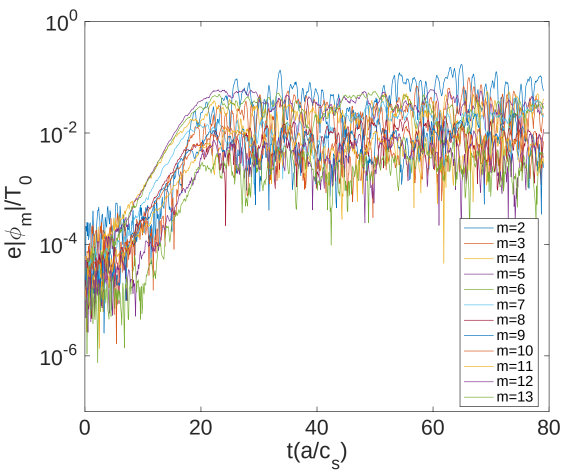
- Comparison of GTS results with results of XGC, GEM and GENE.

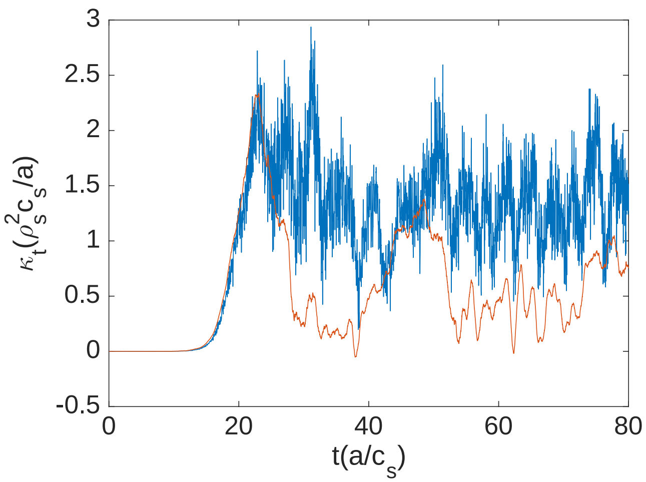




Nonlinear simulations of CBC case with $a/\rho_i = 50$ and $\beta_e = 0.03$

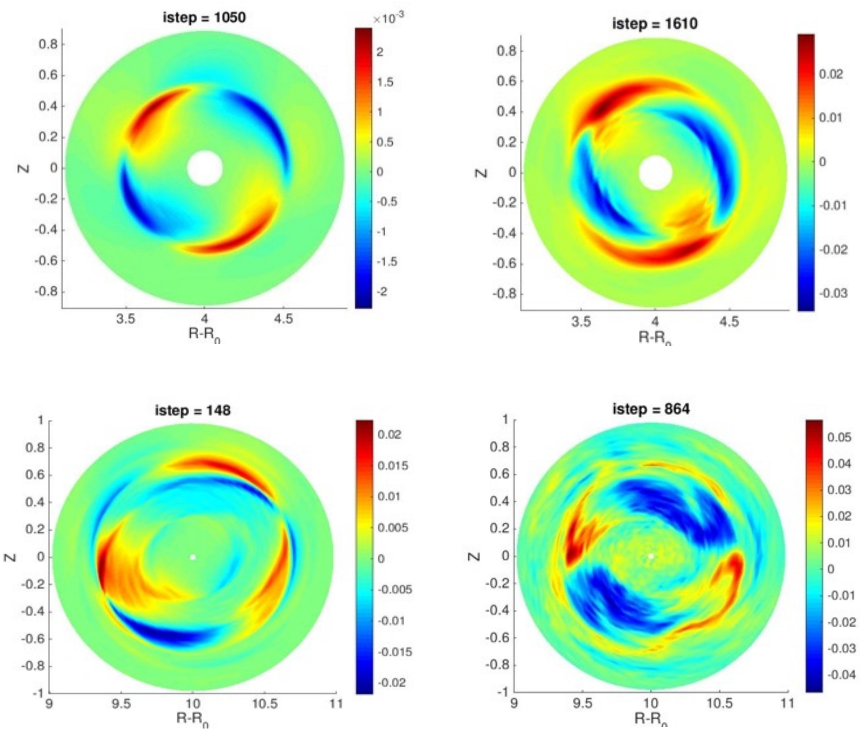
- Above threshold for KBM instability at $\beta_e = 0.013$.
- Heat flux is mostly in electron channel due to parallel transport along fluctuating magnetic field.
- Fluxes are 5x higher compared to $\beta_e = 0.005$ CBC case.
- Fluxes oscillate due to periodic recovery of the linear mode structure which can be a sign of predator-pray dynamics.





Nonlinear simulation of (2,1) tearing and double-tearing

- Possibility to study NTM since trapped particles are included.
- (2,1) –double tearing mode responsible for anomalous current penetration during fast current ramp-up.
- May explain internal disruptions in cases with q¿1.
- Maybe responsible for ITB in advanced tokamaks with negative shear.



Conclusion and Future work

- We have presented the implementation of EM pull-back scheme in GTS code.

- Numerous benchmarks against the eigenmode code and published results showcase the new capabilities of the code to simulate wide range of EM phenomena relevant to modern tokamaks.

- We also presented the nonlinear simulations of the kinetic ballooning instability in a model Cyclone plasma showing that KBM turbulence can be robustly simulated with δf gyrokinetic codes.

- Finally, we have also shown that the code is now capable of simulating non-linear, low-n, reduced MHD (low-beta) physics relevant for thermal quench. That includes kink and tearing modes which are considered as one of the major MHD perturbations that may lead to tokamak disruption.

- This new capability represents a major advance in studying these instabilities which up to now has been exclusively studied using fluid MHD codes with ad- hoc models for plasma resistivity and heat conductivity invalid for the hot and highly anisotropic fusion plasmas.

- On the hand our EM-GTS simulations can be considered as the first-principial simulation approach to studying these instabilities with all kinetic effects properly included.

Drift-Tearing Mode scaling

- For cylindrical plasma, drift-tearing mode scaling are

$$\frac{\delta_c}{\rho_i} \sim \frac{a\Delta'}{\beta(m_i/m_e)}\rho_*,$$
$$\frac{\gamma_c}{c_s/R} \sim n\rho_*^2 \frac{s(r_s\Delta')}{\beta\sqrt{m_i/m_e}}\left(\frac{a}{r_s}\right)^2,$$
$$\frac{\omega_*}{c_s/R} \sim m\rho_*\left(\frac{R}{L}\right)\left(\frac{a}{r_s}\right),$$

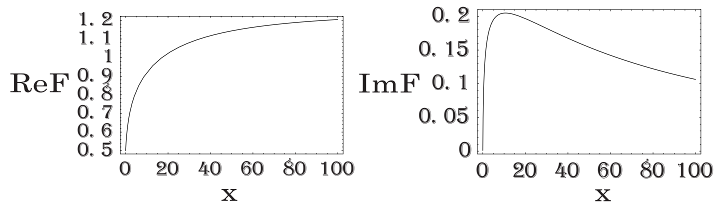
- where $s = d\ln q/d\ln r$, $\rho_* = \rho_i/a$, (n, m) are toroidal and poloidal mode numbers, and r_s is the location of the rational surface.

- For moderate β of several percent, $\delta_c/\rho_i \sim \rho_*$ and $\gamma_c/(c_s/R) \sim \rho_*^2$, which are quite small for $\rho_* \ll 1$.

- For the mode with $\nu \sim \omega_*$, the width of the current layer is

$$\frac{\delta}{\rho_i} \sim \delta_c \frac{\omega_*}{\gamma_c} \sim \left(\frac{q}{s}\right)\sqrt{\frac{m_e}{m_i}}.$$

Micro-tearing Mode Properties, cont'd



- If $\omega_{Te}^* \neq 0$ the mode is unstable. For example for $\omega_n^* = 0$

$$\frac{\omega}{\omega_{Te}^*} = F\left(\frac{\bar{\nu}}{\omega_{Te}^*}\right).$$

- Function $F(x)$ is plotted below for Maxwellian plasma and $\nu(v) = \bar{\nu}(v_{te}/v)^3$

- For given ω^*_{Te} the growth rate is maximized $Im\omega \approx 0.2\omega^*_{Te}$ for $5 < \bar{\nu}/\omega^*_{Te} < 20$.

- For given ω^*_{Te} the the mode frequency $1/2 < Re\omega/\omega^*_{Te} < 5/4$.

Micro-tearing Mode Properties, cont'd

- The role of neglected $\gamma_c/(c_s/R) \sim -m^2\rho_*^2/\beta$ (for $m \gg 1$) is to provide stabilization of the mode with sufficiently large m.

- Therefore the most unstable mode scales as

$$m \sim \frac{\beta}{\rho_*}\sqrt{\frac{m_i}{m_e}}, \quad \text{or,} \quad k_\theta \rho_i \sim \beta\sqrt{\frac{m_i}{m_e}},$$

- Provided that $\bar{\nu}/(c_s/R) \sim \omega_{Te}^*/(c_s/R) \sim m\rho_*(R/L_{Te})$.

- Localized micro-tearing mode can exist only if $\omega \sim \omega_{Te}^* < \omega_A(\rho_i)$, or

$$\omega_{Te}^* < \rho_i \frac{d}{dr}\left(\frac{k_\parallel c_s}{\sqrt{\beta}}\right), \quad \text{or,} \quad \beta\left(\frac{R}{L_{Te}}\right)^2 < \left(\frac{s}{q}\right)^2 \text{ (Gladd, 1980.).}$$