50 mins

Magnetic Reconnection Rate in Collisionless Plasmas

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Outlines

- ★ Theory of Reconnection Rate; Past —Present
- ★ From non-relativistic to relativistic
- ★ From symmetric to asymmetric
- ★ Toward the high-β limit
- ★ Short X-line limit
- ★ Controlling the 3D nature? — X-line orientation!
- ★ Summary & Future (Unsolved questions)

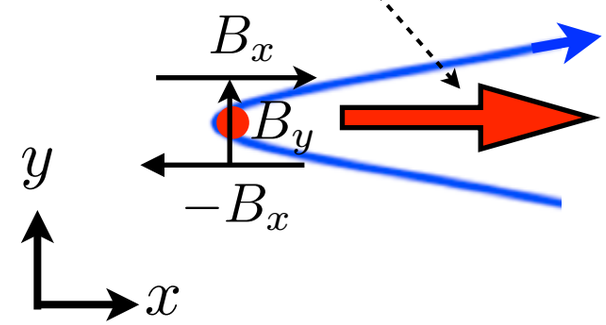
*Sorry…I am not going to focus on plasmoid instability in this talk

Theory of Reconnection Rate
Past — Present

Magnetic tension & Alfvén waves

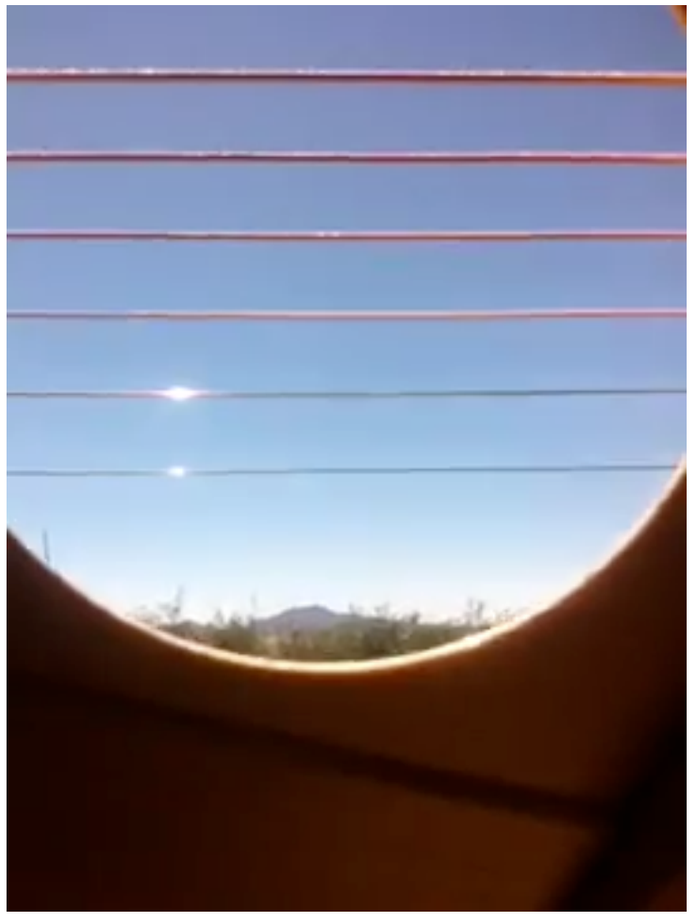
$$\frac{B_y(\partial_y B_x)}{4\pi}\hat{x}$$

magnetic tension: $\dfrac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi}$



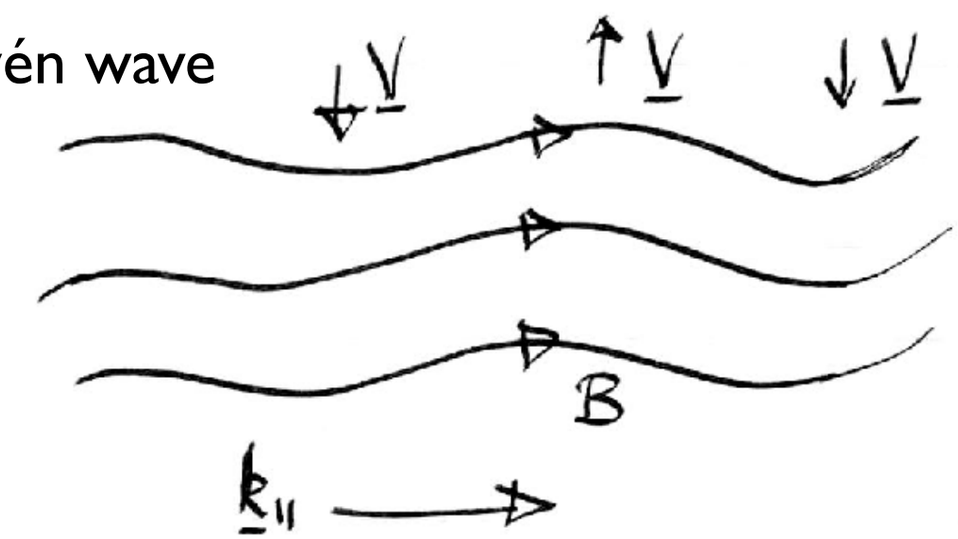
plasma inertia: $nm_i\dfrac{d\mathbf{V}}{dt}$

vibration of guitar strings



(Youtube: iphone 4 inside a guitar oscillation! VERY COOL!)

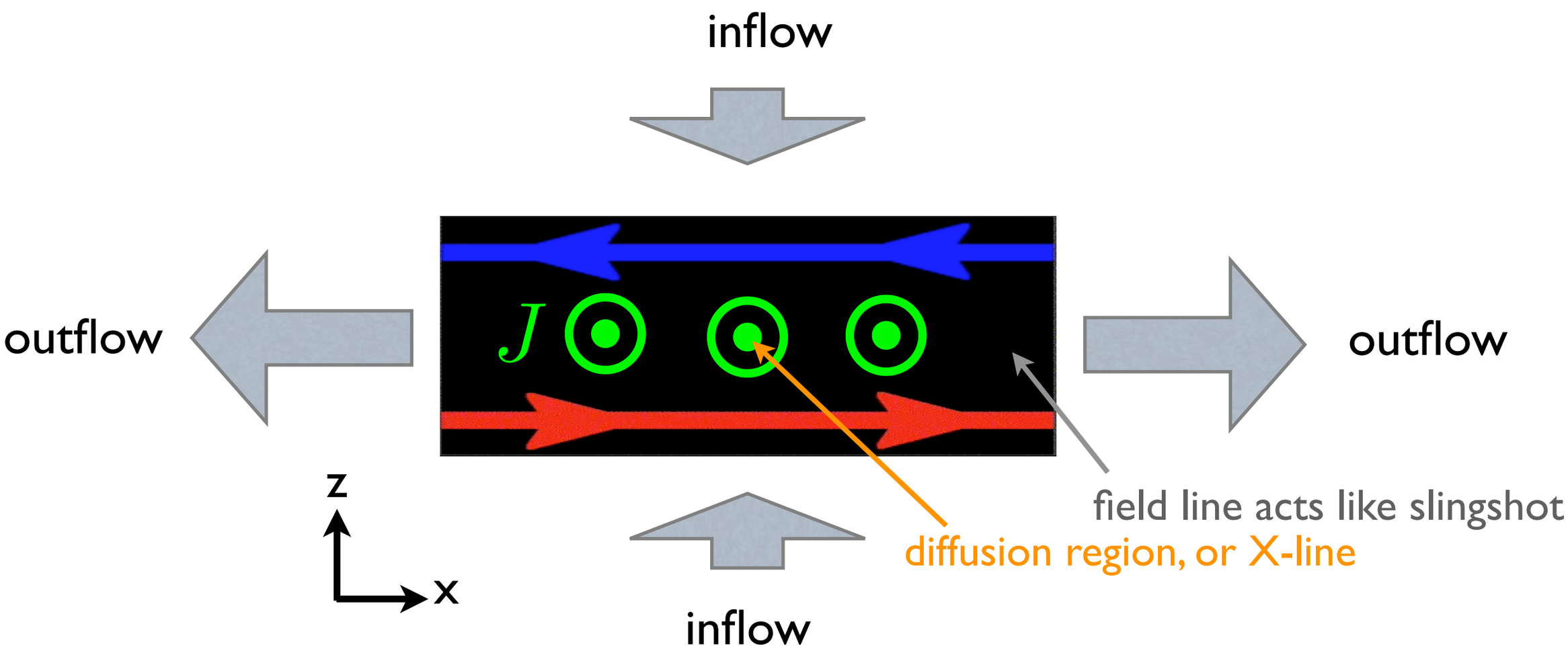
Alfvén wave



Alfvén speed

$$\rightarrow V_A \sim \frac{B}{\sqrt{4\pi n m_i}}$$

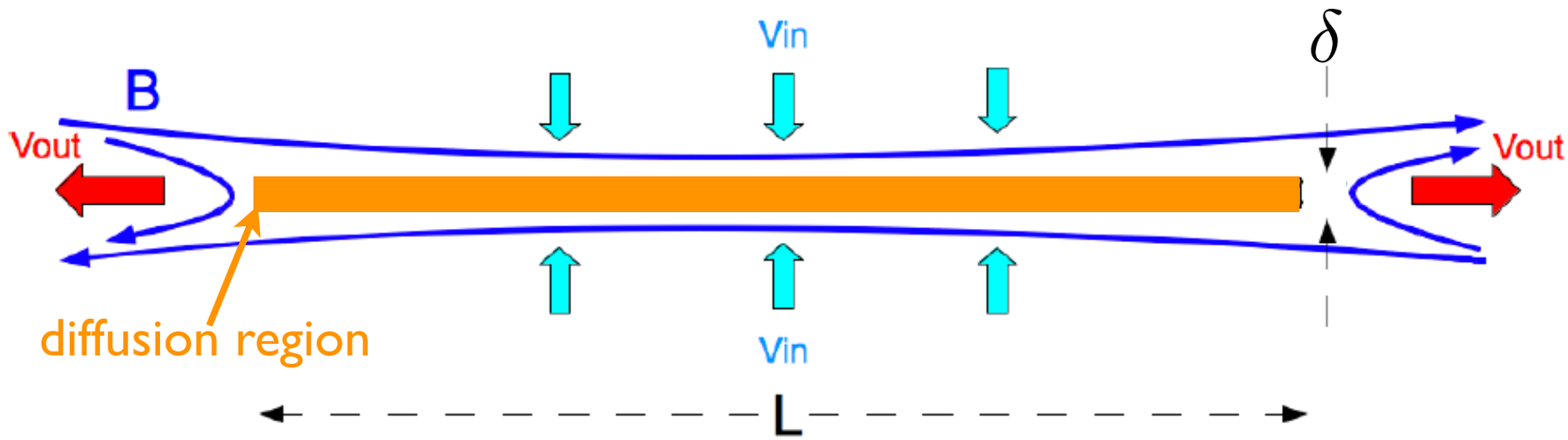
Magnetic Reconnection



1. Inflow brings in magnetic flux (*frozen-in*)
2. Field lines break & reconnect (*frozen-in* is violated !!)
3. Reconnected field line shoots out plasma (*frozen-in*)
4. Pressure drop sucks in plasma inflow
1. Inflow brings in magnetic flux (*frozen-in*)
2. .....
3. ...

A self-driven process!!!

Sweet-Parker solution (1957)



mass conservation: $\nabla \cdot (n\mathbf{V}) \simeq 0$ $\rightarrow V_{in} L \simeq V_{out} \delta$

momentum eq.: $\dfrac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} \simeq n m_i \mathbf{V} \cdot \nabla \mathbf{V}$ $\rightarrow V_{out} \simeq \dfrac{B}{\sqrt{4\pi n m_i}} = V_A$!

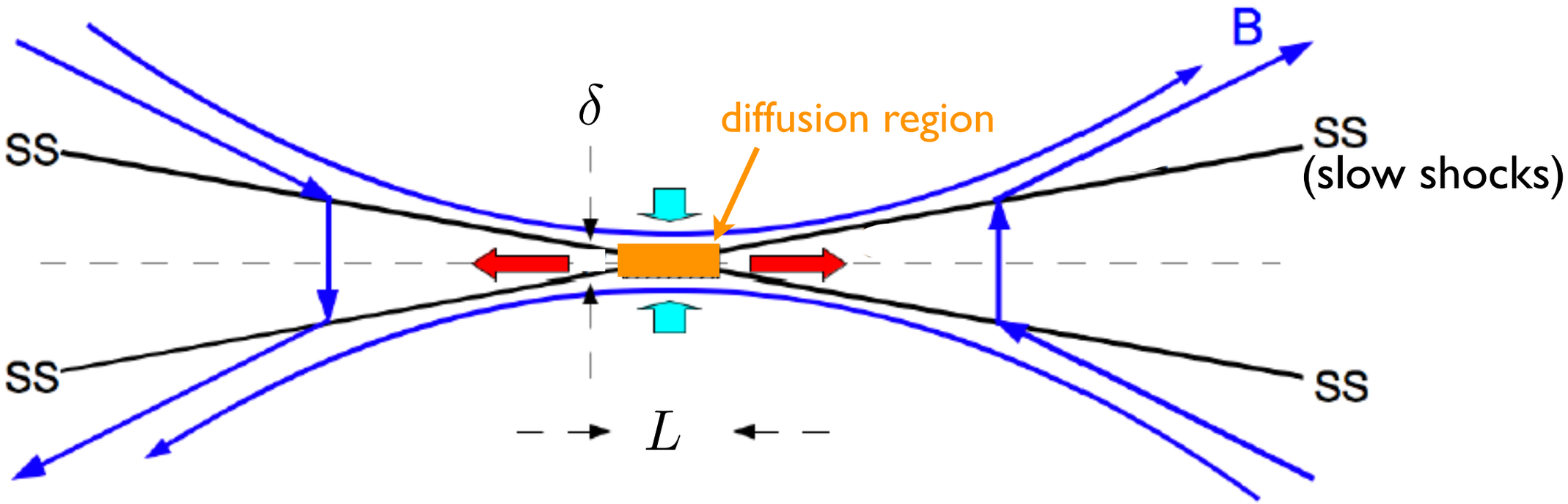
tension inertia

normalized reconnection rate $\rightarrow$ $\boxed{R \equiv \dfrac{V_{in}}{V_A} \sim \dfrac{\delta}{L}}$

- However, this model has a small δ/L ,
  the rate is also too small to explain the time-scales in solar flare. (Parker 1963)

- To explain the flares, it requires $R \simeq \mathcal{O}(0.1)$ (Parker 1973)

Petschek solution (1964)

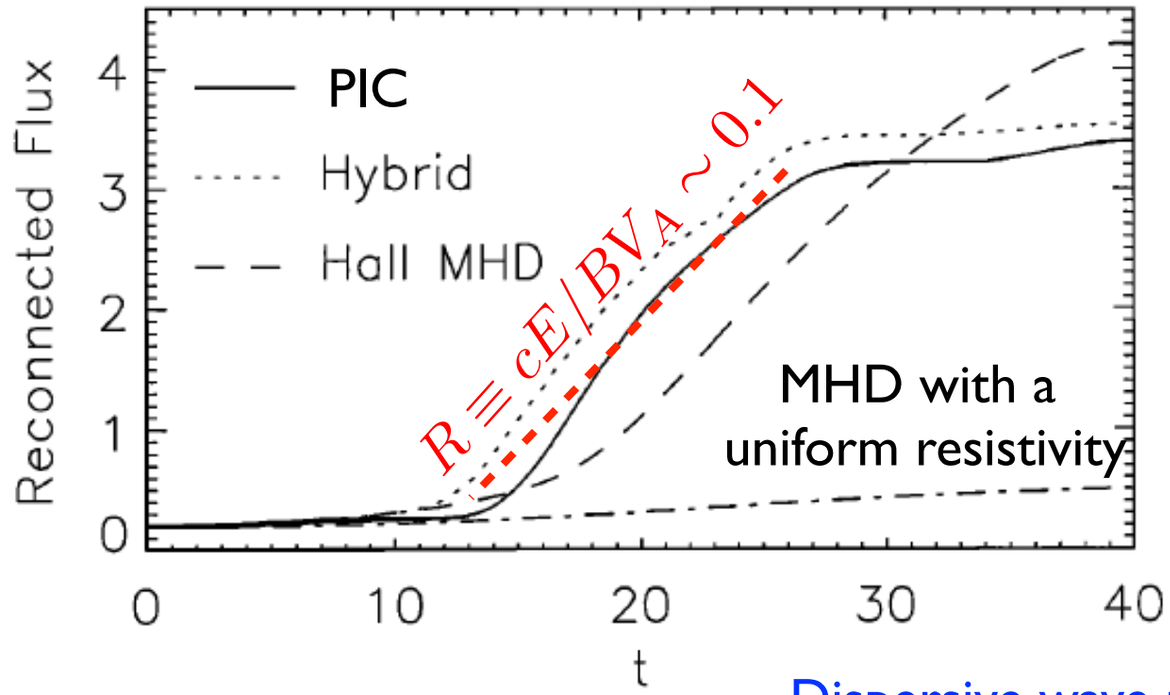


Reconnection rate is much larger if $R \sim \frac{\delta}{L} \uparrow$

- However, this is not a self-consistent solution. (Sato & Hayashi, 79; Biskamp, 86; Kulsrud 01)

*aspect ratio = aspect ratio of the diffusion region

GEM Reconnection Challenge (2001)



(Birn+ 2001)

Dispersive wave picture
(Sonnerup 79; Shay+ 98; Rogers+ 01; Drake+ 08)

Ohm's Law in plasmas: $\mathbf{E} + \frac{\mathbf{v}_i \times \mathbf{B}}{c} = \eta \mathbf{J} + \frac{1}{nec}\mathbf{J} \times \mathbf{B} - \frac{1}{ne}\nabla \cdot \mathbf{P}_e + \frac{m_e}{e^2}\frac{d\mathbf{J}/n}{dt}$

	convection	resis.	Hall term	pressure	inertia	
MHD:	✓					slow
Hall MHD:	✓		✓			↑
Hybrid:	✓		✓		✓	fast with R~0.1
PIC:			✓	✓	✓	↓
However, electron-positron (PIC): (Bessho & Bhattacharjee, 05; Daughton+ 07; Swisdak+ 08; Liu+ 09)				✓	✓	also fast with R~0.1
strong guide field limit (PIC): (Liu+14; TenBarge+14; Stainer+15; Cassak+15)				✓	✓	

Q1: Why is the fast rate $R \simeq \mathcal{O}(0.1)$? Q2: What is the localization mech.?

To be solved.

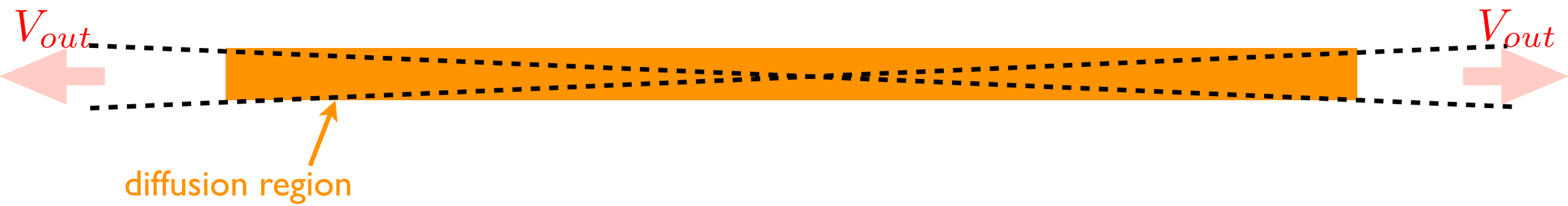
Q1: How to explain the fast reconnection rate value of order 0.1 in different systems?

—including PIC, hybrid, Hall-MHD, MHD with a localized resistivity, even MHD with numerical resistivity only……etc

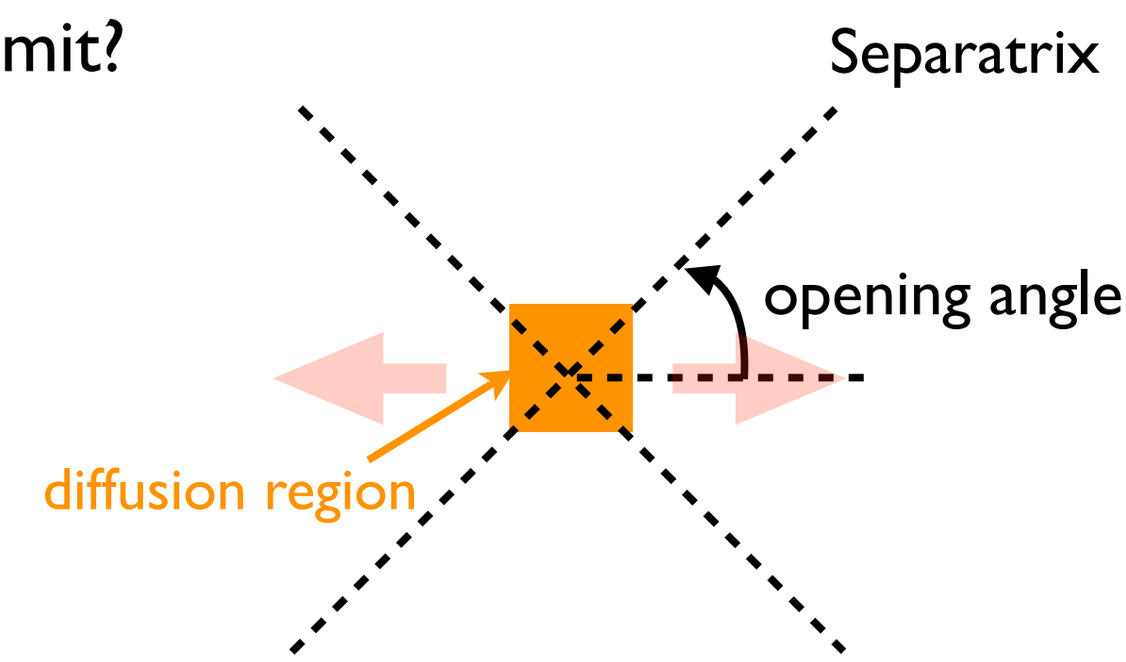
*clue: can not be the diffusion-scale physics!!!

Two extreme limits…

1. In the small δ/L limit, $R \sim \delta/L \to 0$



2. How about the large δ/L limit?



It turns out that when $\delta/L \to 1\,, \quad R \to 0$!

✦ Hey! there must be a maximum in between these two limits~

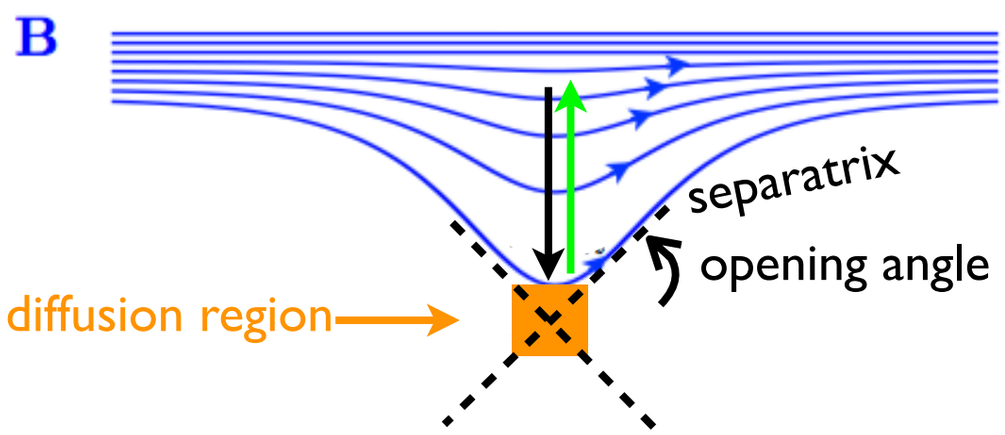
The Key: Geometry & Force balance!

In the large diffusion region aspect ratio, δ/L , limit .....

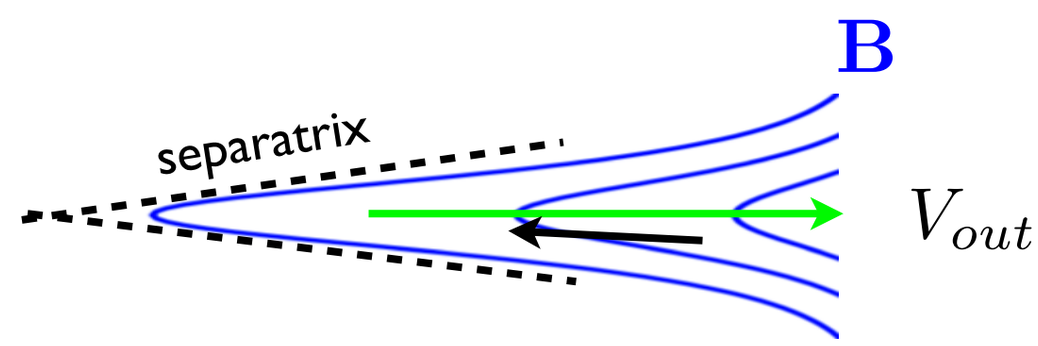
tension magnetic pressure inertia

$$\frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi} \simeq \frac{\nabla(B^2)}{8\pi} + nm_i\mathbf{V}\cdot\nabla\mathbf{V}$$

@ inflow region



@ outflow region



$\rightarrow$ reduction of the reconnecting field!!!

$\rightarrow R\downarrow$

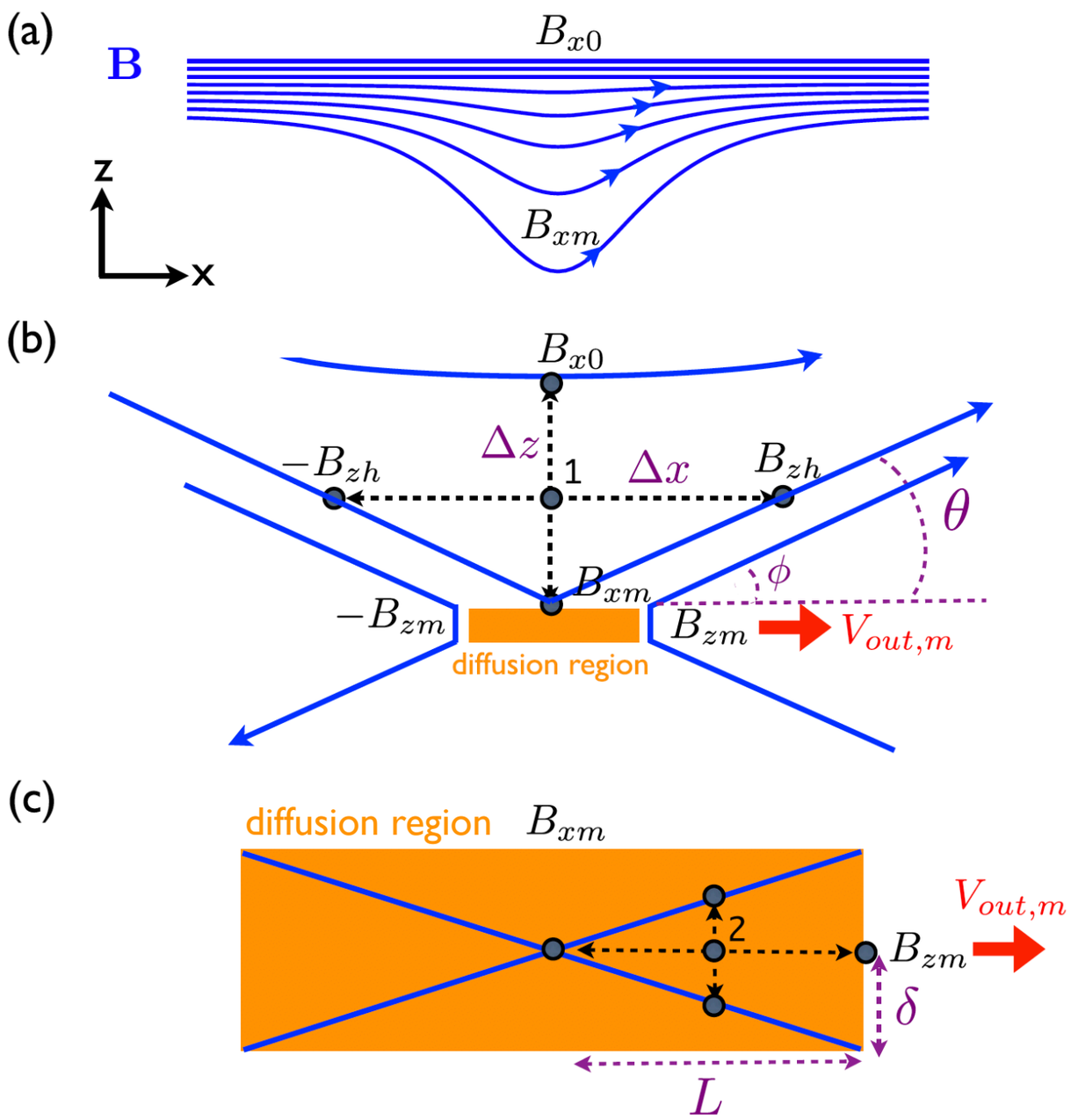
$\rightarrow$ reduction of the outflow speed!!!

$\rightarrow R\downarrow$

- Constraints imposed at the inflow & outflow regions (upper) bound the rate!

Back-of-the-envelope calculation…

(Liu+ PRL 2017)



step1: Introduce the scale-separation~

→ micro-scale ("m") vs. mesoscale ("0")

step2: analyze the inflow force-balance at point 1

$$\frac{B_{x0}^2 - B_{xm}^2}{8\pi\Delta z} \simeq \left(\frac{B_{x0} + B_{xm}}{2}\right)\frac{2B_{zh}}{4\pi\Delta x}$$

$$\rightarrow B_{zm}(S)$$

$Slope = \tan\theta$

step3: analyze the outflow force-balance at point 2

$$\rightarrow V_{out,m}(S)$$

step4: connect these two quantities to get the rate~

$$\rightarrow E_y(S) = B_{zm}V_{out,m}/c$$

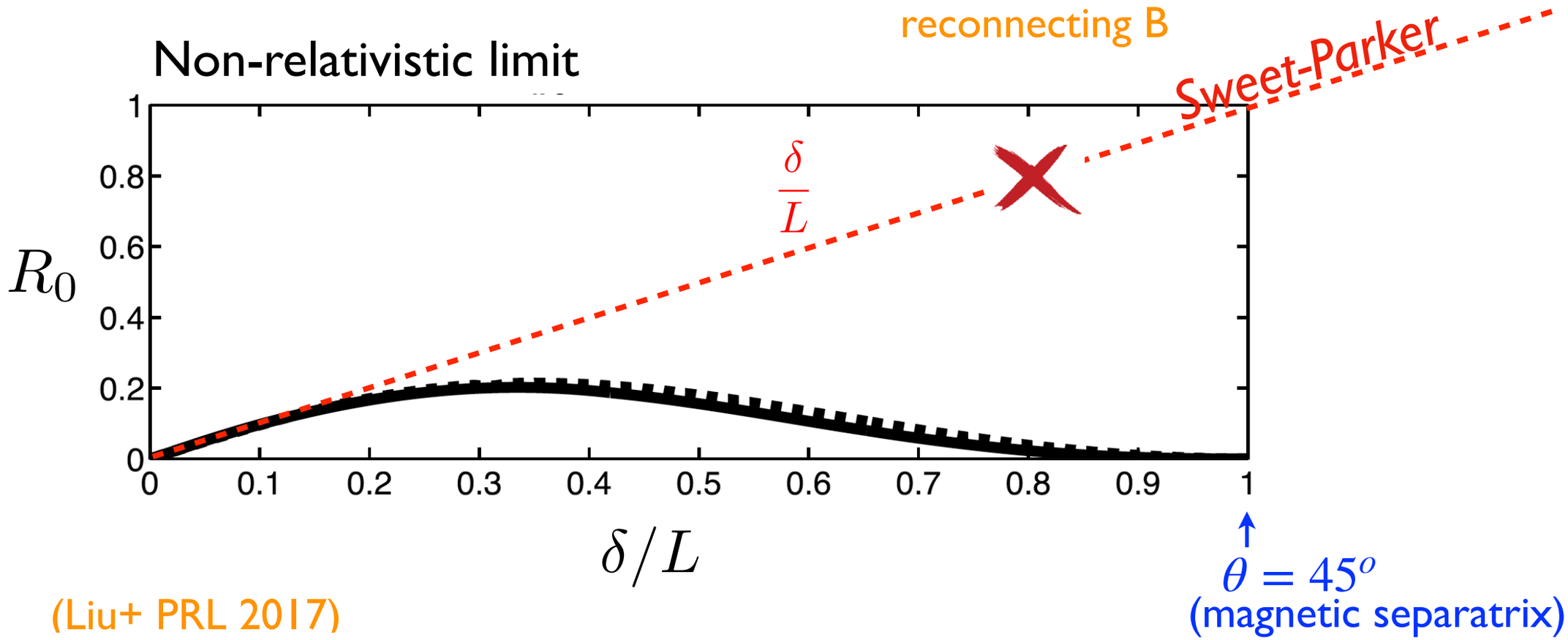
Explanation of the fast rate $\sim \mathcal{O}(0.1)$

-- Geometrical consideration!

reduction of V_{out}

$$R_0 \equiv \frac{cE_y}{B_{x0}V_{A0}} = \left(\frac{B_{zm}}{B_{x0}}\right)\left(\frac{V_{out,m}}{V_{A0}}\right) \simeq \frac{\delta}{L}\left[\frac{1-(\delta/L)^2}{1+(\delta/L)^2}\right]^2 \sqrt{1-\left(\frac{\delta}{L}\right)^2}$$

reduction of reconnecting B

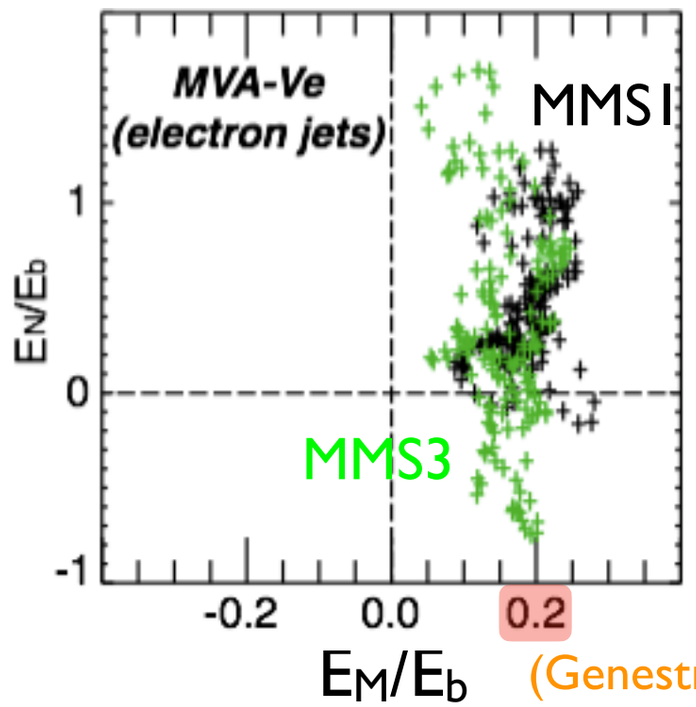


(Liu+ PRL 2017)

- $R \simeq 0.2$ is the predicted maximum plausible rate.
- Reconnection tends to proceed near this most efficient state.
- Nicely, $R \simeq \mathcal{O}(0.1)$ over a wide range of δ/L near this extreme.

Magnetospheric-Multiscale mission (MMS) Observations

7/11/2017 magnetotail event

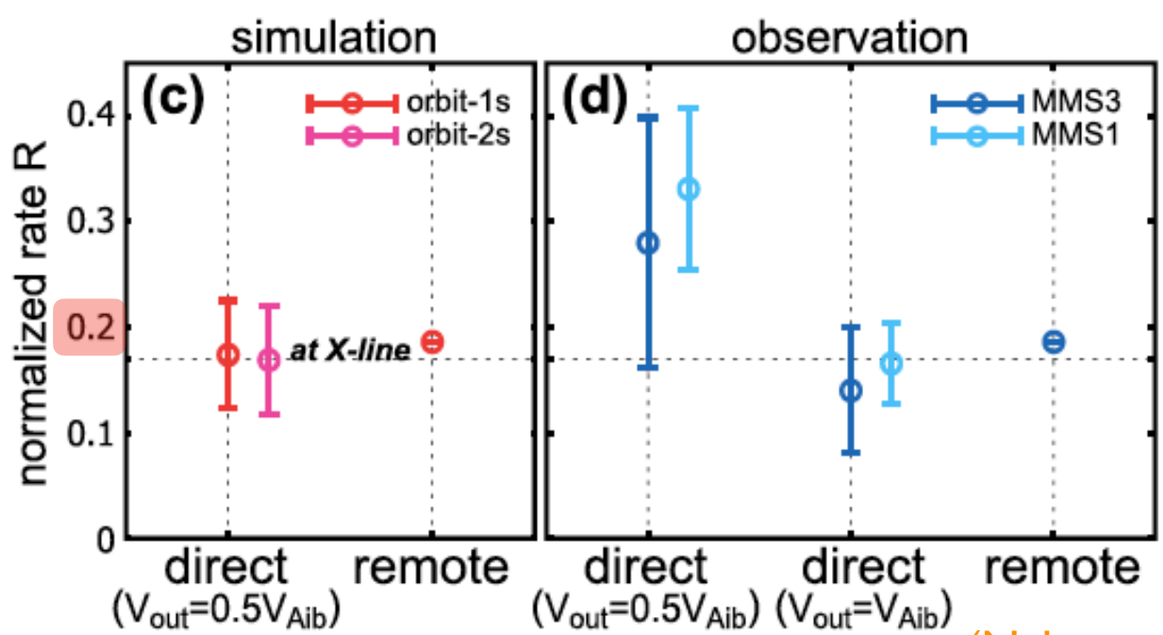


(Genestreti+ JGR 2018)

Measuring E_R=E_M

—Tried 14 different LMN coordinate systems

$\rightarrow R \simeq 0.18 \pm 0.035$



(Nakamura+ JGR 2018)

Measuring E_R=E_M

— Took advantage of the close comparison with 2D PIC simulations~

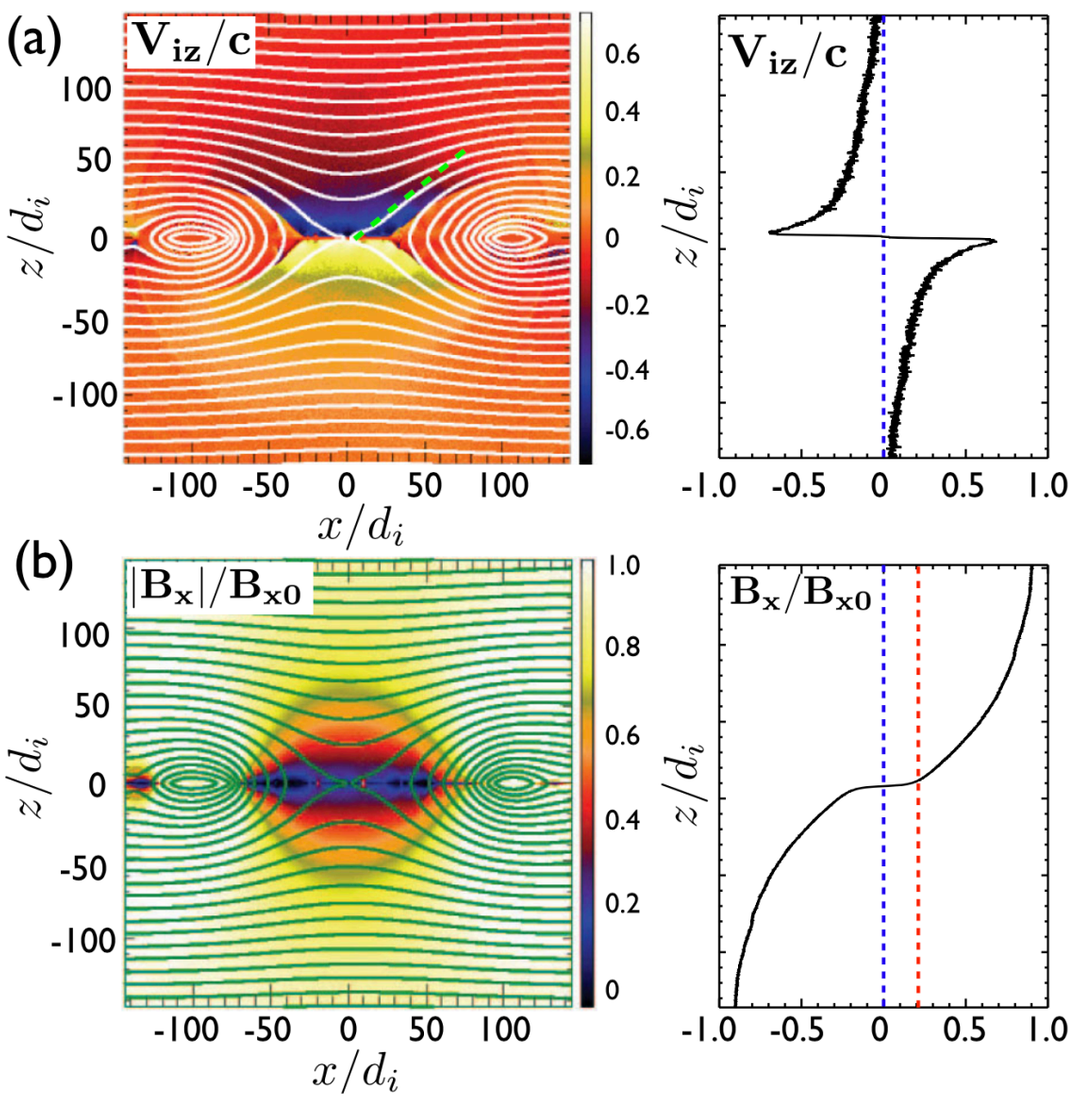
How about the relativistic limit?

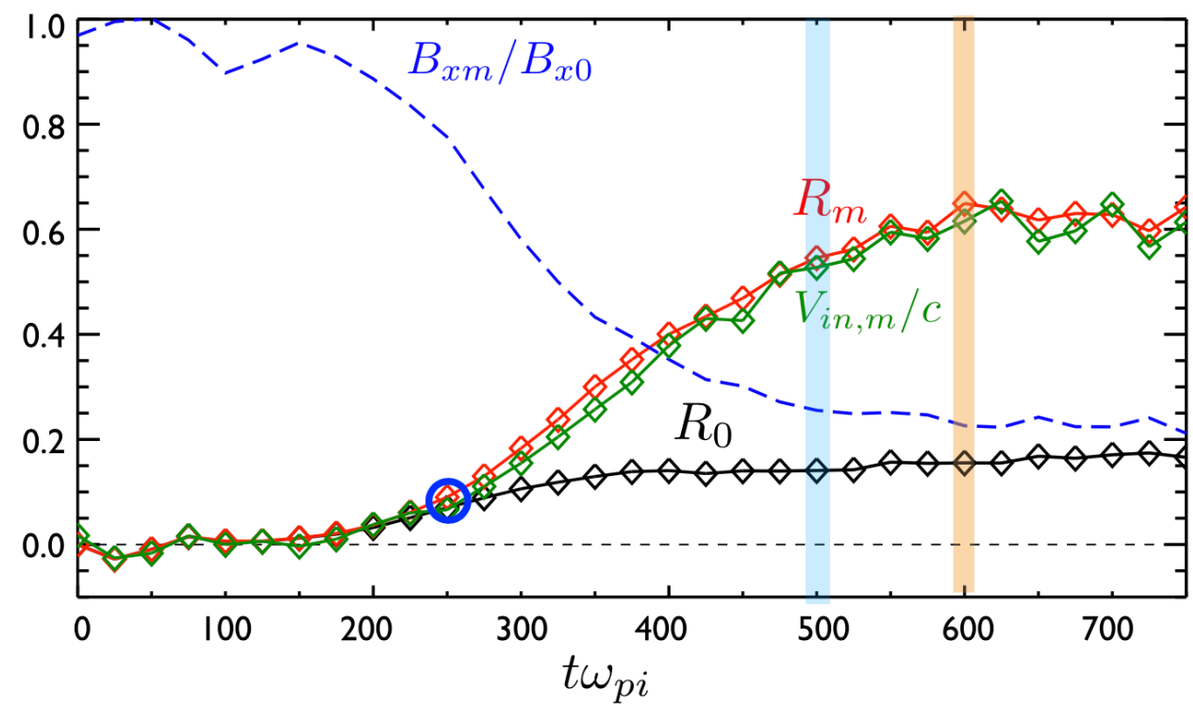
(e.g., Crab Nebula's superflares)

Credit: Hubble/Spitzer/Chandra observatories

Relativistic Reconnection

2D PIC simulation $\sigma = \dfrac{B_x^2}{4\pi n m c^2} \simeq 100$





(Liu+ PRL 2017, 2015
Guo, Liu+ PoP Invited Review, 2020)

- Just need a correction on V_A
- Clear "scale-separation" is observed! (i.e., $B_{xm}/B_{x0} \downarrow$)

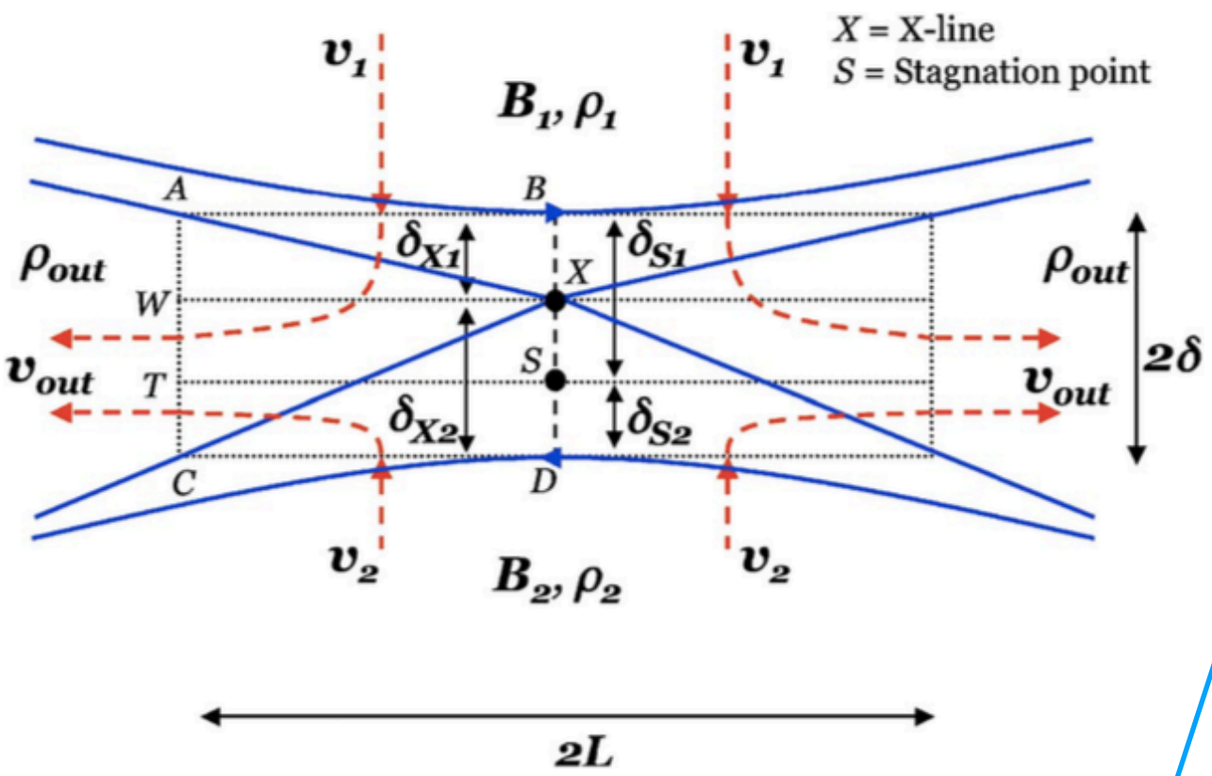
— explained why the rate normalized to microscopic quantities ($R_m \simeq V_{in,m}/c$) can go up to $\mathcal{O}(1)$, while the rate normalized to the asymptotic quantities (R_0) is still bounded by $\mathcal{O}(0.1)$.

Okay~ how about asymmetric reconnection?

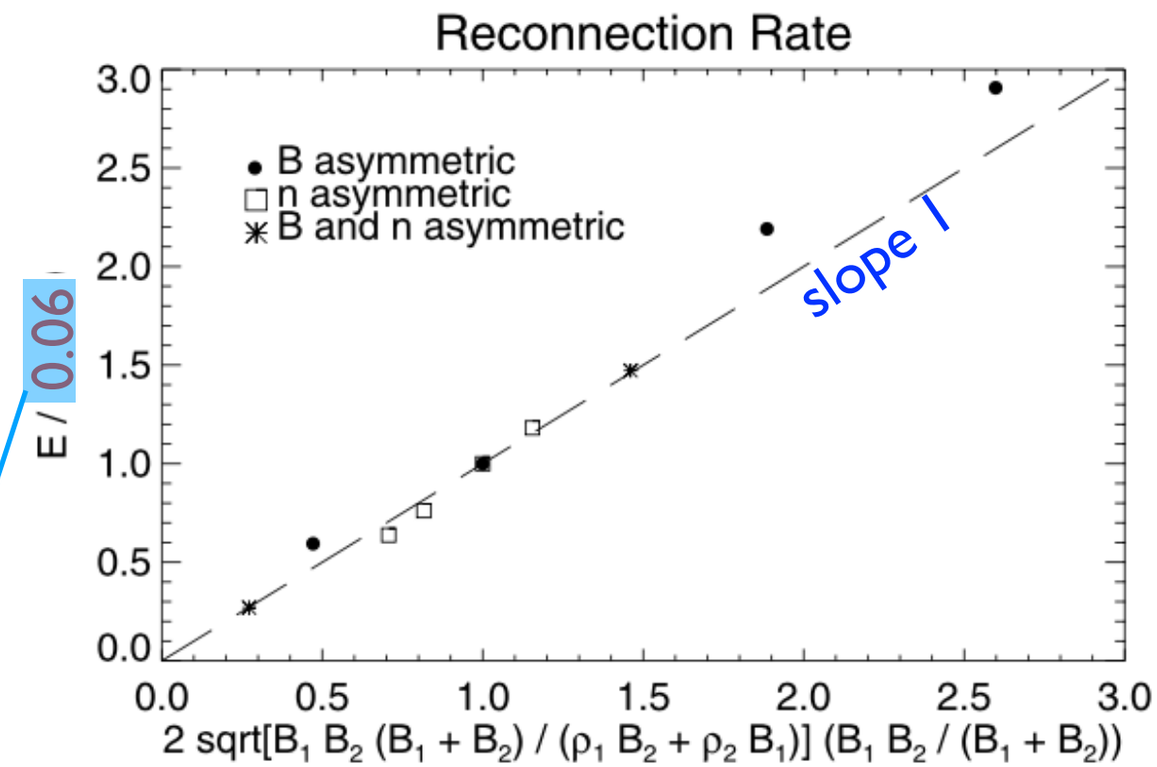
(e.g., Earth's magnetopause)

Credit: MMS /NASA website

Previous theory of Asymmetric Reconnection



Two-fluid simulations



Cassak & Shay GRL (2008)

Cassak-Shay formula

$$E_{CS} = 2\left(\frac{B_1 B_2}{B_1 + B_2}\right)\left(\frac{V_{out}}{c}\right)\left(\frac{\delta}{L}\right)_{eff}$$

Cassak & Shay, PoP (2007)

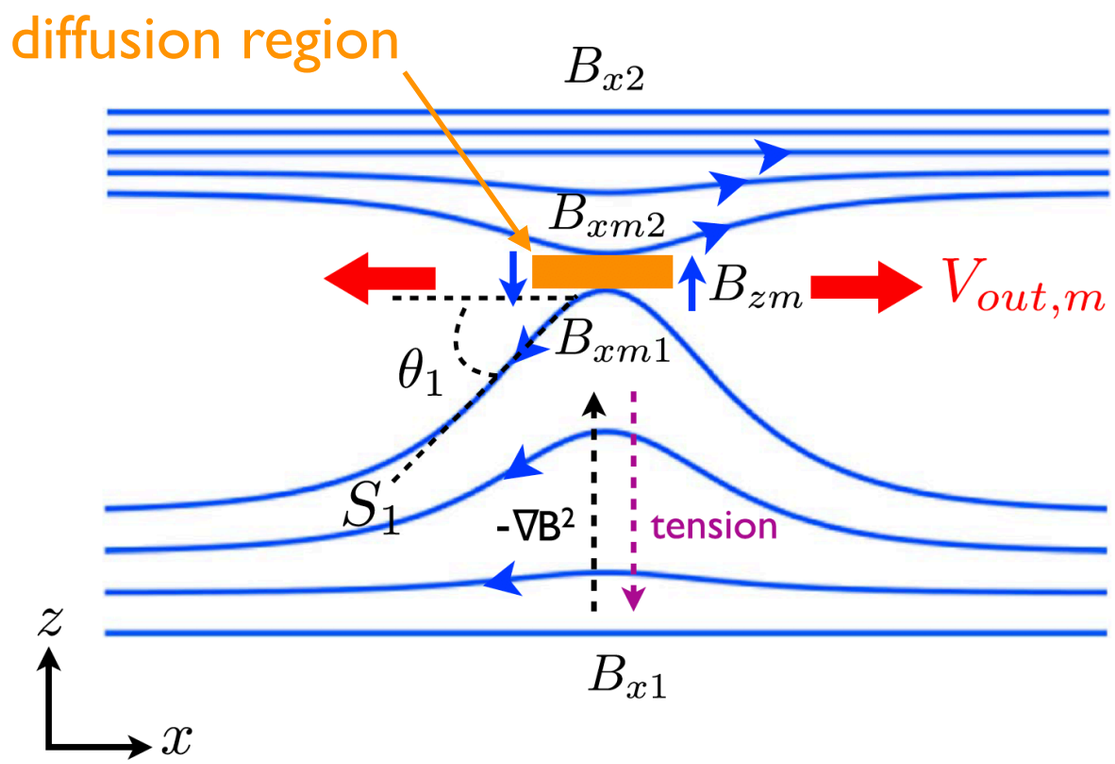
where

$$\frac{V_{out}}{c} = \sqrt{\frac{B_1 B_2}{4\pi}\left(\frac{B_1 + B_2}{B_1\rho_2 + B_2\rho_1}\right)}$$

Swisdak & Drake, GRL (2007)

$\rightarrow (\delta/L)_{eff} \sim \mathcal{O}(0.1)$! but why?

Constraint along the inflow (step1&2)



'*m*' indicates quantities at the micro-scale

From the force-balance along the inflow,

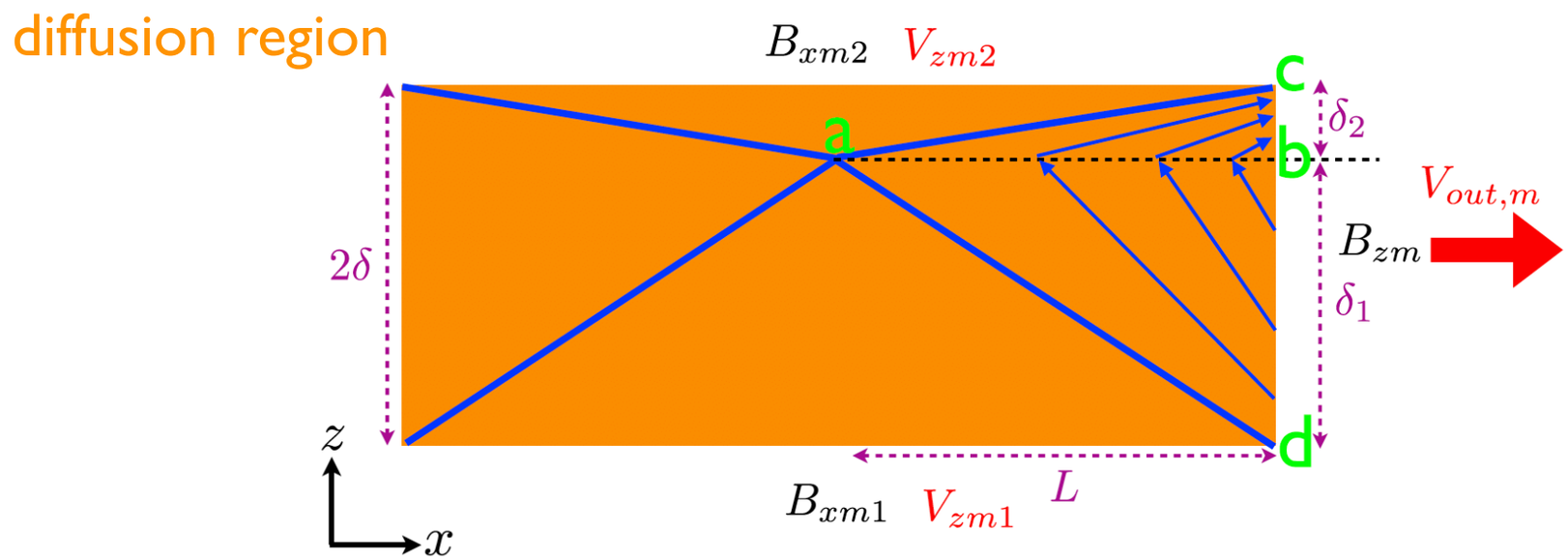
$$B_{xmi} \simeq B_{xi}\frac{1-S_i^2}{1+S_i^2}$$

slope of the upstream field lines

$$i = 1,2$$

- The reduction of the reconnecting field primarily comes from the weaker field side in the strong B-asymmetry limit.

Constraint along the outflow (step 3)



From the momentum equation along the outflow,

$$V_{out,m} \simeq \sqrt{\frac{B_{xm1}B_{xm2}}{4\pi\bar{\rho}} - \frac{1}{\pi\bar{\rho}}\left(\frac{B_{xm1}B_{xm2}}{B_{xm1}+B_{xm2}}\right)^2\left(\frac{\delta}{L}\right)^2}$$

tension

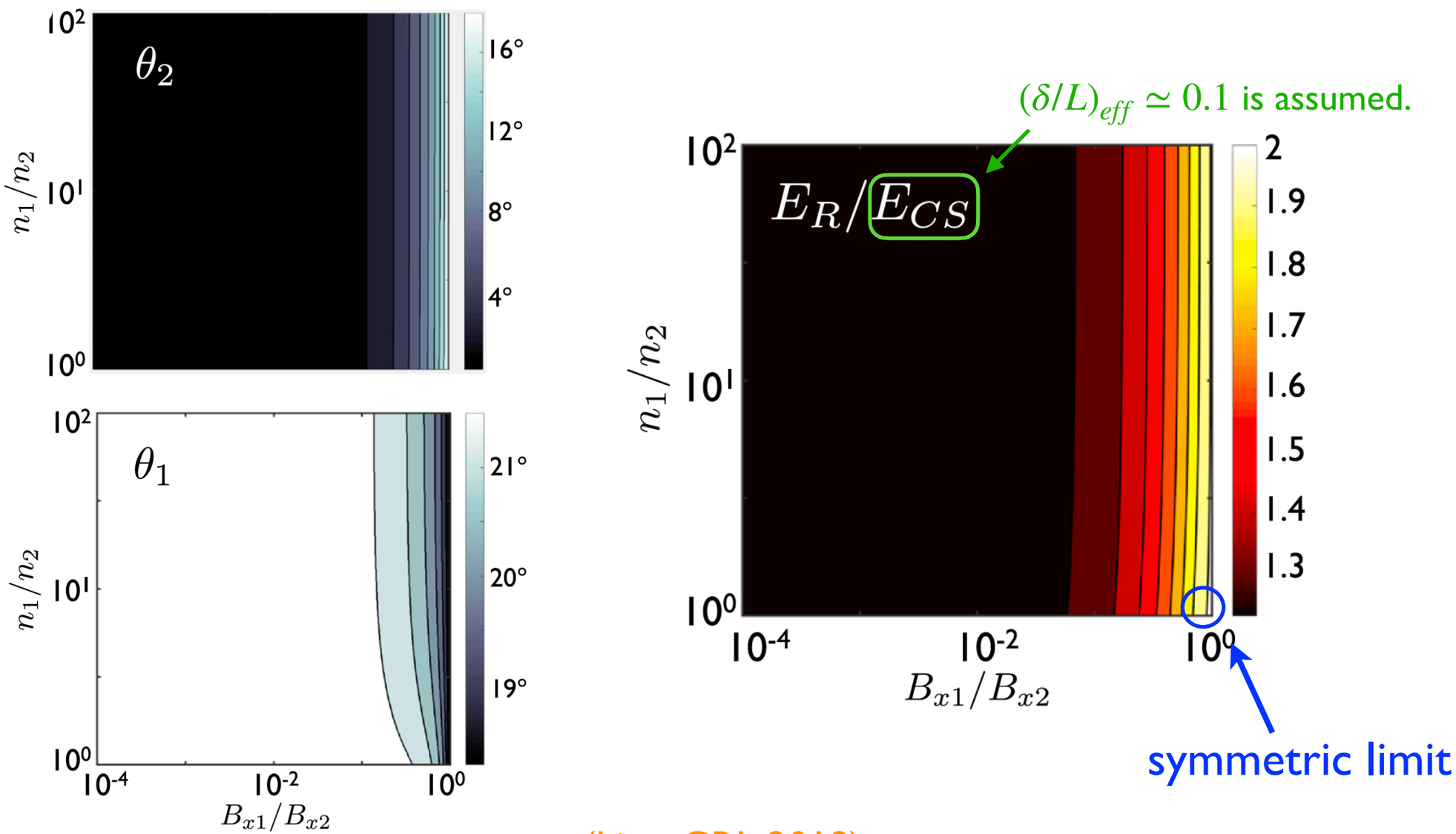
magnetic pressure

Swisdak & Drake, GRL (2007)
Cassak & Shay, PoP (2007)

- Outflow can be reduced in the large δ/L limit
  — but its effect is (B_{xm1}/B_{xm2}) times smaller in the strong B-asymmetry limit.
  — thus the reduction will be negligible in this limit.

Predicted asymmetric reconnection rate (step 4)

By maximizing $E_R(S_1) \simeq B_{zm}(S_1)V_{out,m}(S_1)/c$ .....



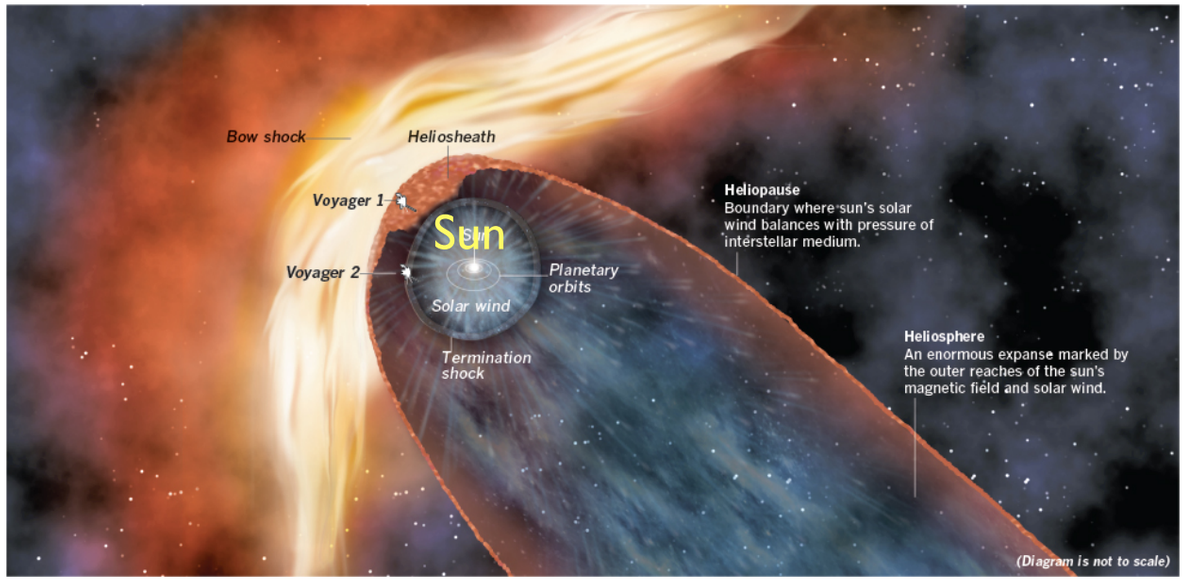
(Liu+ GRL 2018)

- This prediction agrees well with the *Cassak-Shay* scaling!

—this further suggests that $(\delta/L)_{eff} \simeq 0.1$ arises from constraints at the inflow & outflow.

Okay~~ how about the high-β limit?

(e.g., outer-heliosphere, intracluser medium, galactic center…etc)



Credit: Voygers/LA Times Graphics

In the high-β limit,
we need to include the thermal pressure effect…

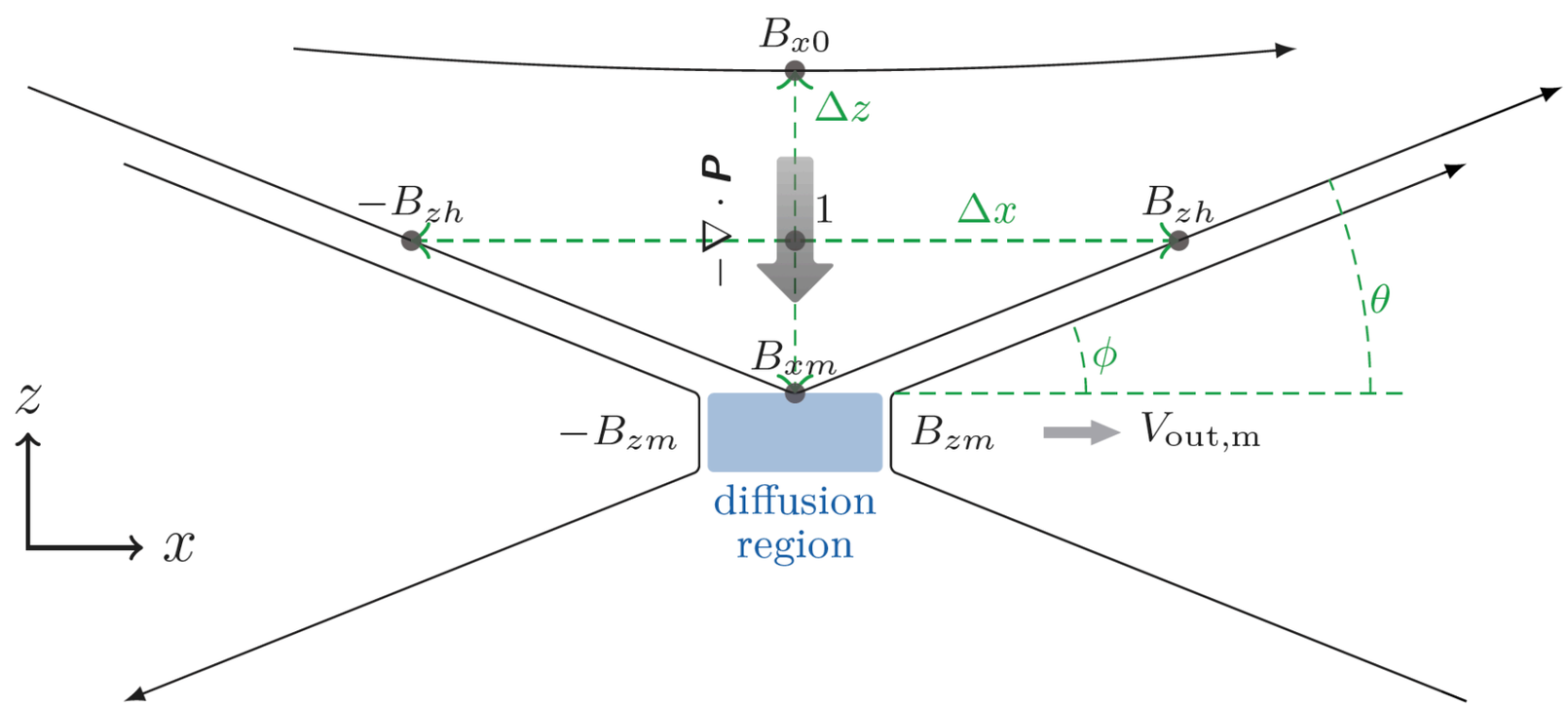
$$nm_i\frac{\partial \mathbf{V}}{\partial t} + nm_i\mathbf{V}\cdot\mathbf{V} + \nabla\frac{B^2}{8\pi} \simeq \nabla\cdot\left(\frac{\mathbf{BB}}{4\pi}\right)$$

$$\rightarrow nm_i\frac{\partial \mathbf{V}}{\partial t} + nm_i\mathbf{V}\cdot\mathbf{V} + \nabla\frac{B^2}{8\pi} + \nabla P_\perp \simeq \nabla\cdot\left(\varepsilon\frac{\mathbf{BB}}{4\pi}\right)$$

where

$\varepsilon = 1 - 4\pi(P_\parallel - P_\perp)/B^2$ the firehose parameter,
which can weaken the magnetic tension!

Constraint along the inflow (step1&2)



'0' indicates quantities at the mesoscale

'm' indicates quantities at the micro-scale

From the force-balance along the inflow, (X. Li & Liu, 2021)

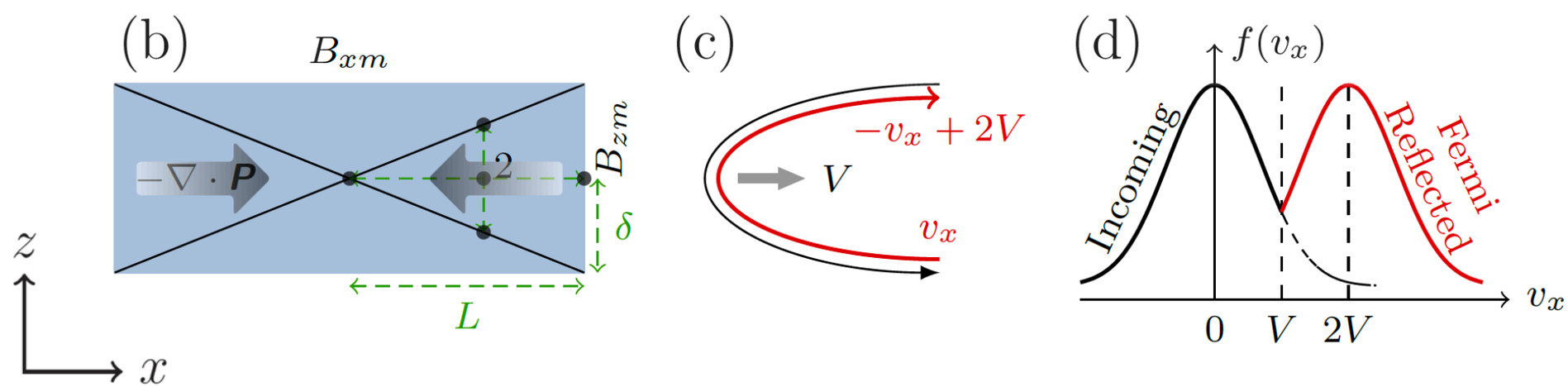
$$\frac{B_{x0}^2 - B_{xm}^2}{8\pi\Delta z} + \frac{P_0 + P_{i0}(\sqrt{2\beta_{i0}} + B_{xm}/B_{x0})}{\sqrt{2\beta_{i0}} + 1}\frac{B_{x0} - B_{xm}}{B_{x0}\Delta z} = \frac{\varepsilon_1}{4\pi}B_{x1}\frac{2B_{zh}}{\Delta x}$$

- The reduction of B_{xm} is less severe with a higher β.

  —because the pressure gradient helps balance the weaker upstream tension.

Constraint along the outflow (step 3)

diffusion region



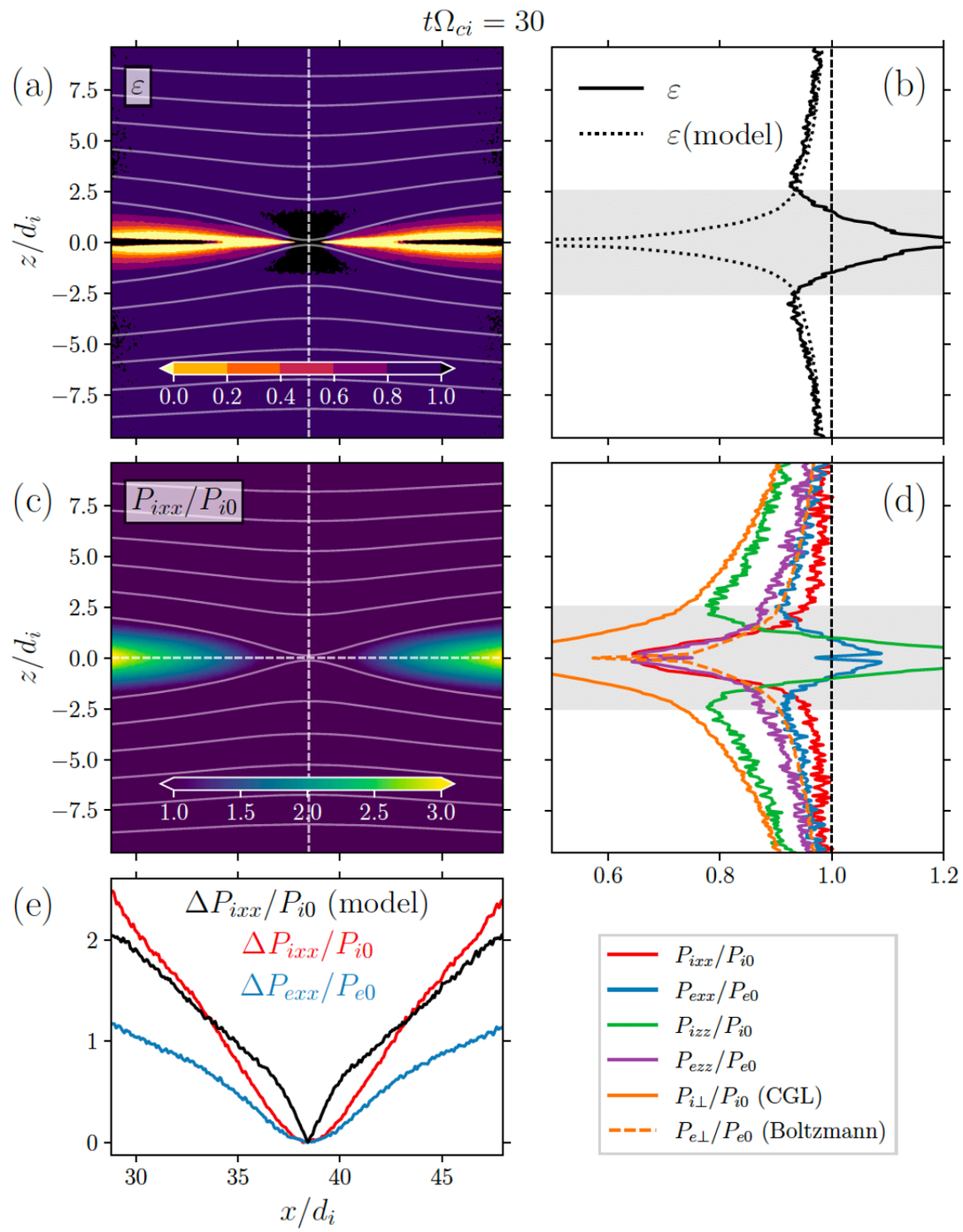
From the momentum equation along the outflow,

$$\frac{n_2 m_i V_{\mathrm{out,m}}^2}{2L} + \frac{B_{zm}^2}{8\pi L} + \frac{\Delta P_{xx,m}}{L} = \frac{1}{4\pi}\frac{B_{zm}}{2}\frac{\varepsilon_m B_{xm}/2}{\delta/2}$$

where $\Delta P_{ixx}(V) = n_0 m_i \left[V^2 + \left(V^2 + \frac{\beta_{i0}}{2} v_{A0}^2 \right) \mathrm{erf}\left(\frac{V}{\sqrt{\beta_{i0}} v_{A0}} \right) + V v_{A0} \sqrt{\frac{\beta_{i0}}{\pi}} e^{-V^2/(\beta_{i0} v_{A0}^2)} \right]$

- Outflows can be slowed down by the back-pressure and weaker tension.
- Predicted scaling is consistent with previous 81 PIC simulations and 14 in-situ observations in the solar wind, the magnetosheath, and the magnetotail! (Haggerty+ PoP 2018)

Finding a reasonable closure



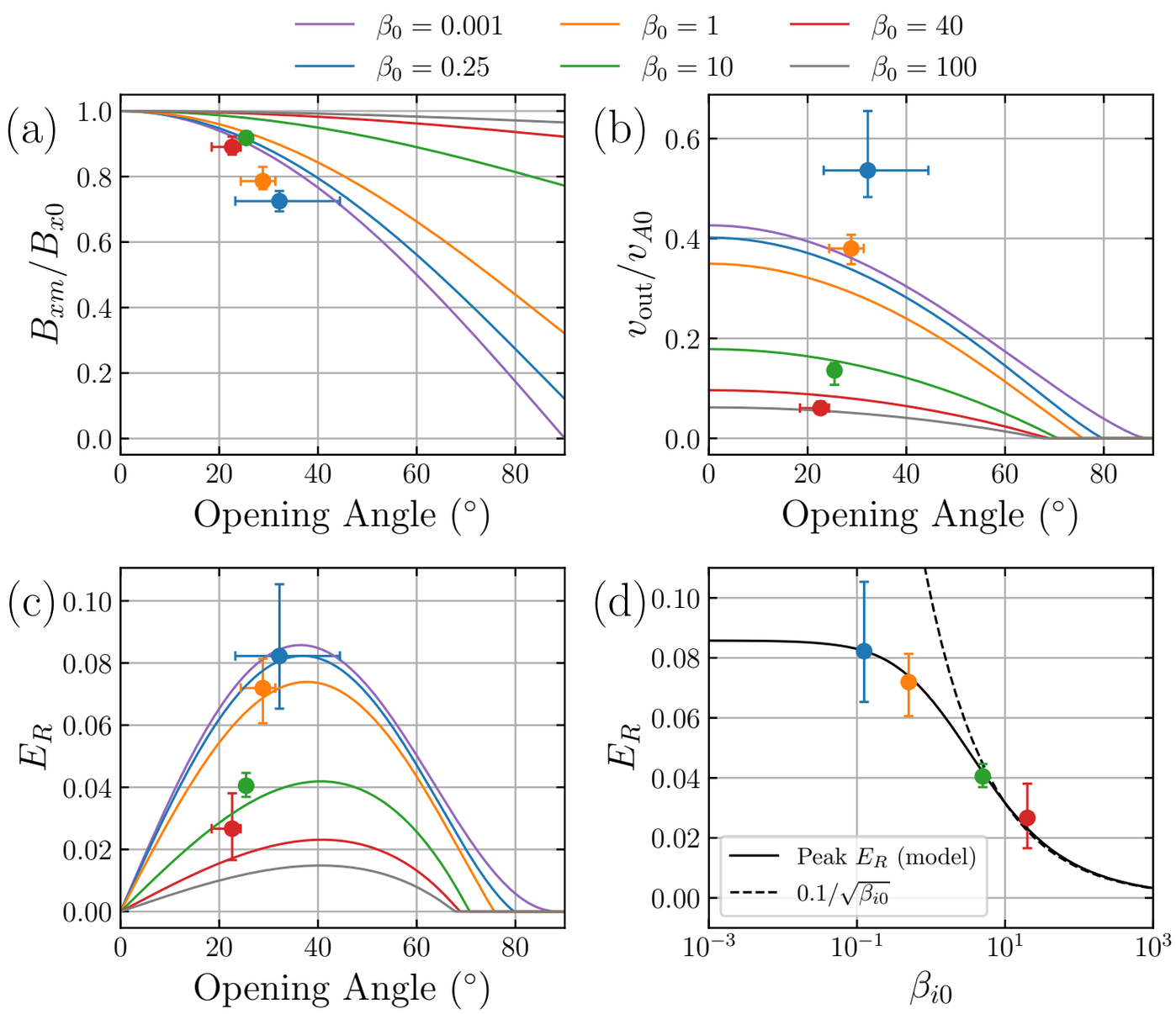
(X. Li & Liu, 2021)

- Fermi-reflection works well to model the back-pressure along the outflow.
- Along the inflow, a combination of CGL, Boltzmann, Le & Egedal closures are considered to model the simulated pressure anisotropy
  — results not very sensitive to the specific choice of closure though~

Predicted reconnection rate in the high-β limit (step 4)

$$E_R = B_{zm} V_{\mathrm{out,m}} / (B_{x0} v_{A0})$$

as a function of the opening angle ($\equiv 2\theta$)

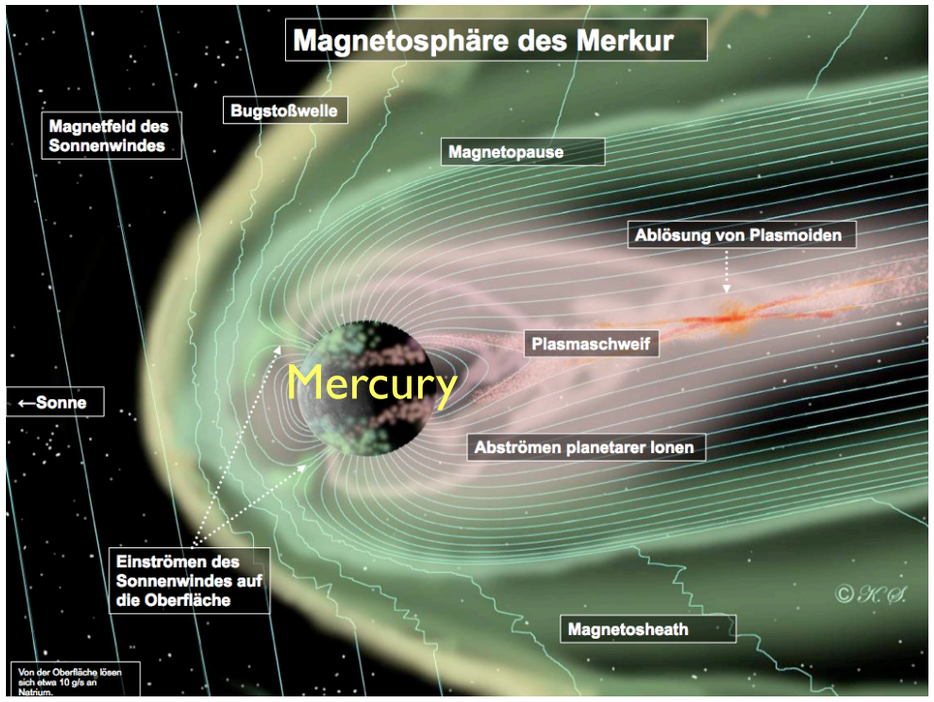


(X. Li & Liu, 2021)

- The predicted reconnection rate $R \simeq 0.1/\sqrt{\beta_i}$ in the high-β limit.

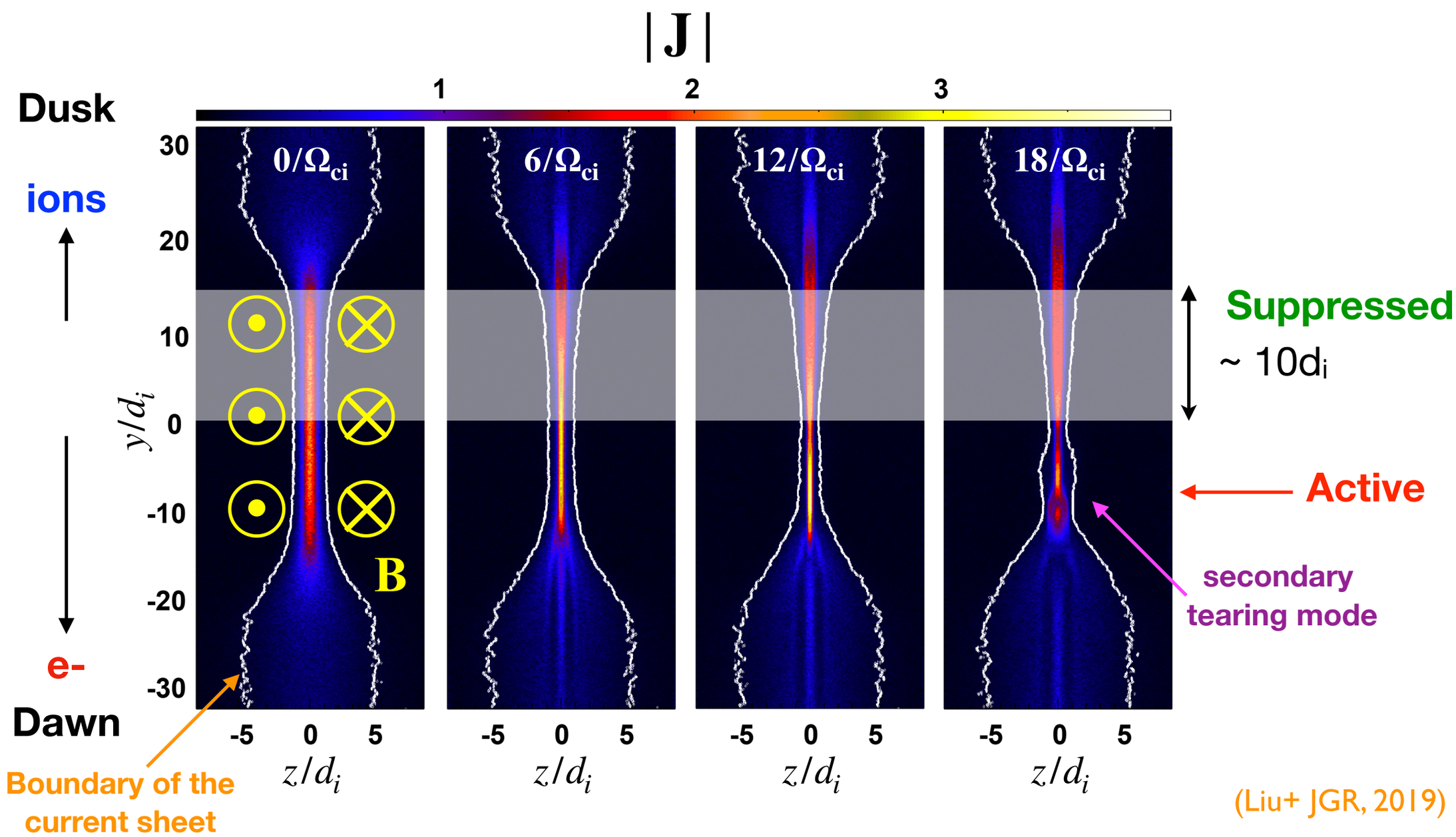
How about the short X-line limit?

(e.g., magnetotail of Mercury, Bursty-Bulk Flows, comets…etc)



Credit: Bepi Colombo /Max-Planck Institute website

A case with the thin current sheet extent similar to the global dawn-dusk extent of Mercury (~ 30d_i)



(Liu+ JGR, 2019)

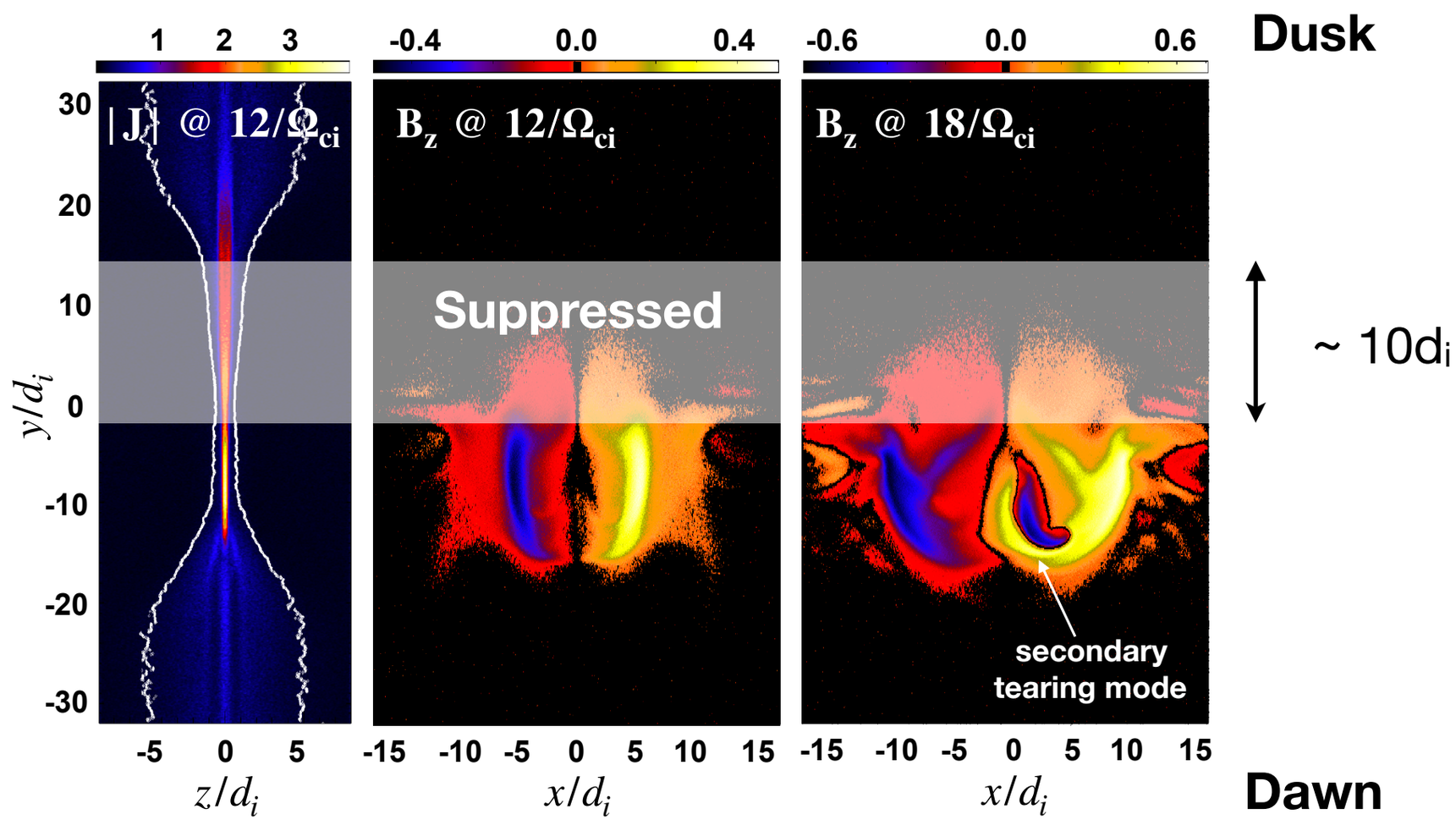
- Asymmetric thinning!

— thinner & more active on the electron-drifting-side (i.e., dawn-side)

— this can explain the dawn-dusk asymmetry observed in Mercury's magnetotail!

(W. Sun 2016; Rong 2018; Y. Chen 2019; C. Dong 2019)

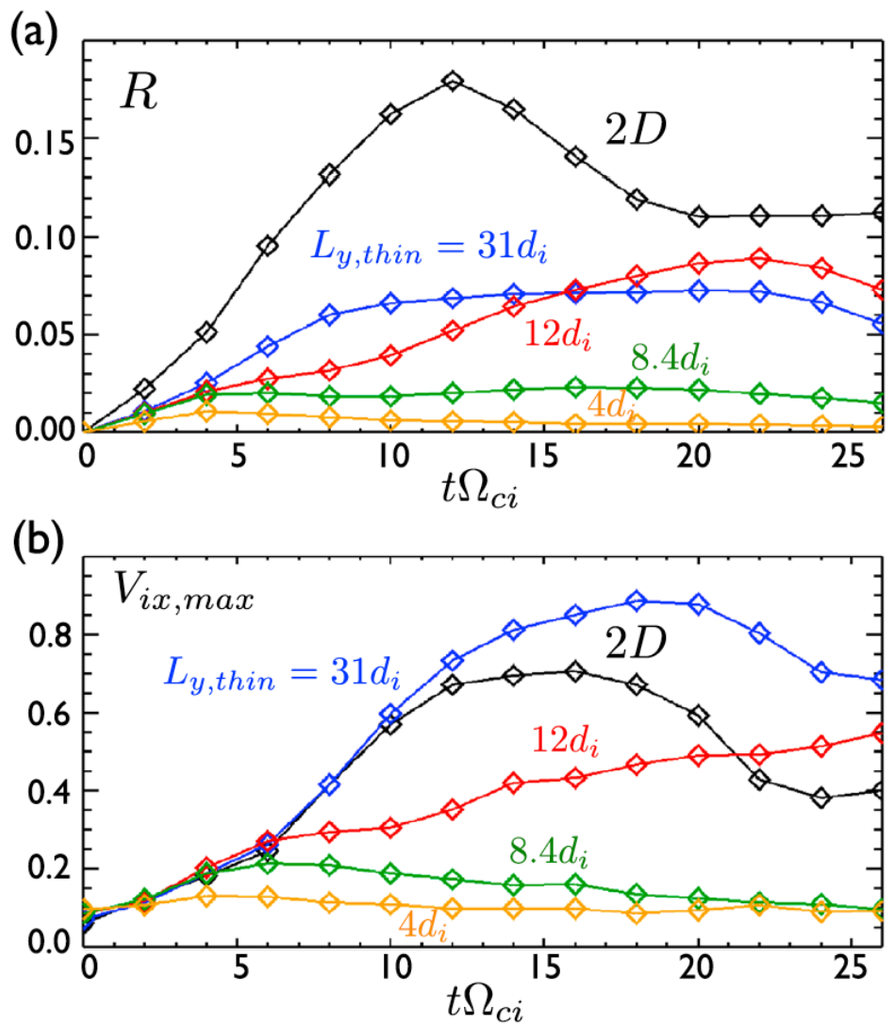
Revealing the "Internal X-line Asymmetry"



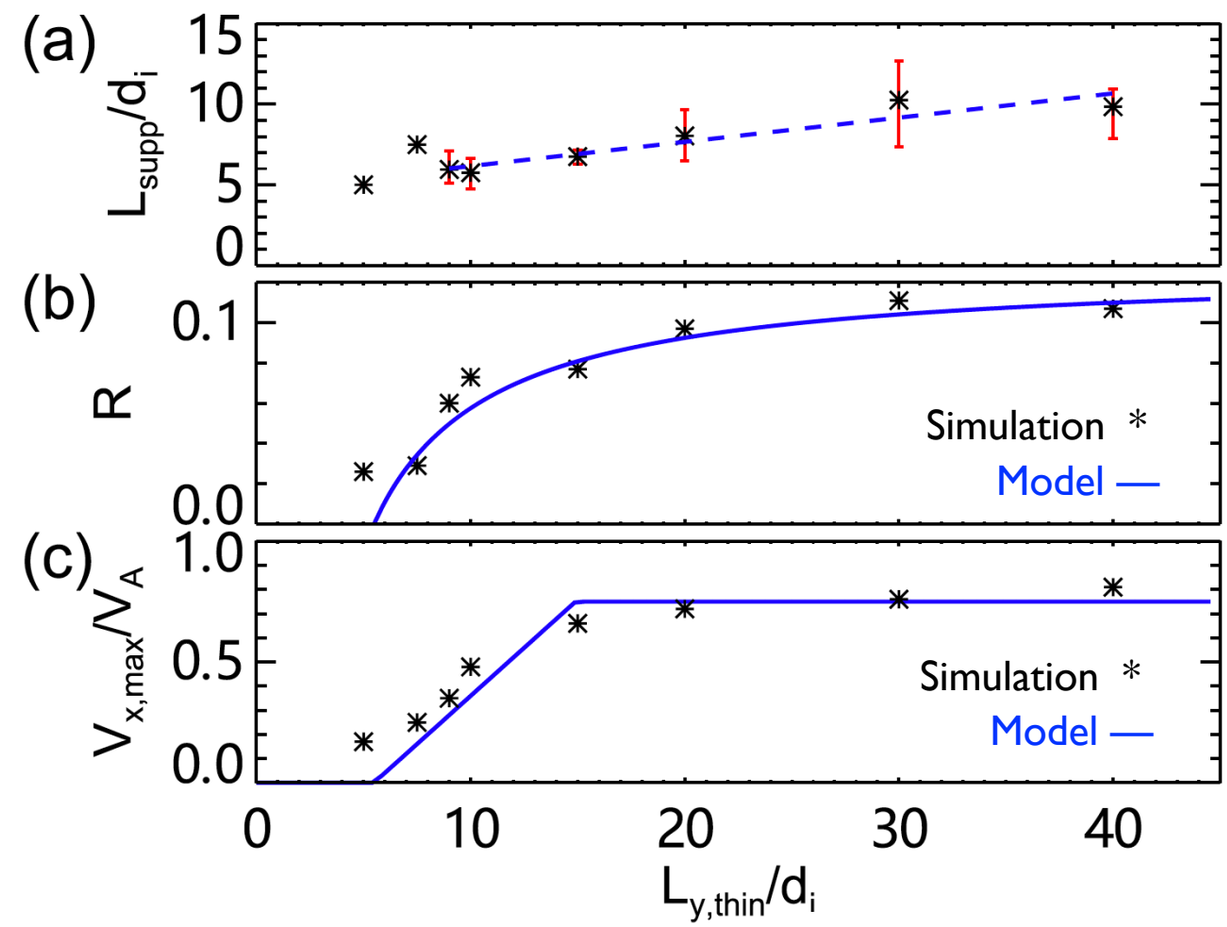
- Reconnection is more active on the electron-drifting-side (i.e., dawn side) because
  (1) reconnected B_z is critical to the tension force, that drives outflows, enables reconnection
  (2) this B_z is transported to the dawn-side by electrons beneath the ion kinetic scale. (i.e., Hall effect!)

Q: What if the x-line is too short (< the suppression region size~ $10d_i$)?

→Reconnection gonna be suppressed!



(Liu+ JGR, 2019)

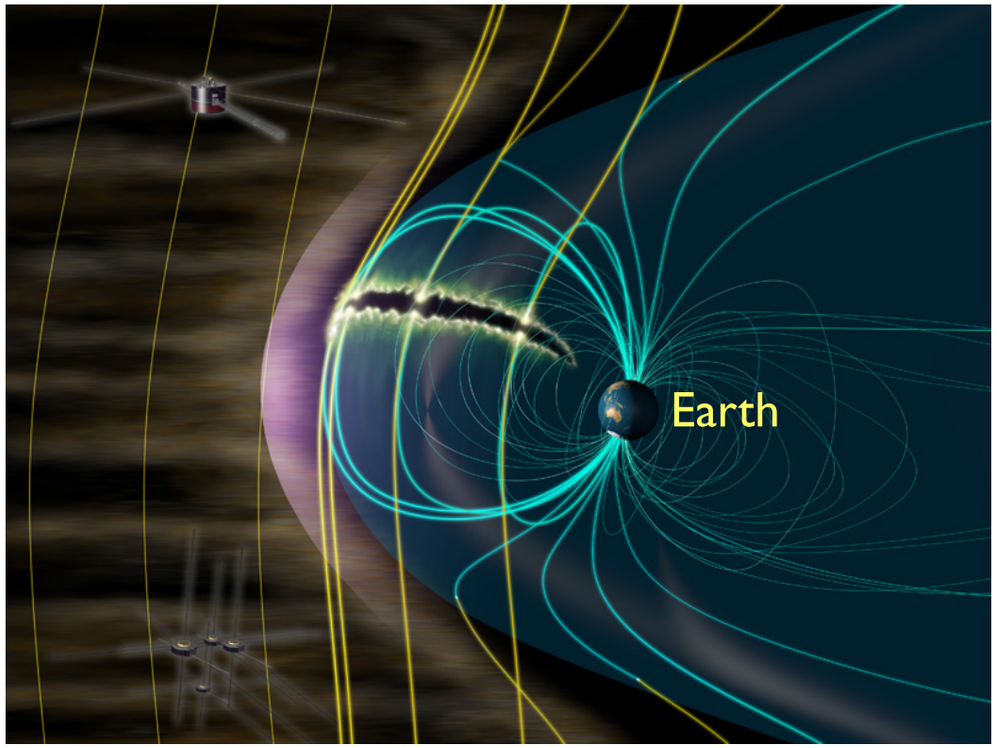


(K. Huang+ GRL 2020)

- When $L_{y,thin} \lesssim 10d_i$, both the rate & outflow speed drop significantly.
- An analytical model based on the dawn-ward flux transport is developed.

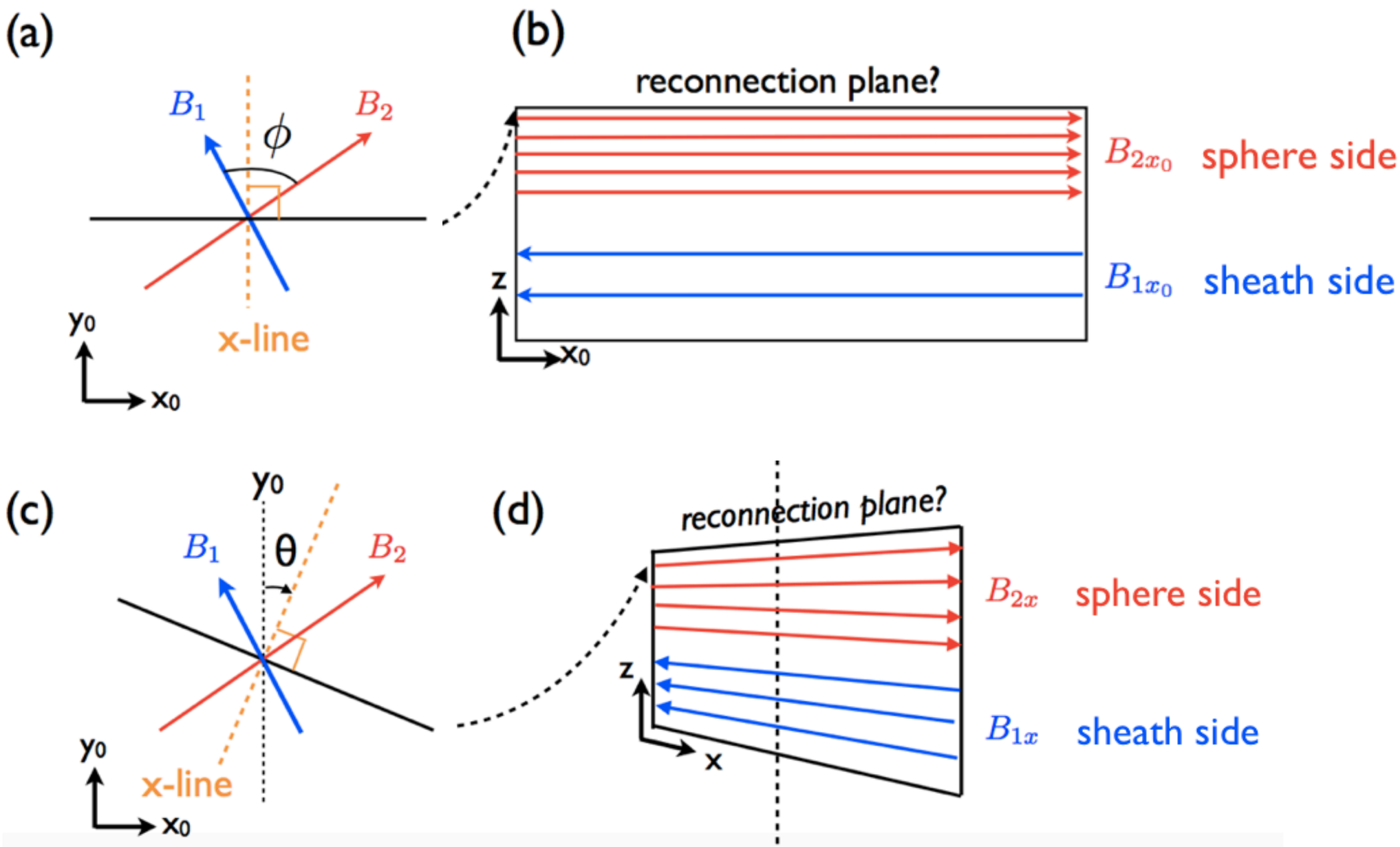
Relate reconnection rate to its 3D nature.?

(e.g., X-line orientation at Earth's magnetopause)



Credit: Geotail+MMS/JAXA/phys.org

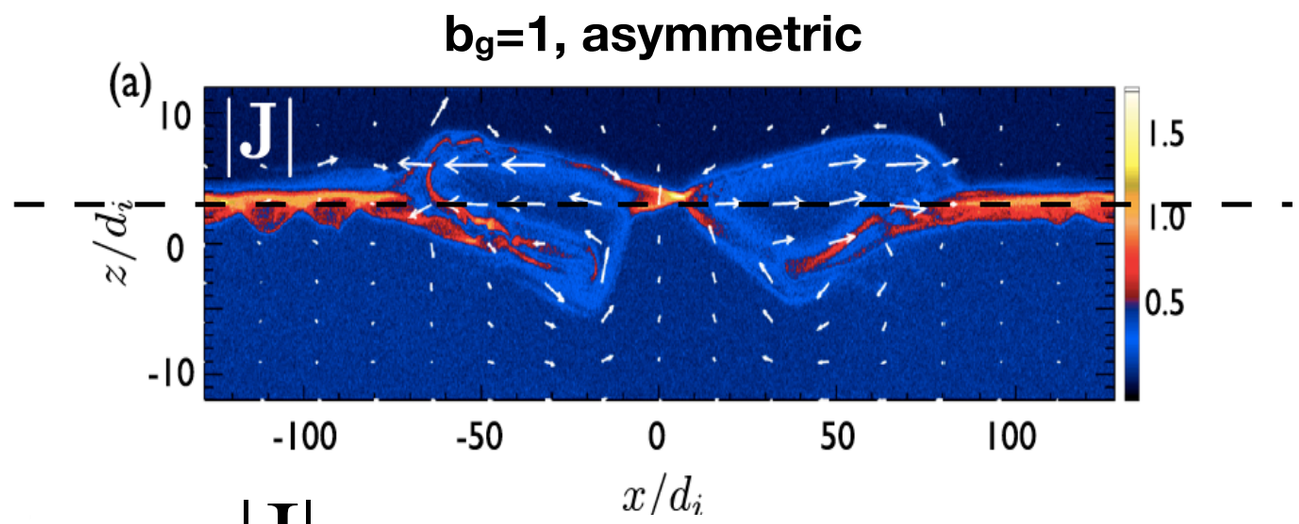
Q: Which plane does reconnection "prefer"??

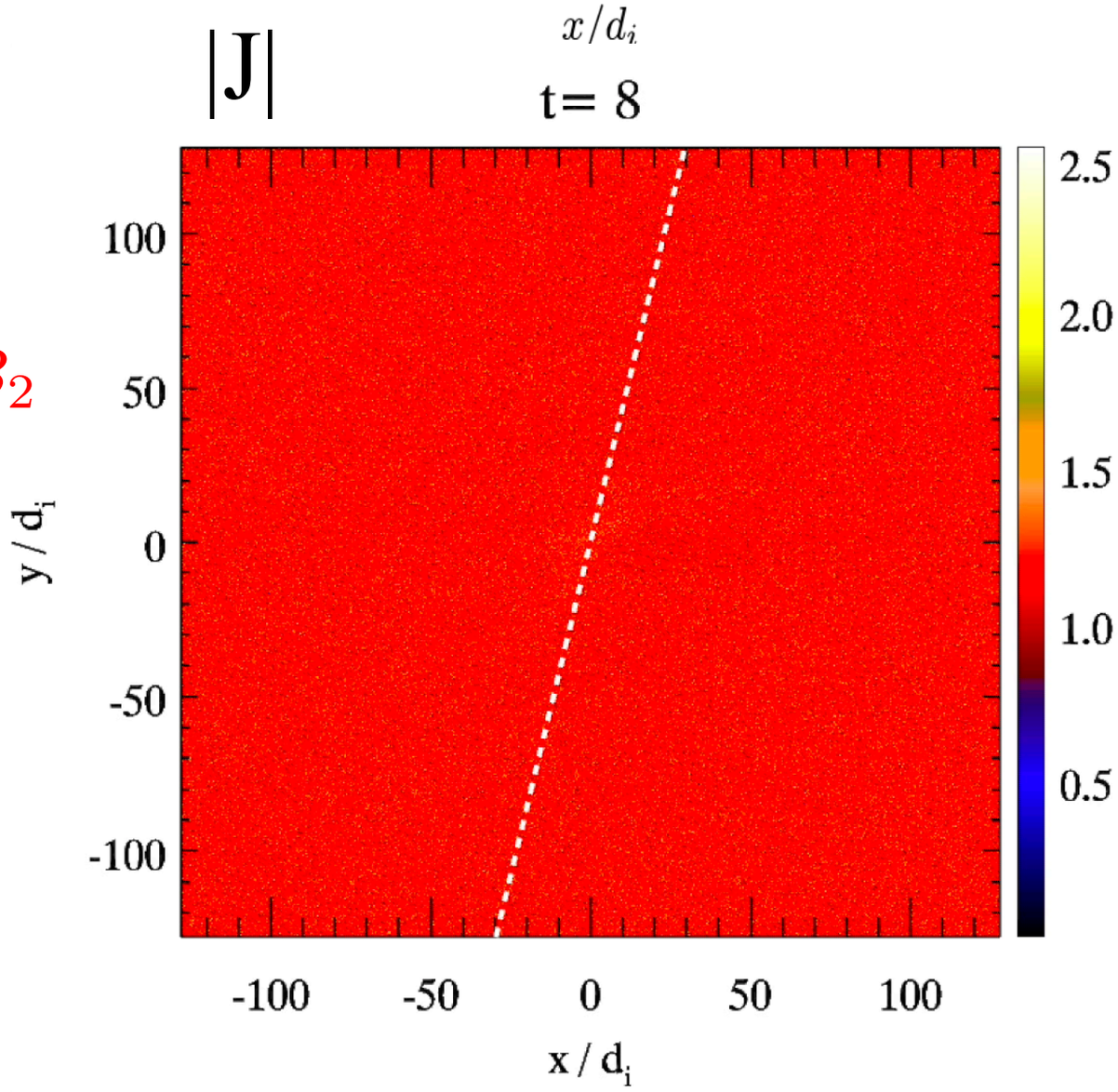


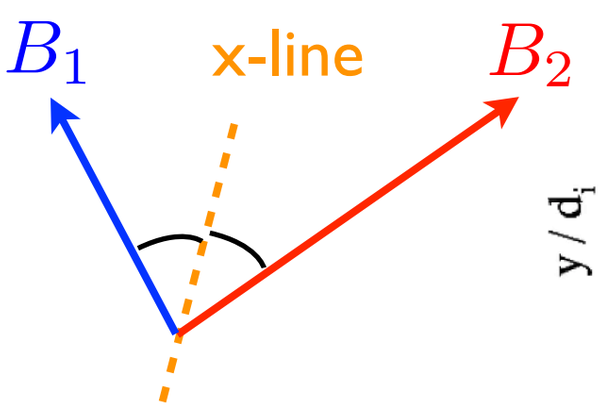
- Once the preferred reconnection plane is determined, the x-line orientation is determined~

Overview of 3D (less-turbulent) reconnection in PIC

mi/me=25
256 x 256 x 24 di





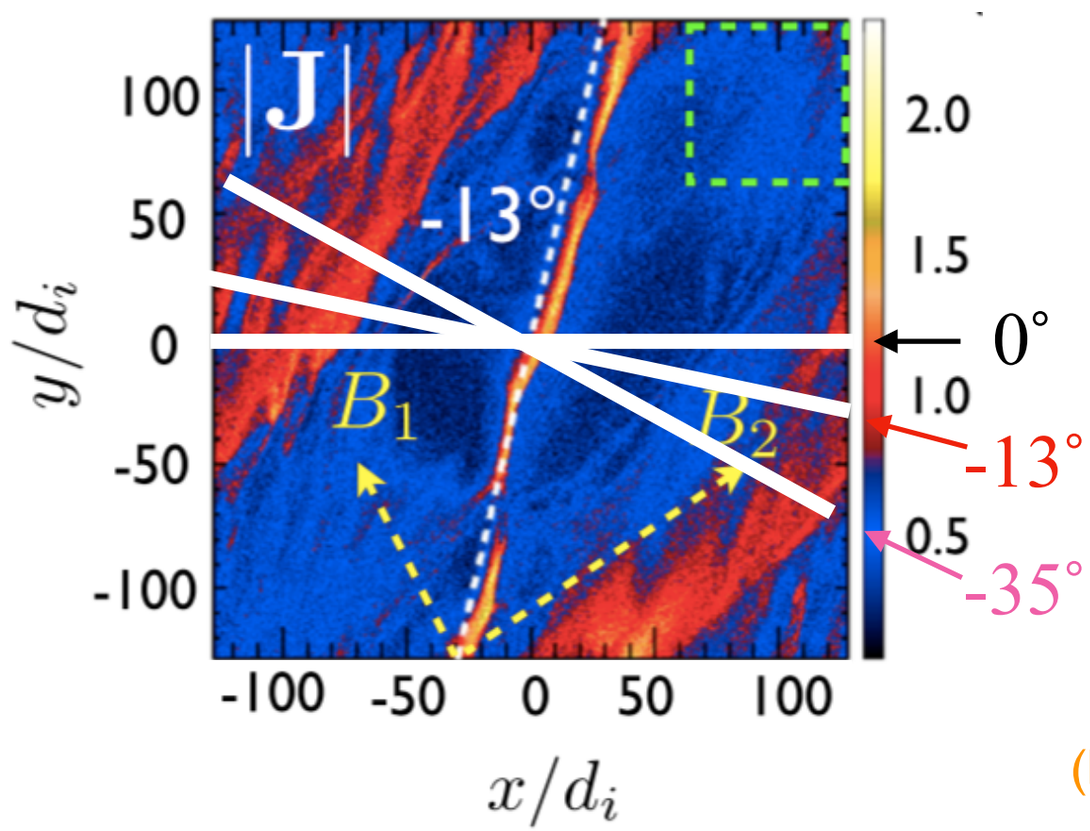


(T. Li+ JGR 2020, Liu+ JGR 2018)

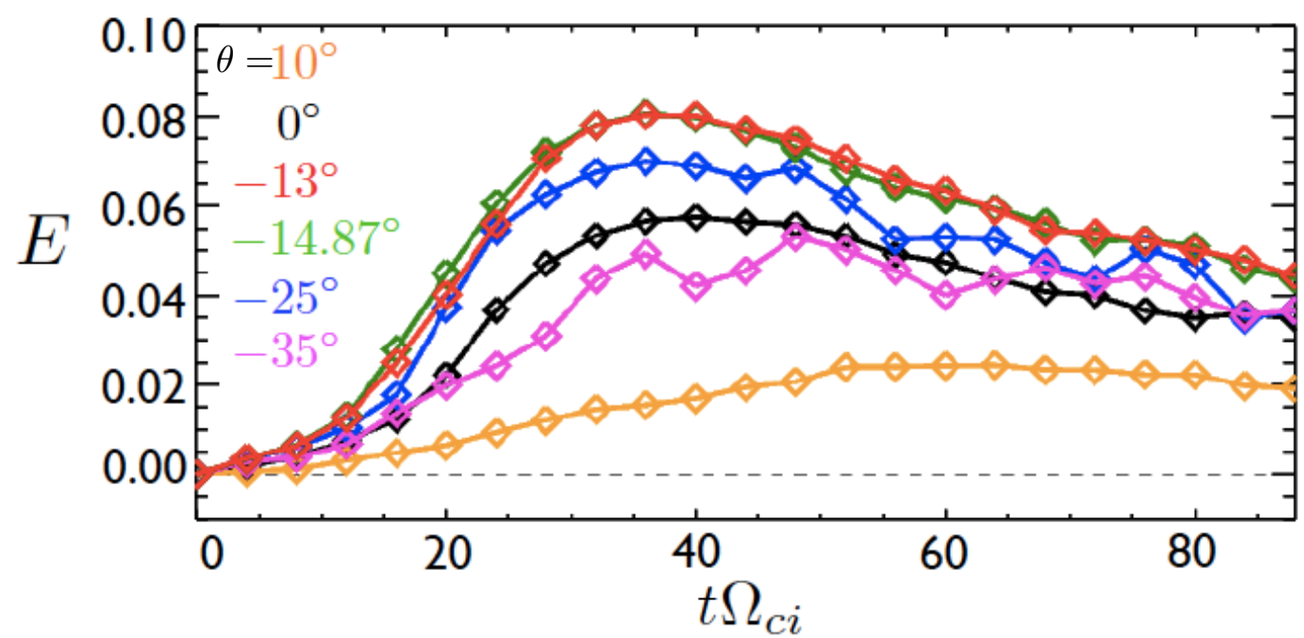
- Check out the orientation & spread of the reconnection x-line!

What can companion 2D simulations tell us?

3D



2D oblique planes



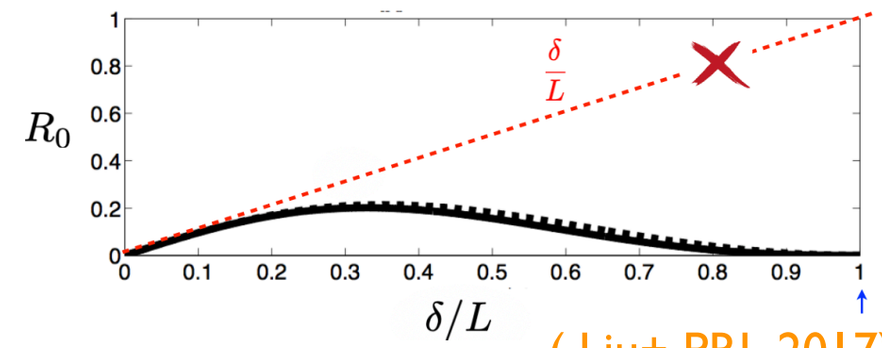
(Liu+ JGR, 2018)

- This 2D vs 3D comparison suggests that the system tends to maximize the reconnection electric field!

— other possibilities include maximizing the outflow speed, energy density, tearing growth rate…etc

(Swisdak & Drake 2007; Schreier+2010; Hesse+ 2013; Liu+JGR 2015,2018)

Summary & future (unsolved questions)



(Liu+ PRL 2017)

★ Fast rate 0.1 is an upper bound value.

— the same approach was extended to relativistic regime, asymmetric geometry and high-β limit

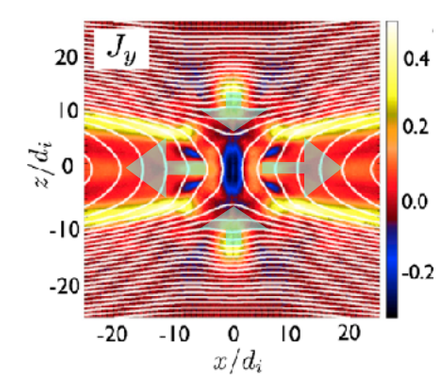
★ Reconnection is suppressed when the X-line extent $\lesssim 10d_i$

★ X-line orientation in 3D can be determined by maximizing the rate.

★ What is the primary localization mechanism?

— Why is the geometry in uniform-resistivity MHD so different? (the only exception.?)

— While a localization mechanism is needed for fast reconnection, different systems may have different localization mechanisms.



(Liu+, PoP, 2018)

★ Turbulence!? if yes, how does it affect reconnection rate?

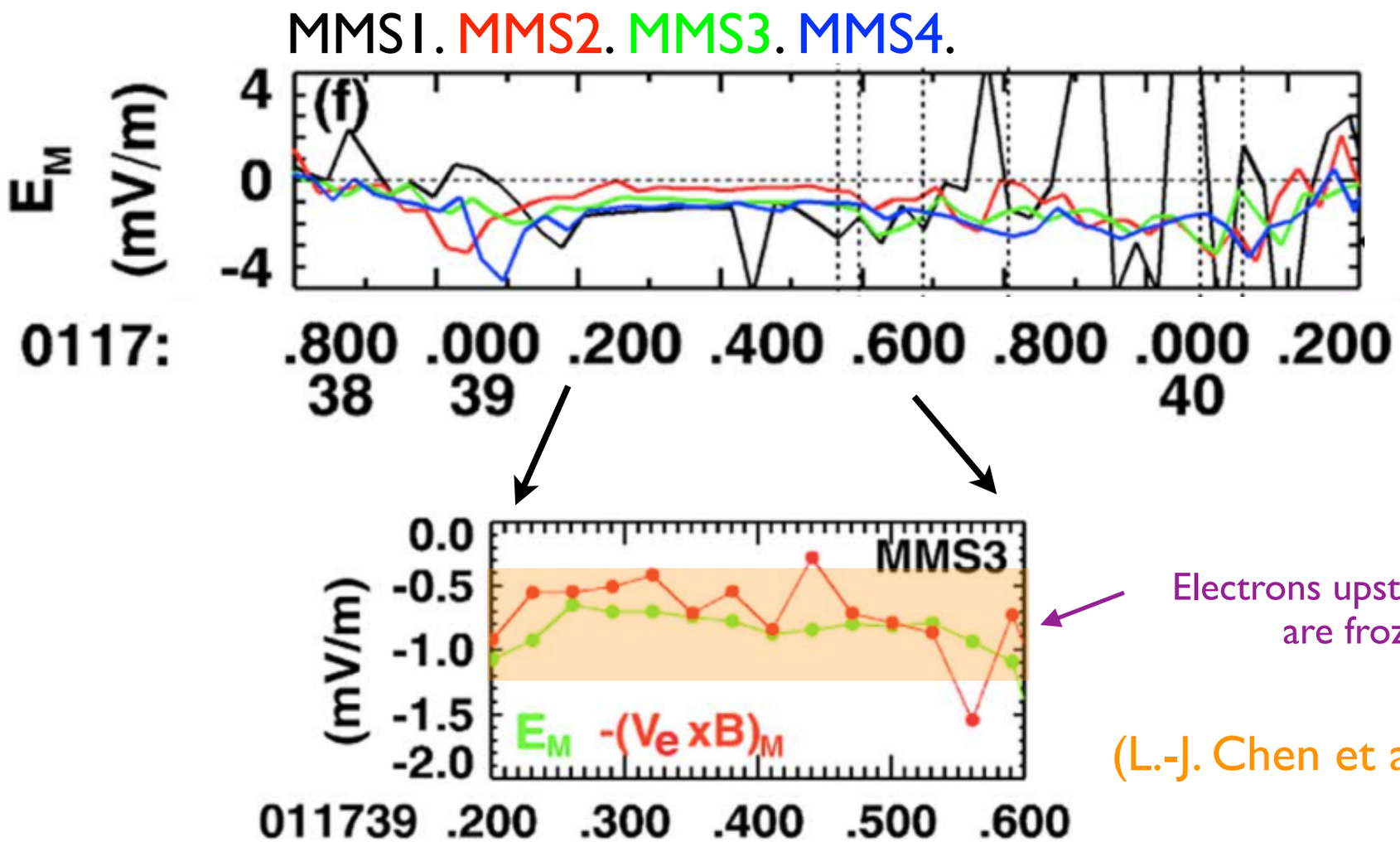
— 3D rate in PIC turbulent-reconnection appears to be similar to its 2D counterpart though~

(Guo+ 2021, Le+ PoP, 2018, Daughton+, PoP 2014)

More on the rate measured by Magnetospheric Multiscale Mission (MMS)

MMS Observations

December 14, 2015 event: $B_g \sim 0.2$, $B_{L2}/B_{L1} \sim 1.3$, $n_2/n_1 \sim 6.8$



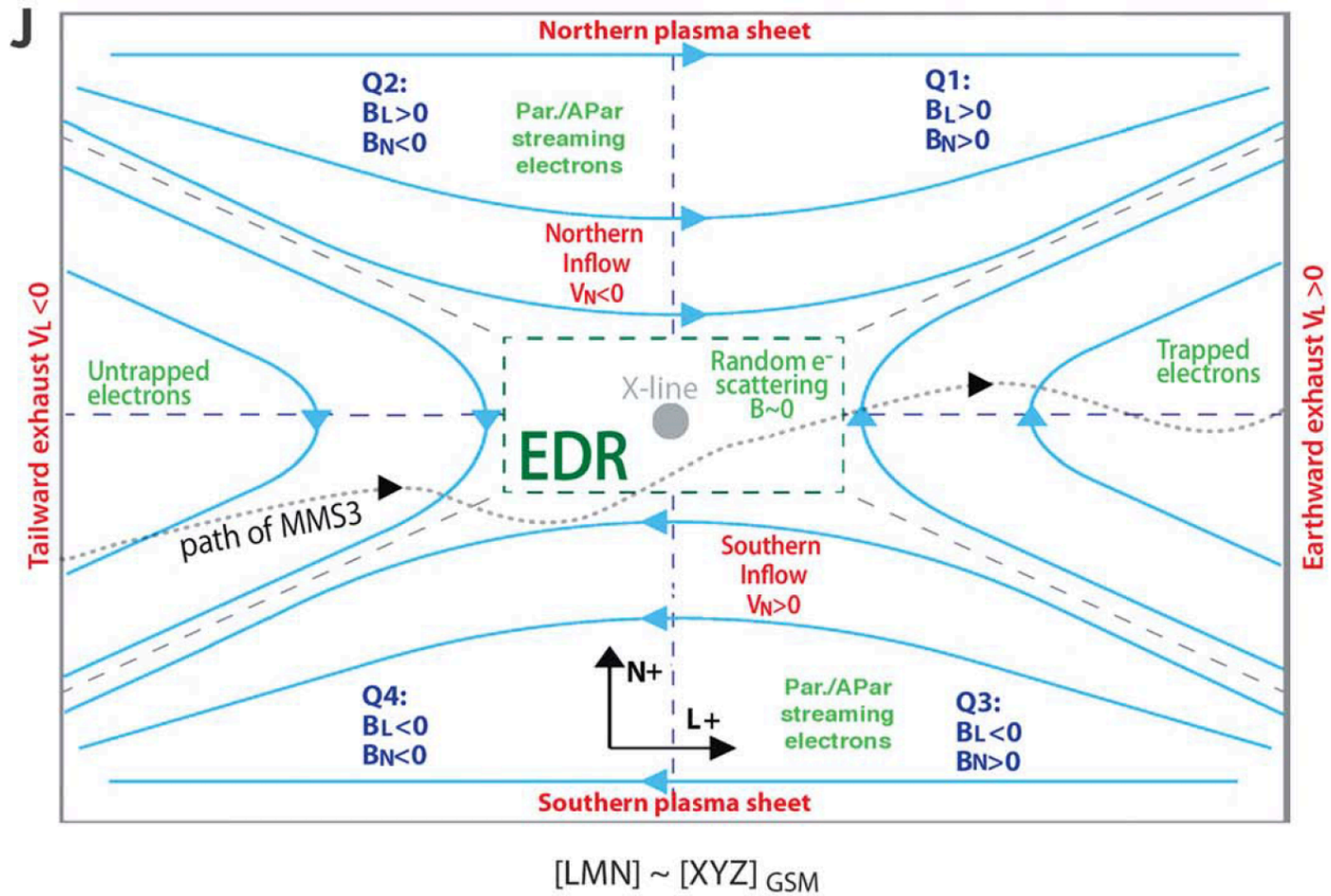
Electrons upstream of EDR are frozen-in!

(L.-J. Chen et al., JGR 2017)

- An uniform electric field over at least 8 electron skin depths corresponds to a normalized rate ~ 0.1.
- The rate of the October 16, 2015 event was estimated to be ~ 0.3. (Burch+ Science 2016)

MMS Observations

7/11 event



(Torbert+ Science 2018)

- Measuring the aspect ratio of EDR~ 0.1–0.2

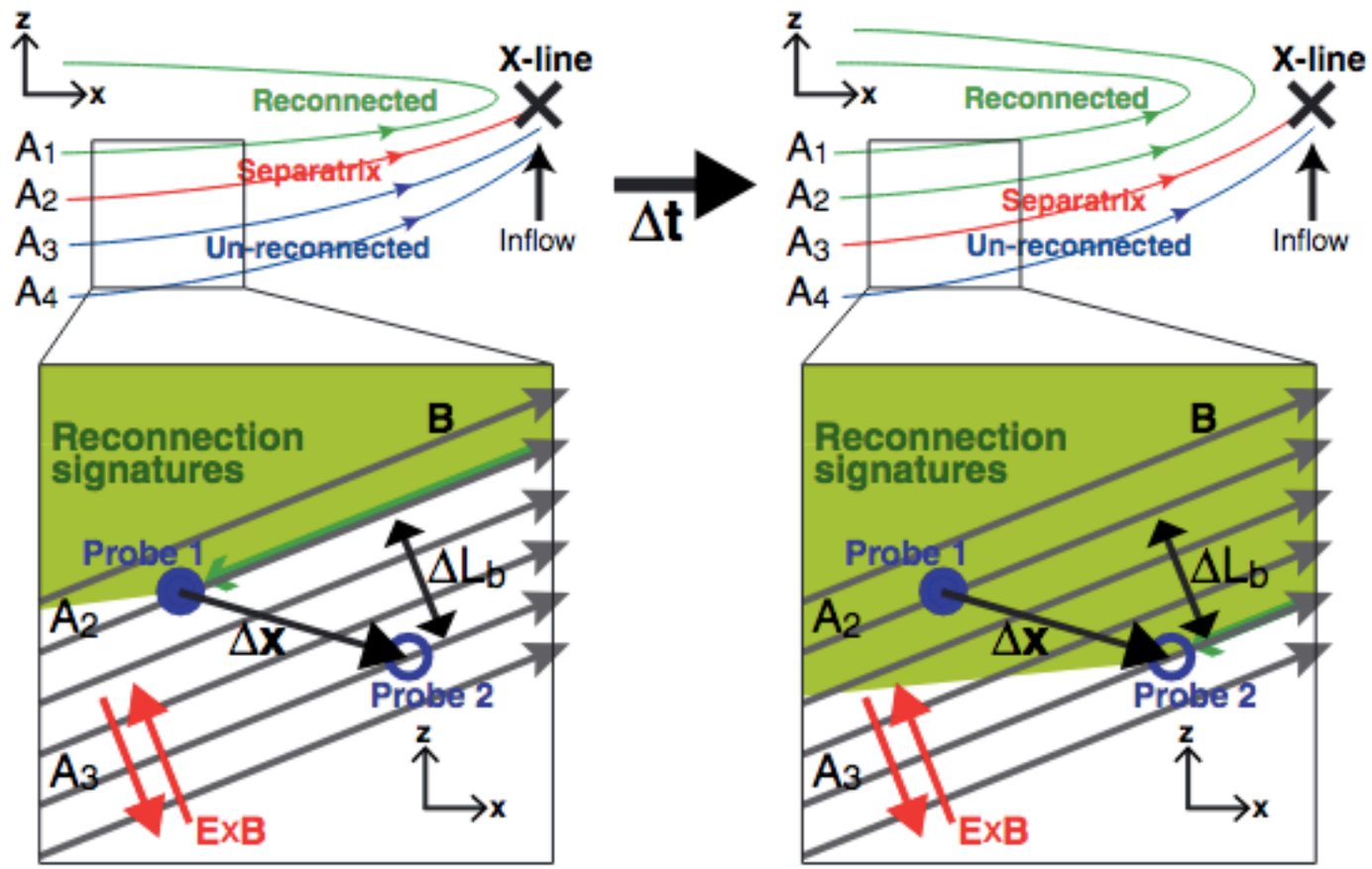
— Using timing analysis to get L.

— Using current density to get δ.

MMS Observations

— new technique in measuring the rate

t_0 $\qquad\qquad$ $t_0 + \Delta t$



(Nakamura+ JGR 2018)

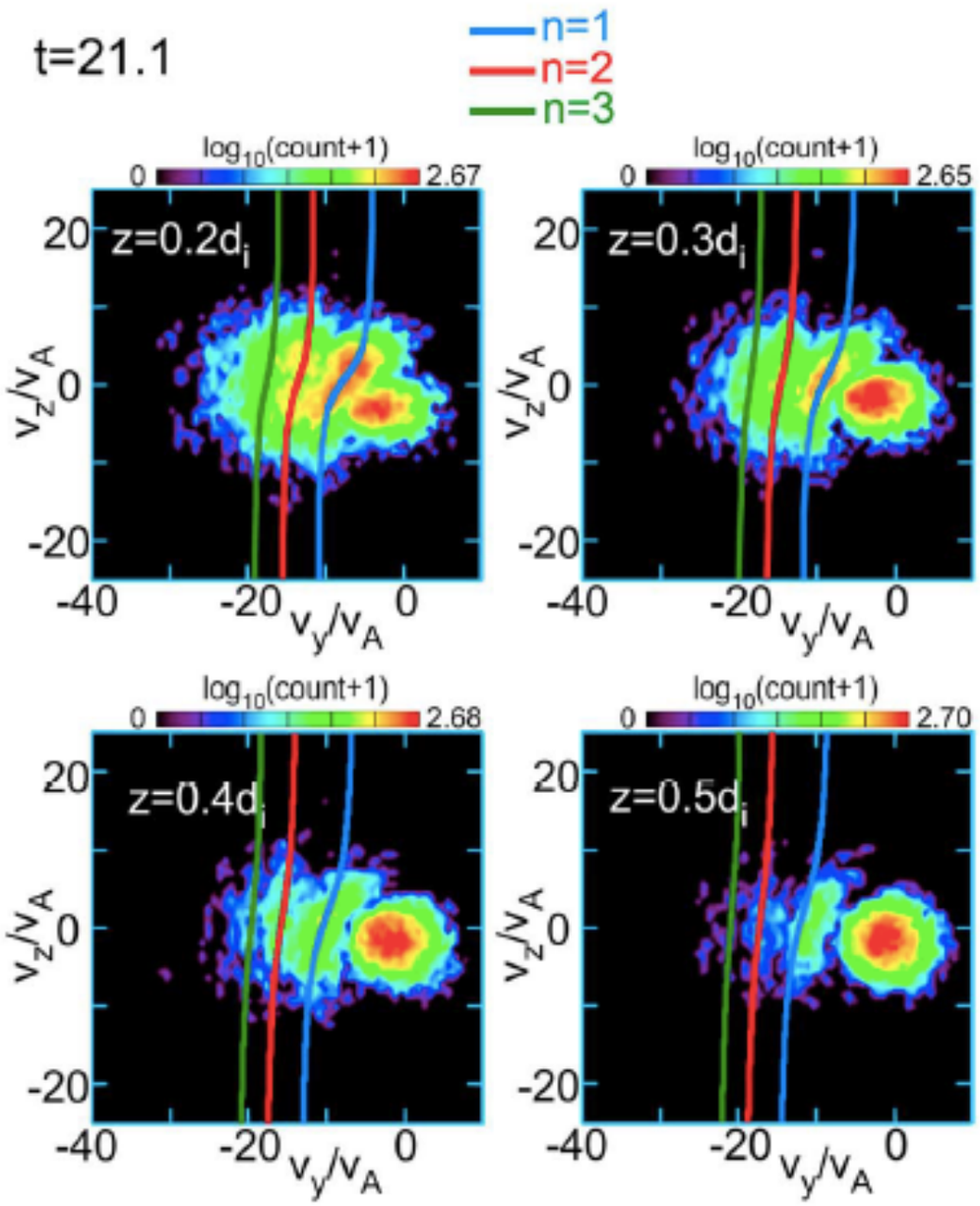
$$E_R \simeq -\frac{\Delta L_b B}{\Delta t} = -\frac{(\Delta \mathbf{X} \times \mathbf{B})_y}{\Delta t} \simeq -\left[(\mathbf{V}_{timing} - \mathbf{V}_{convection}) \times \mathbf{B}\right]_y$$

convection of the magnetic flux respect to the probe

- Measuring the flux difference at separatrix to infer the reconnection rate remotely!

MMS Observations

— new technique in measuring the rate



(Bessho+ GRL 2018)

$$b = dB_x/dz$$

$$k = dE_z/dz$$

$$\Delta v_y = \left[\left(-v_{y0} - \frac{ck}{b}\right)^{3/2} + \frac{3\pi}{4}(2n+1)\left(\frac{mc}{eb}\right)^{1/2}\frac{eE_r}{m}\right]^{2/3} - \left[\left(-v_{y0} - \frac{ck}{b}\right)^{3/2} + \frac{3\pi}{4}(2n-1)\left(\frac{mc}{eb}\right)^{1/2}\frac{eE_r}{m}\right]^{2/3}.$$

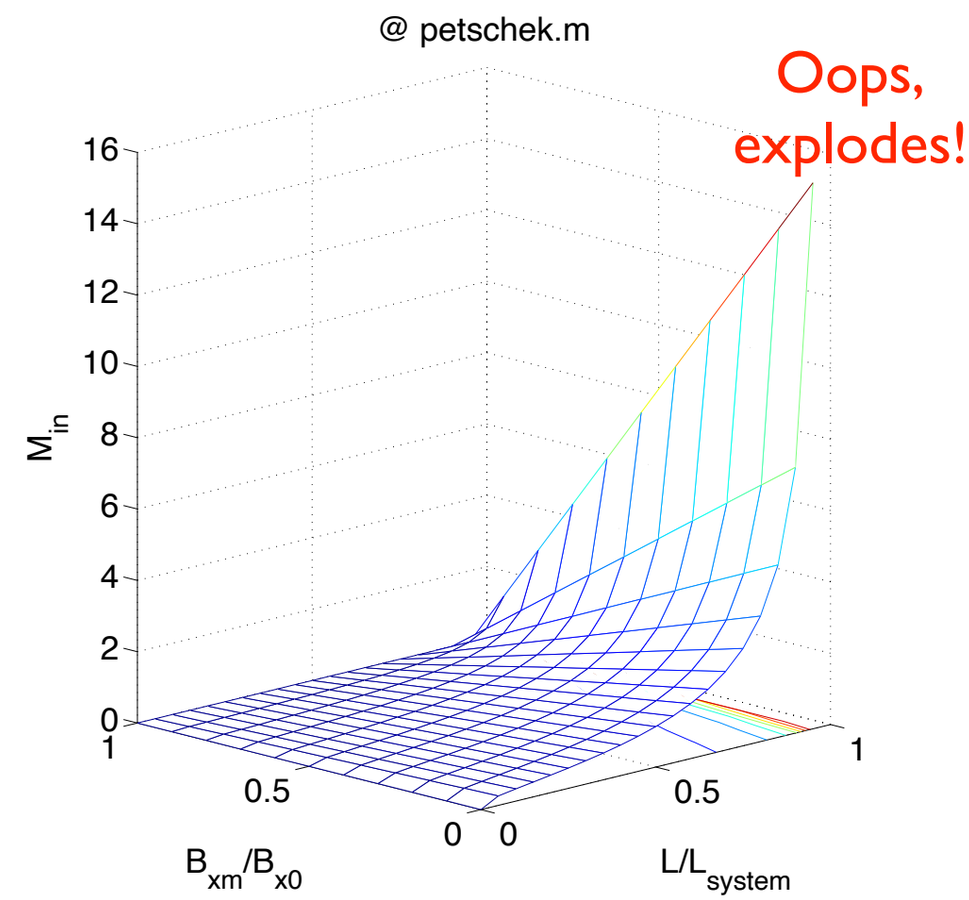
- Inferring reconnection rate from particle distributions at the diffusion region.
  - — E_R accelerates electrons in the out-of-plane (-y) direction.
  - — R~ 0.22-0.28 for the 7/11 event.

Differences from Petschek's model

$$M_{in}^{*} \approx \frac{\pi}{8\log R_{eynolds,mag}}$$

where $R_{eynolds,mag} = LV/\eta$

eq. (4.43), Priest & Forbes 2000



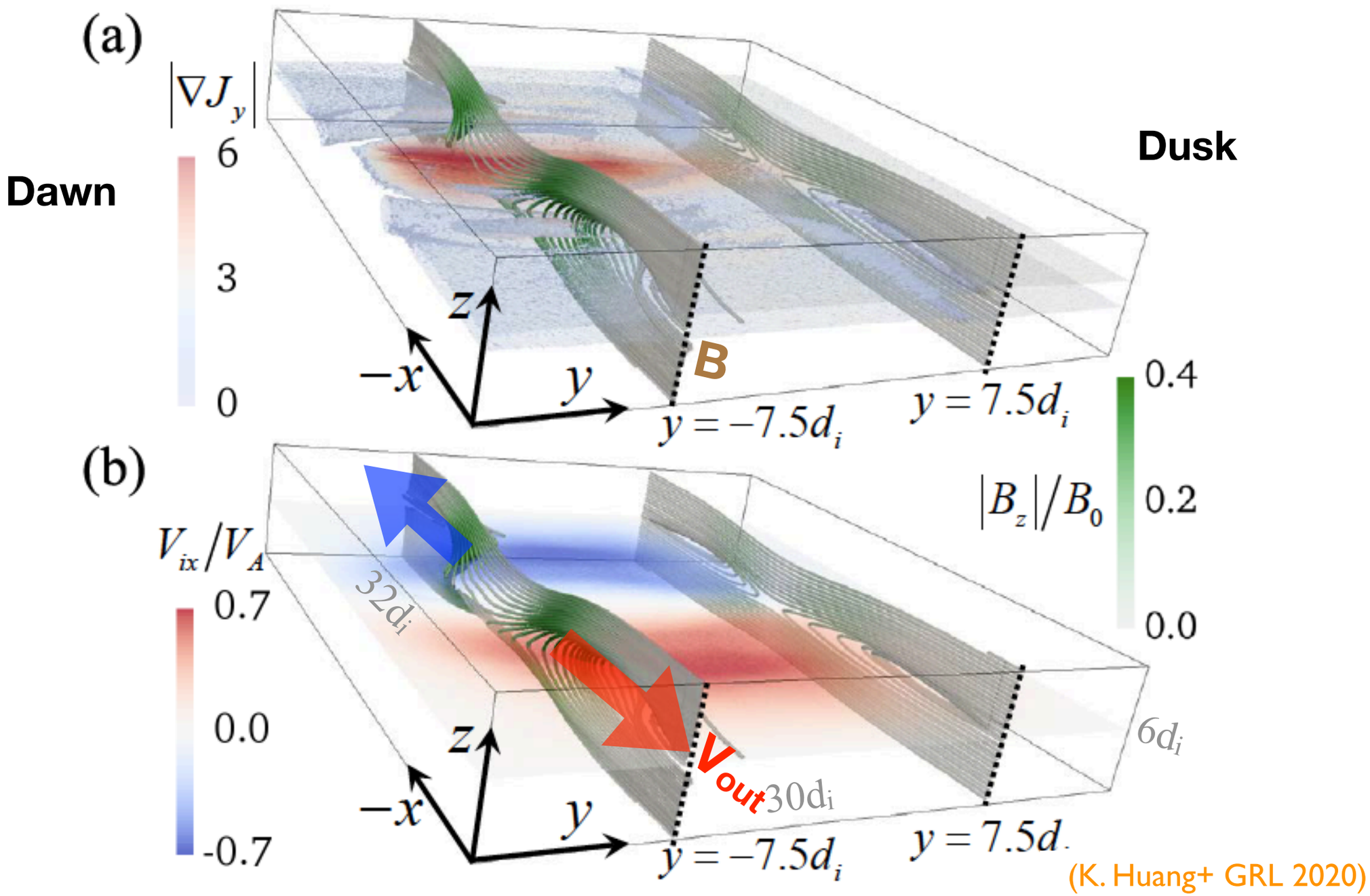
$$M_{in} = \frac{\pi(1 - B_{xm}/B_{x0})}{4\log(L_{system}/L)}$$

eq. (4.41), Priest & Forbes 2000

In Petschek's model:

- B_{xm}/B_{x0}=0.5 is assumed. (we investigated R_0 vs. B_{xm}/B_{x0})
- Small opening angle is assumed. (we investigated R_0 vs. opening angle)
- The resulting rate is not bounded in between 0 & 1. (ours is well-bounded)

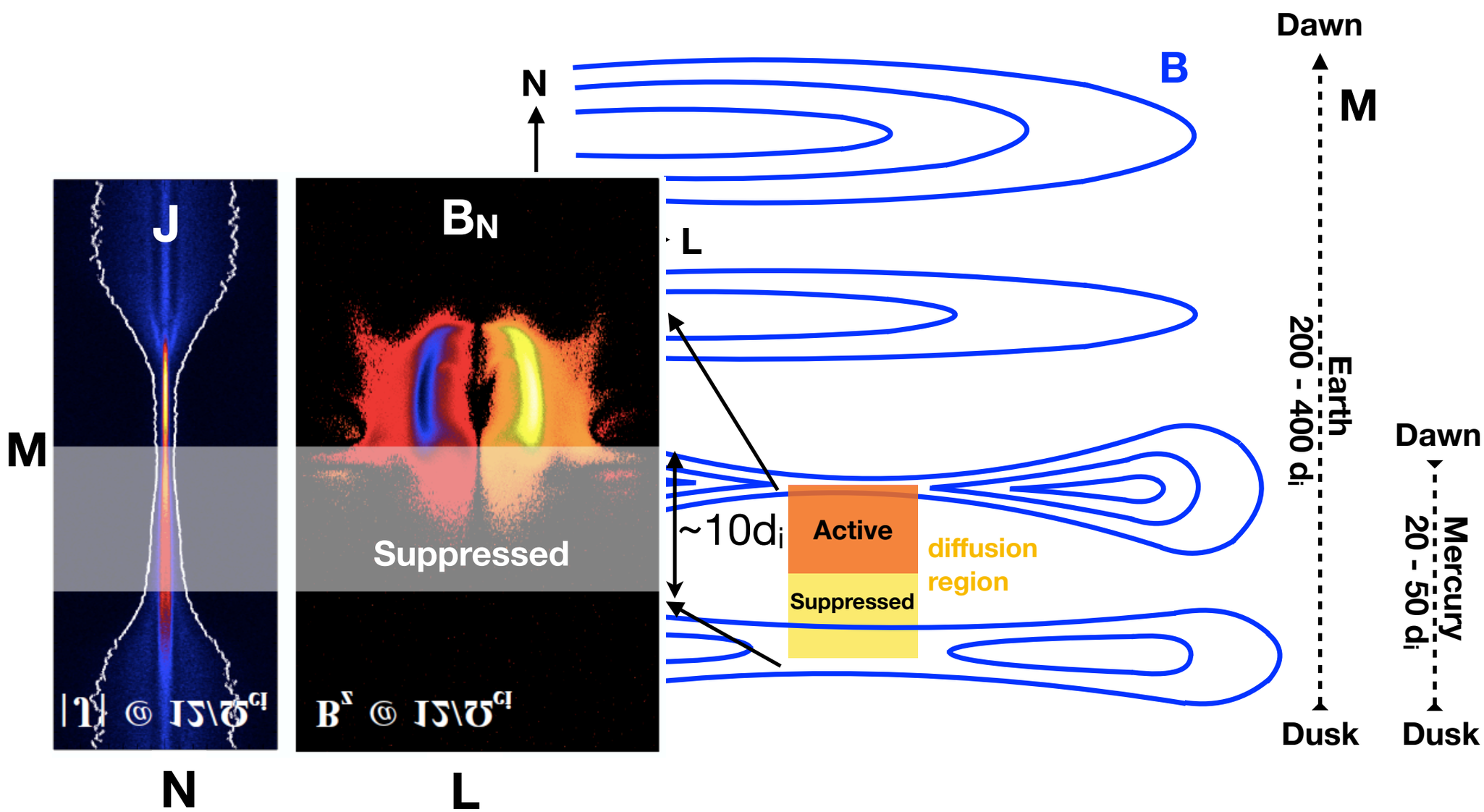
3D visualization of this short X-line



(K. Huang+ GRL 2020)

- Cooool!

“An” explanation of the opposite Dawn-Dusk asymmetry at Magnetotails of Mercury vs. Earth



(S Lu+ 2016) (Liu+ 2019)

- “Onset location asymmetry” vs. “internal X-line asymmetry” !!!

—both are based on the dawn-ward magnetic flux (i.e., B_N) transport~

—the size of the global magnetotails determines which asymmetry to be revealed!

—the critical cross-tail size for the asymmetry reversal is ~ 20-30 di