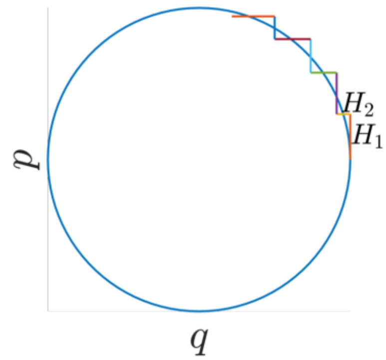
A THEORY ON HOW TO BUILD GOOD SIMULATION CODES FOR FUSION PLASMAS

$$H = \frac{1}{2}(q^2 + p^2) = H_1 + H_2$$

$$H_1 = \tfrac{1}{2}q^2$$
$$H_2 = \tfrac{1}{2}p^2$$




The Hamiltonian splitting method is depicted above using the simplest Hamiltonian system. Each subsystem can be exactly solved on a computer, and high order algorithms are constructed by composing these subsystem solutions together. The Hamiltonian system for plasma dynamics involves infinitely many degrees of freedom and is considerably more complicated. Nevertheless, the main idea remains the same. For particle-in-cell simulations, it is shown that Hamiltonian splitting algorithms can be designed to preserve electromagnetic gauge symmetry and the local conservation of electromagnetic charge.

**The Science**

A large number of simulation codes have been built to study fusion plasmas. But not all codes are created equal in terms of their accuracy and fidelity. Good simulation codes are built from advanced algorithms that preserve fundamental laws of physics, such as charge conservation and energy-momentum conservation. What are the common characteristics of such algorithms? Using the example of charge conservation, physicists have recently developed a systematic theoretical framework that links conservative algorithms with their discrete gauge symmetries.

**The Impact**

This new theoretical framework provides guidance to plasma physicists for the design of locally charge-conserving algorithms from first principles, without any need for ad hoc techniques. Simulations adopting this type of algorithm will lead to a more accurate understanding of the complex dynamics of fusion plasmas. Furthermore, this new framework is widely applicable to the simulation of more general gauge theories and provides a blueprint for designing algorithms that admit exact conservation laws.

**Summary**

A comprehensive theoretical framework is developed for charge conservation in gauge-symmetric Lagrangian and Hamiltonian particle-in-cell (PIC) algorithms. It is shown that the charge conservation property of these algorithms is a consequence of their discrete electromagnetic gauge symmetry. Like its continuous counterpart, a discrete gauge symmetry is described by a Lie algebra with infinitely many degrees of freedom. For PIC methods based on variational Lagrangian theory, Noether's second theorem is applied to demonstrate that discrete gauge symmetry gives rise to a local charge conservation law as an off-shell identity. For Hamiltonian splitting algorithms, the momentum map establishes the connection between charge

conservation and discrete gauge symmetry. As an application of this general theory, a novel, explicit, symplectic, gauge-compatible splitting PIC method is designed. The momentum map of this method yields an exact local charge conservation law.

The study also confirms the adaptability of gauge theories to the discrete structures of simulation algorithms. Gauge symmetries, as internal symmetries, transform foreground physical fields defined against a background space-time. Their smooth, internal geometric structure can be maintained even when the space-time is discretized in an algorithmic setting. The present work demonstrates that preserving these gauge symmetries in algorithms is beneficial for the accuracy and fidelity of numerical simulations.

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**Publications**
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