Accurate calculation of Singular Current Densities in non-linear, ideal MHD equilibrium solutions

1. For decades, singular current densities have been predicted in ideal 3D MHD equilibria,

   - From $\nabla \cdot \mathbf{j} = \nabla \cdot (\sigma \mathbf{B} + \mathbf{j}_\perp) = 0$, where $\mathbf{j}_\perp \equiv \mathbf{B} \times \nabla p / B^2$, derive $\mathbf{B} \cdot \nabla \sigma = -\nabla \cdot \mathbf{j}_\perp$.
   - Solution for $\sigma = \dfrac{\mathbf{j} \cdot \mathbf{B}}{B^2}$, assuming $\mathbf{B} = \nabla\psi \times \nabla\theta + \iota(\psi) \nabla\zeta \times \nabla\psi$, is $\sigma_{m,n} = \underbrace{\dfrac{g_{m,n}(\psi)\, p'}{\iota\, m - n}}_{Pfirsch-Schlüter} + \Delta_{m,n}\, \delta(\psi - \psi_s)$

2. It is essential to resolve these singularities for accurate equilibrium calculations, and these currents play a vital role in both linear stability theory and tearing mode theory.

   - Conventional equilibrium codes (such as NSTAB, VMEC) cannot resolve these currents because of numerical methods that assume continuous functions, . . .

3. The recently developed STEPPED PRESSURE EQUILIBRIUM CODE (SPEC) [Hudson et al., PoP 19, 112502 (2012)] *does* allow for discontinuous equilibrium solutions and *can* resolve the singular currents!

4. Shown (below) is a soon-to-be published [Loizu et al., PoP, 2015] sequence of equilibria, where an island is successively "shielded" by ideal currents on so-called "ideal"-interfaces, which are brought closer to the rational surface, and (right) is the resonant pressure-driven $1/x$ current-density, as computed by SPEC red squares) and an analytic calculation (black stars).