SC Web Highlights Template
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Fractals, Plasma Physics and Fusion

Esoteric mathematical methods applied to ideal plasmas reveal that "pathological" profiles are just fractal.

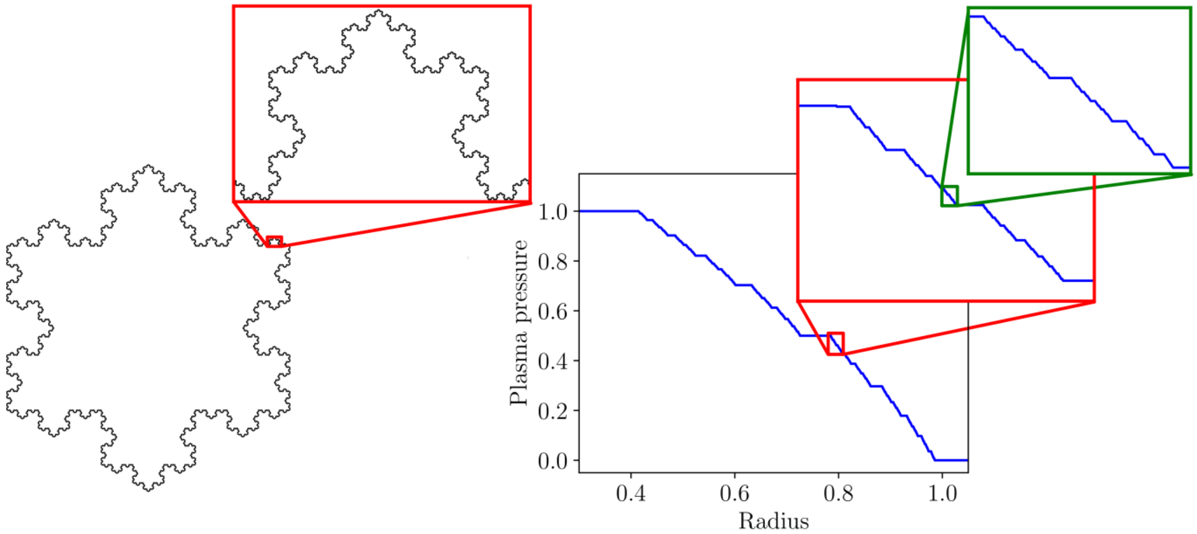

Image courtesy of Brian Kraus
A classic Koch snowflake fractal (left) shows repeated structure at small scales, a property that has been modeled for plasmas with staircase pressure profiles (right).

The Science

Fractal objects have similar structure at small and large scales, and are useful for modeling many interesting patterns in nature, such as branching on trees, turbulence in the ocean, and rugged coastlines. These fractals are infinitely complicated, and can display counter-intuitive properties. For instance, the Koch snowflake pictured above has a finite area but infinite circumference!

Difficulties like this have plagued numerical simulations of the magnetically confined, hot ionized gases known as plasmas studied in fusion research. The magnetic fields used to confine the plasmas produce an infinite hierarchy of resonances; and this introduces the fractality to numerical simulations.

New computational techniques have applied an explicitly fractal numerical simulation to accurately describe the fractal properties of the physics, opening up a new path toward simulations of ideal fusion plasmas.

The Impact

Even though fractal solutions to ideal plasma equations have been anticipated since 1967, no research has adequately investigated the impact of fractal pressure profiles on plasma equilibria in which the confining magnetic field pressure balances the plasma pressure. In fact, most calculations ignore fractality, but this leads to physically impossible infinite currents near the resonances that must be artificially suppressed. New work embraces the fractal plasma profiles from the beginning, and the infinite currents vanish.

A key development involves simulating ideal plasmas with fractal grids, which serves as a step toward incorporating the numerically challenging fractal structure into equilibrium codes. Using these techniques, a new set of equilibria has been produced with smooth and non-smooth regions, discontinuous but physically valid current density, and a Devil's staircase pressure profile, as shown in the above figure.

Summary

To avoid infinite currents in ideal magnetohydrodynamics (MHD), a toroidal plasma cannot maintain pressure gradients in the wrong places. These problematic regions are associated with "resonances". To avoid this physical impossibility, numerical simulations often truncate results and thus ignore the fractal distribution of rational surfaces.

New work by Kraus and Hudson incorporates fractality into the numerical grid from the start, leading to more self-consistent treatment of the full ideal problem.

By using the fractal grid, new types of ideal MHD equilibria can be calculated that have the plasma pressure flattened on every rational surface. These equilibria behave very differently on rational and irrational flux surfaces, so that the partitioning of the torus into more or less irrational locations is crucially important. Results from these studies indicate that even the simplest plasma systems must address the foundational problem of infinite currents on rational surfaces.

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Publications

B. F. Kraus and S. R. Hudson, "Theory and discretization of ideal magnetohydrodynamic equilibria with fractal pressure profiles," Physics of Plasmas **24**, 092519 (2017); https://doi.org/10.1063/1.4986493