Introduction to Energetic Particles Physics in Magnetic Fusion: Part I classical confinement and equilibrium distribution function

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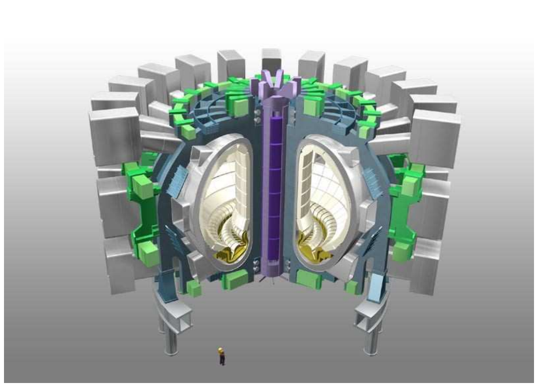
Princeton Plasma Physics Laboratory, Princeton University

ITER - $10bln (wikipedia) *NSTX (nstx.pppl.gov)* *Solar corona*



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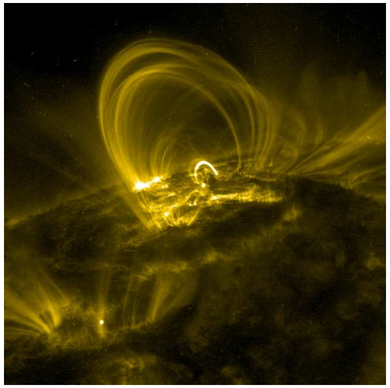

PPPL, Graduate Student AST558 Seminar, April 6, 2009

Burning plasma and energetic particles

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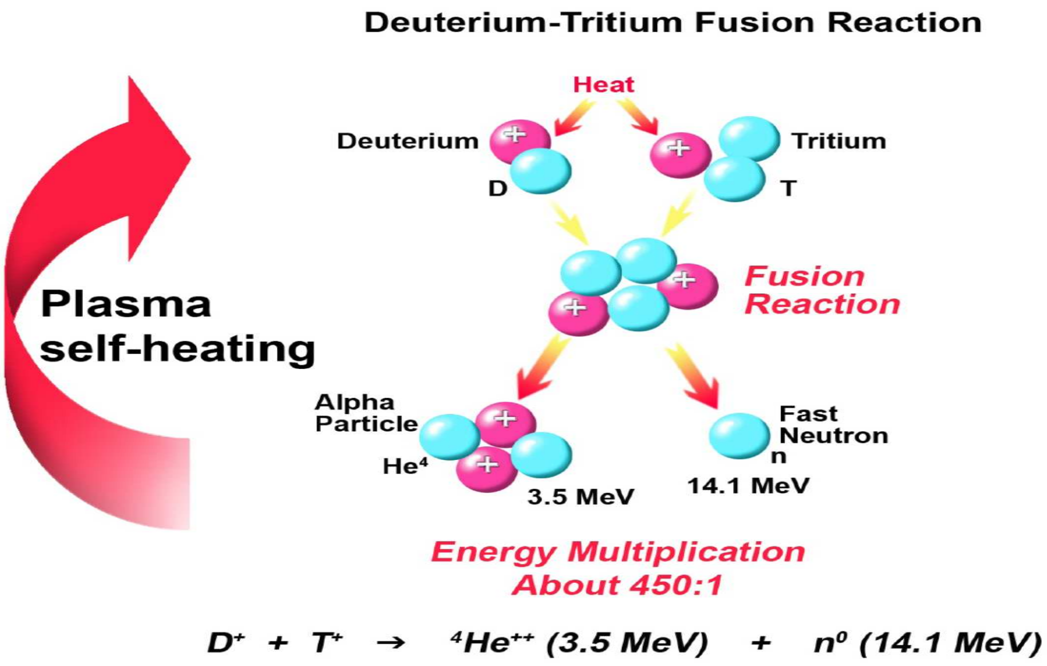
Consider self-sustained, self-heated burning plasma - α's (EP) provide heating



Huge gain in energy from $\sim 20keV$ Deuterium and $\sim 20keV$ Tritium to $3.52MeV$ in α and $14.1MeV$ in neutron per (D-T, $m_{D+T} = 5m_n$) reaction, $3.52MeV$ per nucleon.

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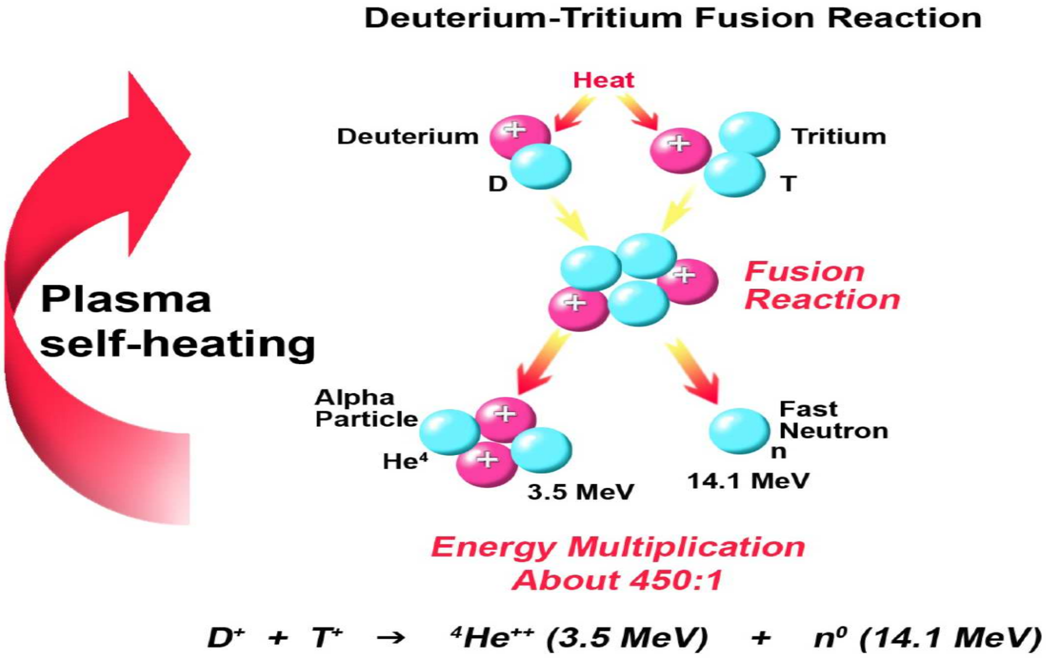

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For comparison fission reaction $\sim 200MeV$ per $m_{n+U_{235}} = 236m_n$, $0.9MeV$ per nucleon.

$1keV$ corresponds to 10^7K - solar core ($1eV \simeq 10^4$ - solar surface)

Motivations to study energetic particle (EP) physics

DT reactor with 1 GW of power should have 200MW (20%) in fusion alphas

Where does this power goes? What are the mechanisms of power transfer? How to control it? How can we design efficient fusion reactor? Can it damage the plasma facing components, diagnostics? Can we make use of EP instabilities for plasma diagnostics?

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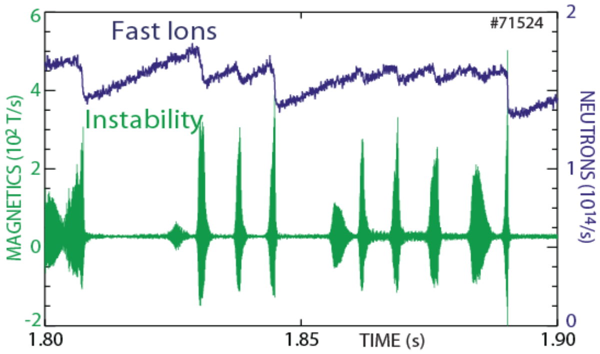

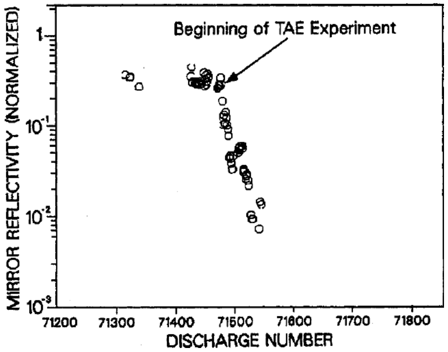

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Need to mitigate the risk: ITER cost is $10bln.

Duong, Nucl.Fusion 33 (1993) p.749

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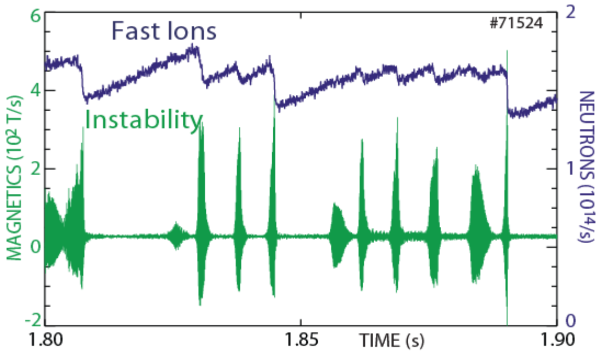

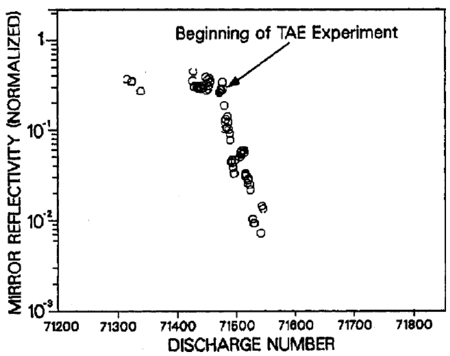

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Positive (for fusion) things are expected/happens - see next lecture

In this first lecture

- Single Energetic Particle (EP) confinement - primarily tokamak applications
- Distribution function
  - collision integral and kinetic equation
  - solutions of kinetic equation
  - experimental validation
  - some example NPA/PCX from NSTX, DIII-D, TFTR

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Second lecture is on collective instabilities due to EP

Some TMA's (Too Many Acronyms)

Some used:

DIII-D (1980-) - d-shaped tokamak, General Atomic, CA

TFTR - (1980-1997) tokamak fusion test reactor, PPPL

NSTX (1998-) - national spherical torus experiment, PPPL

ITER (2018-) international thermonuclear test reactor

NPA - neutral particle analyzer

PCX - particle charge exchange

NBI - neutral beam injection

ICRH - ion cyclotron resonant frequency heating

RF - radio frequency

What kinds of EPs are present fusion plasmas?

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Energetic particles (super-thermal, fast - f ions) by definition $E_f \gg T_i$:

$$v_{Ti} \ll v_f \ll v_{Te}.$$

Relation to the Alfvén wave phase velocity, v_A (here $\beta_i = 8\pi n_i T_i / B^2$)

$$v_f = \frac{v_f}{v_{Ti}} \sqrt{\frac{2T_i}{m_i}} \frac{v_A}{\sqrt{B^2/4\pi m_i n_i}} = \left(\frac{v_f}{v_{Ti}} \sqrt{\beta_i} \right) v_A \sim (0.3 - 5)\, v_A$$

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- for $E_{EP} > 20T_e$, electron drag dominates
- characteristic drift frequencies are much larger than those of thermal ions
- EPs are “decoupled” from background ions

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Sources of EPs:

- resonant cyclotron frequency heating (RF): H, $50 - 1000keV$
- beam injection (NBI): D,H, $50 - 350keV$, $1MeV$ in ITER
- fusion reactions: $\alpha\,(3.52MeV, DT)$, $p\,(3MeV, DD)$, $T\,(1MeV, DD)$

When it all started and some references

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L.V. Korablev and L.I. Rudakov, Zh.Eksp.Teor.Fiz. (Sov.Journal Exp.Theor.Phys.) **54** (1968) 818

A.B. Mikhailovskii, Nucl.Fusion. **68** (1975) 727

M.N. Rosenbluth and P.H. Rutherford, PRL **34** (1975) 1428

Some review papers on EP physics:

A.B. Mikhailovski, Rev. Plasma Physics **9**

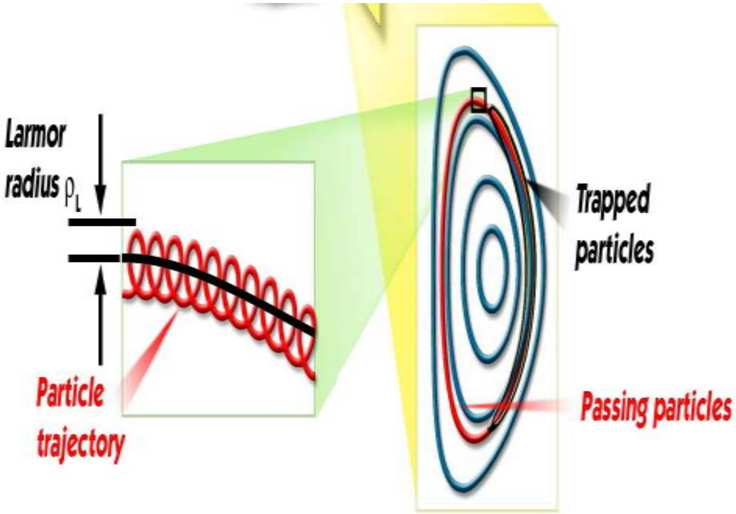
Ya.I. Kolesnichenko, Nucl. Fusion. **20** (1980) 727

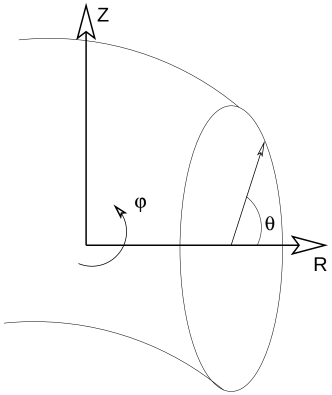
S.V. Putvinski, Rev. Plasma Physics **18**

W.W. Heidbrink and G.J. Sadler, Nucl. Fusion. **34** (1994) 535

How EPs are confined? What is special?

Schematic of charged particle
orbit in a tokamak $\rho_{Li} \ll \rho_{Lf}$





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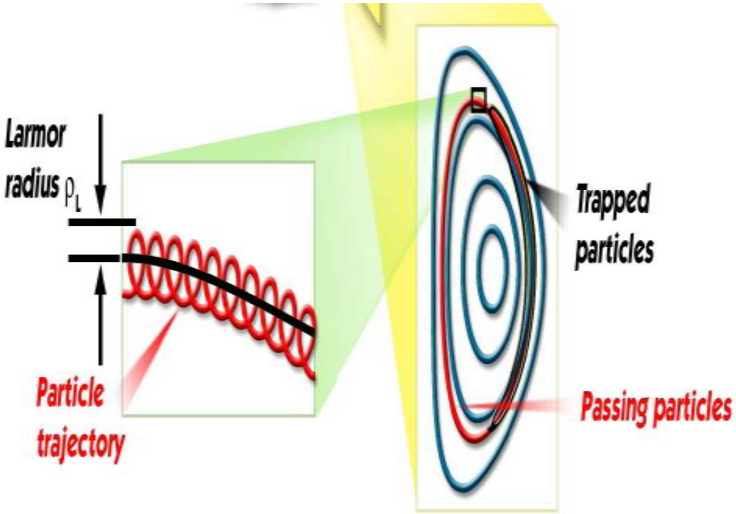

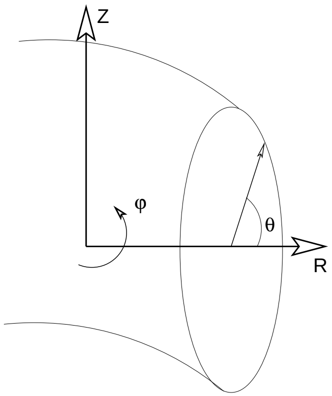

- Larmor radius should be small enough
  (1) $\rho_{Lf} \ll a$, $\rho_{Lf} = v/\omega_c = vmc/eB_\varphi$
- Poloidal Larmor radius (drift orbit width) determines confinement $\Rightarrow$
  (2) $a > \rho_{Lf\theta} = v_\parallel mc/eB_\theta$.

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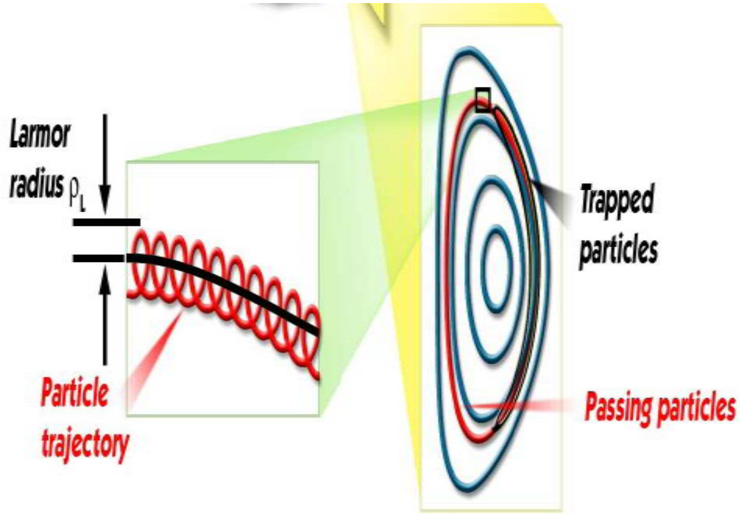

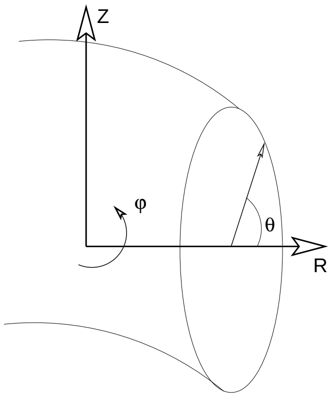

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- Poloidal Larmor radius (drift orbit width) determines confinement ⇒
  (2) $a > \rho_{Lf\theta} = v_\parallel mc/eB_\theta$.
- Let's find required "critical" plasma current:
  $B_\theta\,[T] = \left(\frac{1}{2}\right) 0.2 I_{pl}[A]/a[cm]$
  adiabatic moment conservation
  $v_\perp^2/B \simeq const \Rightarrow v_\parallel \simeq v\sqrt{2a/R}\ \ / \Rightarrow$
  $I_{pl\text{crit}}\ [A] = \left(10\rho_{Lf}\,[cm]\,\sqrt{2a/R}\right) B\,[Gauss] \simeq$
  2×10^6 for ITER
  More accurate estimate gives $3MA$.
- In ITER plasma $I_{pl} = 10MA \gg I_{pl\text{crit}}$.

Why this topic?

Distribution function of energetic particles

Why this topic?

Equilibrium distribution function affects:

- stability properties of plasma, EP - plasma interactions
- power balance
- confinement
- diagnostic needs

What are the fundamental physical processes which determine the EP distribution function?

EP distribution function: 90^o *degree scattering*

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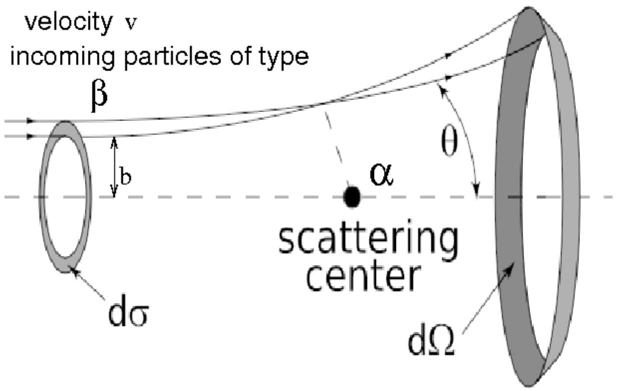

B.A. Trubnikov, In Rev.Plasma Physics, ed. M.A.Leontovich, v.1 (1963)

Perpendicular force at the closest approach (impact paramter b):

$$F_\perp = \frac{z_\alpha z_\beta}{b^2}.$$

Duration of the encounter

$$\Delta t = b/v$$

$\Rightarrow$ change in perp velocity

$$\Delta m v_\perp = F_\perp \Delta t = \frac{z_\alpha z_\beta}{vb}.$$

Maximum of the momentum change is mv
$\Rightarrow$ 90^o degree scattering parameter

$$b_\perp = \frac{z_\alpha z_\beta}{m_\alpha v_\alpha^2}.$$

EP distribution function: 90^o *degree scattering*

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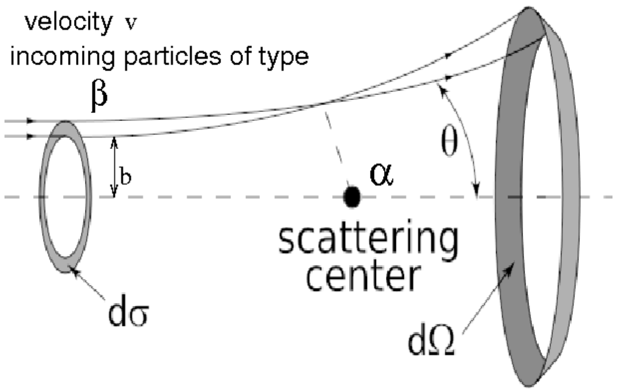

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More detailed expression:

$$\tan\frac{\theta}{2} = \frac{b_\perp}{b}$$

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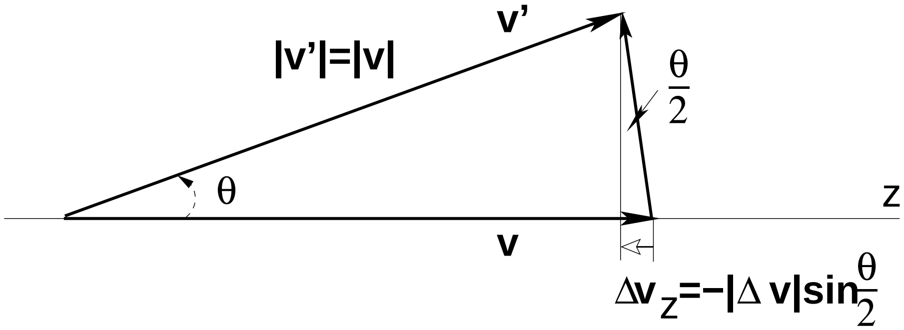
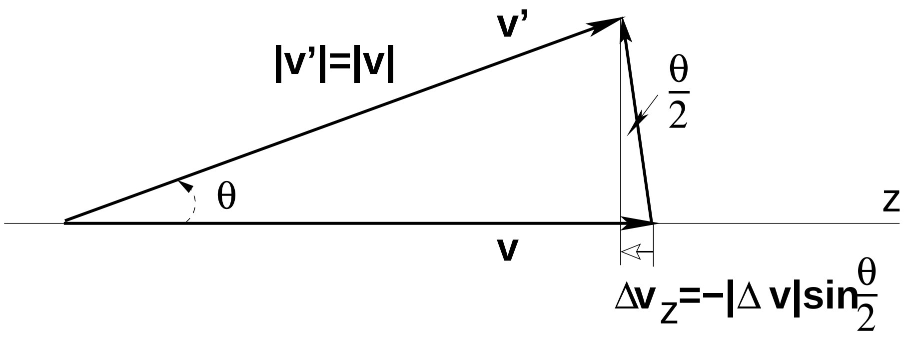


Simple geometrical considerations give $\tan\frac{\theta}{2} = \frac{b_\perp}{b} \Rightarrow \Delta v_z = -2v\sin^2\frac{\theta}{2} = -2v\frac{b_\perp^2}{b_\perp^2+b^2}$.



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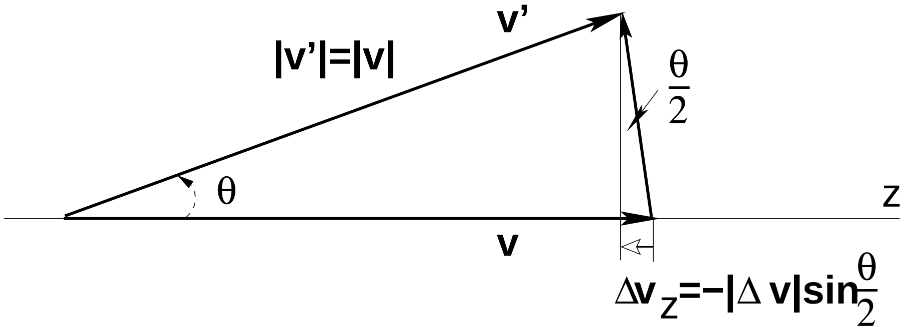
Compute the force on an particle α:

$$\mathbf{F} = -\frac{\mathbf{v}}{v}\int (m\Delta v_z)\left(n_\beta v d\sigma\right) = \frac{\mathbf{v}}{v}\left(m2vb_\perp^2 n_\beta v2\pi\right)\int_0^{b_{max}}\frac{bdb}{b_\perp^2+b^2} = \lambda\frac{4\pi}{m}z_\alpha^2 z_\beta^2 n_\beta\frac{\mathbf{v}}{v^3}.$$



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Or for particle α going through particles β:

$$\mathbf{F}_\alpha(\mathbf{v})=-\lambda 4\pi\frac{m_\alpha+m_\beta}{m_\alpha m_\beta}z_\alpha^2 z_\beta^2\int\frac{\mathbf{v}-\mathbf{v}'}{\left|\mathbf{v}-\mathbf{v}'\right|^3}f_\beta\left(\mathbf{v}'\right)d\mathbf{v}'.$$

Drag force - How it looks like?

$$F_\alpha(\mathbf{v}) \sim \int \frac{\mathbf{v}-\mathbf{v}'}{|\mathbf{v}-\mathbf{v}'|^3} f_\beta(\mathbf{v}')\, d\mathbf{v}'$$

Important analogy

Drag force - How it looks like?

$$F_\alpha(\mathbf{v}) \sim \int \frac{\mathbf{v}-\mathbf{v}'}{|\mathbf{v}-\mathbf{v}'|^3} f_\beta(\mathbf{v}')\, d\mathbf{v}'$$

Electric field:

$$E(\mathbf{r}) = \int \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} \rho(\mathbf{r}')\, d\mathbf{r}' = -\nabla\phi_E(\mathbf{r})$$

E-potential:

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Effectively “conservative”/potential drag force

$$F_\alpha(\mathbf{v}) = -\lambda \frac{m_\alpha + m_\beta}{m_\alpha m_\beta} \left(4\pi z_\alpha z_\beta\right)^2 \nabla_v \phi_\beta$$

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Trubnikov potentials are related to Rosenbluth potentials via

$$h_\alpha(\mathbf{v}) = -4\pi \sum_\beta \left(1 + \frac{m_\alpha}{m_\beta}\right) \phi_\beta(\mathbf{v})\,.$$

Compute “potential” ($\beta = i$)

$$\phi_i(\mathbf{v}) = \frac{-1}{4\pi}\int \frac{f_i(\mathbf{v}')}{|\mathbf{v}-\mathbf{v}'|}d\mathbf{v}'$$

EP drag on thermal ions

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For fast ions $\mathbf{v} \gg \mathbf{v}'$ and

$$\phi_i(\mathbf{v}) \simeq \frac{-1}{4\pi}\frac{\int f_i(\mathbf{v}')\,d\mathbf{v}'}{|\mathbf{v}|} = \frac{-n_i}{4\pi v}.$$

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Average velocity change is

$$m_\alpha \bar{\dot{\mathbf{v}}} = F_\alpha = -\lambda \frac{m_\alpha + m_i}{m_\alpha m_i} (4\pi z_\alpha z_i)^2 \nabla_v \phi_i = -4\pi\lambda \frac{m_\alpha + m_i}{m_\alpha m_i} n_i z_\alpha^2 z_i^2 \frac{\mathbf{v}}{v^3}.$$

Energy change is

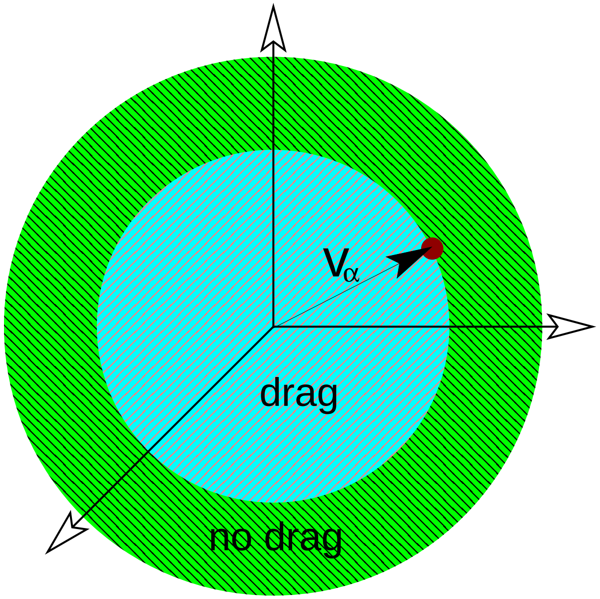
$$\dot{\mathscr{E}}_\alpha = -4\pi\lambda n_i \left(\frac{m_\alpha}{m_i} + 1 \right) z_\alpha^2 z_i^2 \frac{1}{v}.$$

EPs have weak interaction with ions.

Effective potential due to electrons

$$\phi_e(\mathbf{v}) = \frac{-1}{4\pi}\int \frac{f_e(\mathbf{v}')}{|\mathbf{v}-\mathbf{v}'|}d\mathbf{v}'$$

Electrostatic analogy and $v_{Te} \gg v_\alpha$



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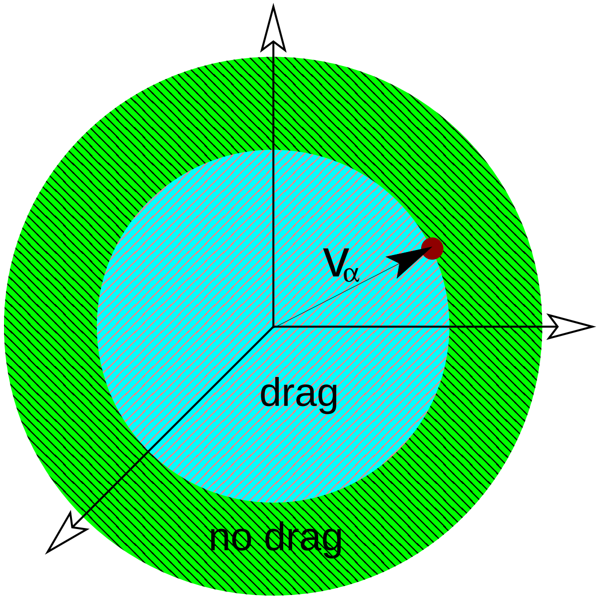

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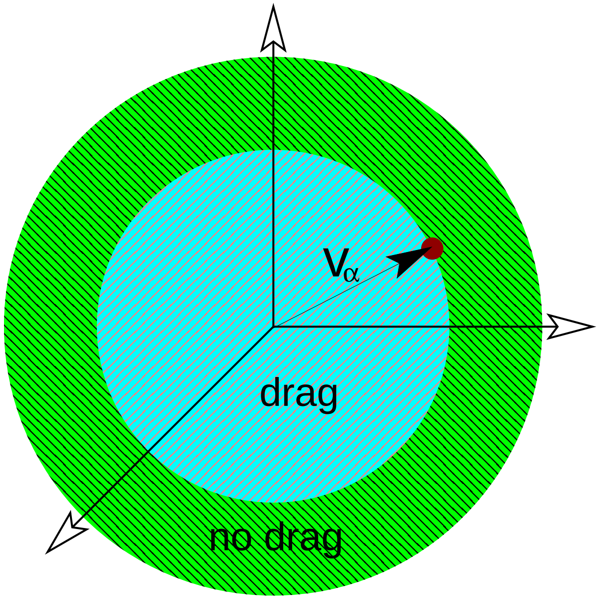

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Near origin

$$f_e = \frac{n_e}{\left(\sqrt{\pi}v_{Te}\right)^3}e^{-v'^2/v_{Te}^2} \simeq \frac{n_e}{\left(\sqrt{\pi}v_{Te}\right)^3}$$

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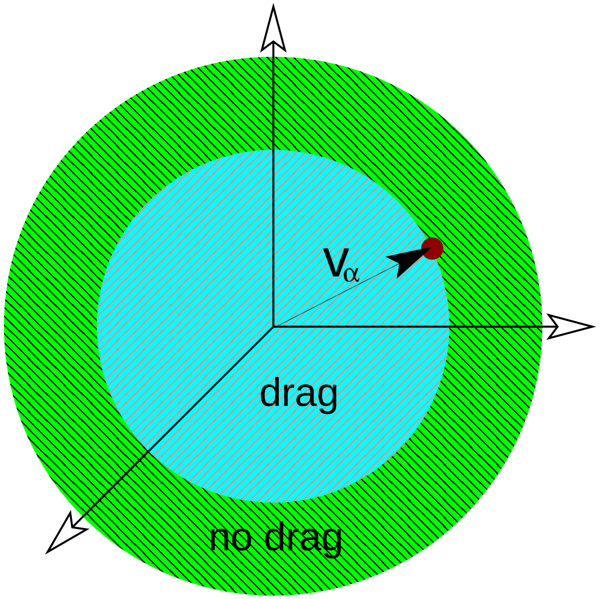

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And we find (change sign)

$$\phi_e(\mathbf{v}) = \frac{1}{4\pi} \frac{n_e}{\left(\sqrt{\pi} v_{Te}\right)^3} \frac{4\pi}{3} \frac{v^2}{2} = \frac{n_e}{\left(\sqrt{\pi} v_{Te}\right)^3} \frac{v^2}{6}.$$

Effective potential due to electrons

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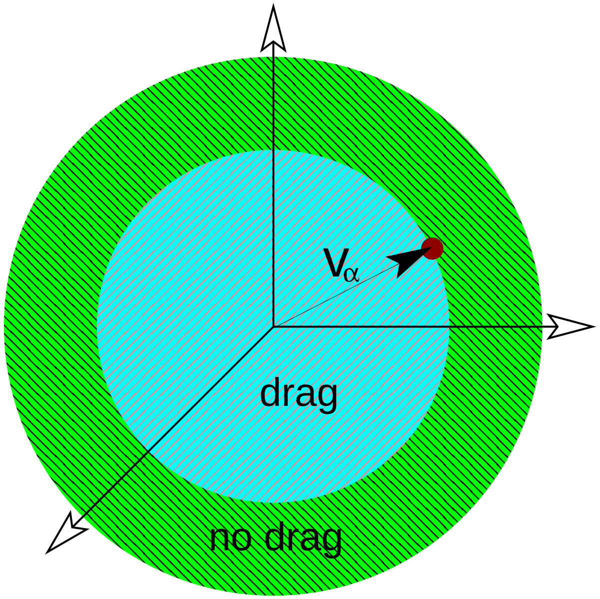
Electrostatic analogy and $v_{Te} \gg v_\alpha$



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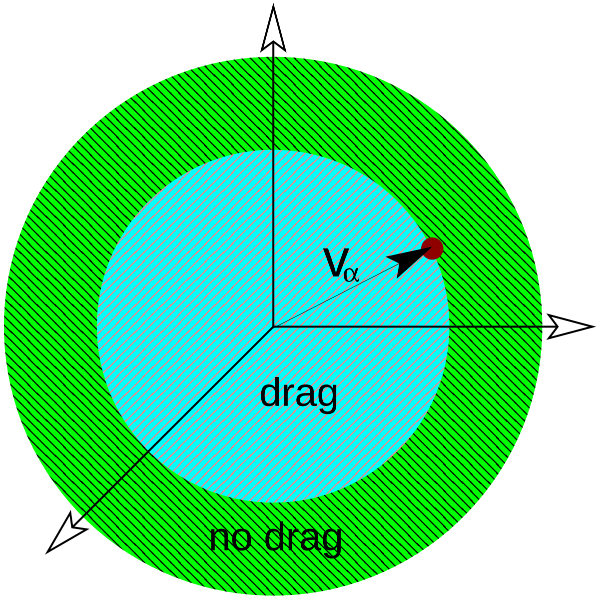

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Energy slowing down on electrons:

$$\dot{\mathscr{E}}_\alpha = -\frac{\lambda}{m_e}\frac{n_e(4\pi z_\alpha z_e)^2 v^2}{3\left(\sqrt{\pi}v_{Te}\right)^3}.$$

Kinetic equation for EP distribution function

Start with the Liouville's continuity equation in the phase space

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla \mathbf{v} f + \nabla_v \mathbf{a} f = -\nabla_v \frac{F_\alpha}{m_\alpha} f + S_\alpha$$

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where we introduced a characteristic slowing down time

$$\tau_{se} = \frac{3 m_\alpha m_e v_{Te}^3}{16 n_e \lambda z_\alpha^2 z_e^2 \sqrt{\pi}}$$

and "critical velocity" (electron and ion drags are comparable)

$$v_*^3 \simeq \frac{3\sqrt{\pi}}{4}\frac{m_e}{m_i} v_{Te}^3$$

Classical steady state EP distribution function

$\partial f/\partial t = 0$

$$\frac{1}{\tau_{se} v^2} \frac{\partial}{\partial v} \left(v^3 + v_*^3\right) f + S\left(v\right) = 0$$

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has a solution

$$f = \frac{S_0 \tau_{se}}{4\pi} \frac{v_0^2 H(v_0 - v)}{v^3 + v_*^3}.$$

H is step function.

What happens if plasma is not in steady state

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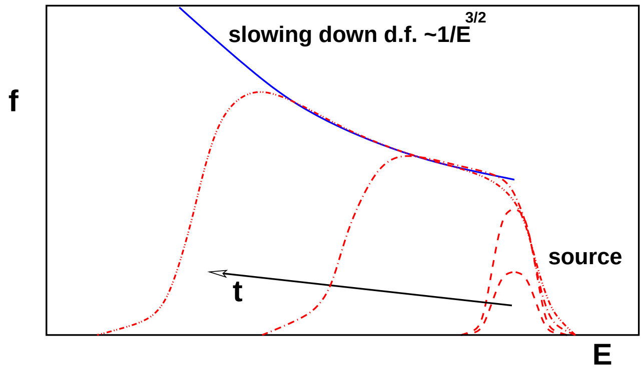

Classical EP confinement *in fusion plasma is demonstrated on first DT full scale laboratory experiments on TFTR*

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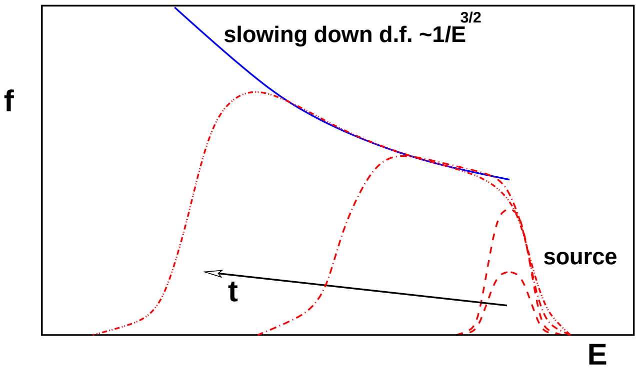
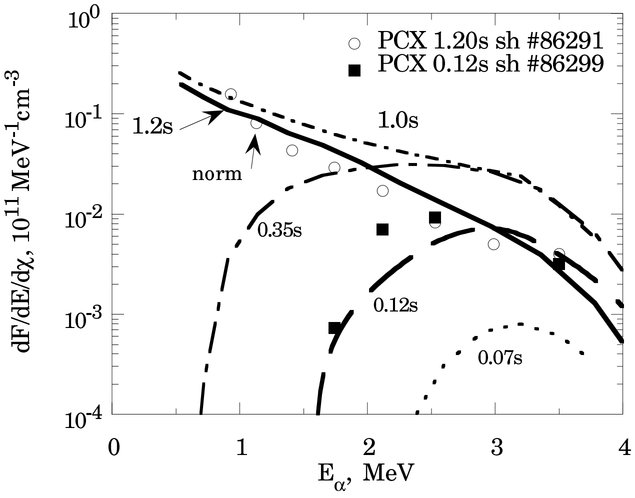
What happens if plasma is not in steady state

Numerical simulations of NPA in TFTR: more effects are included: (1) realistic geometry, (2) $S = S(v,t)$ *(Gorelenkov, NF'97)*





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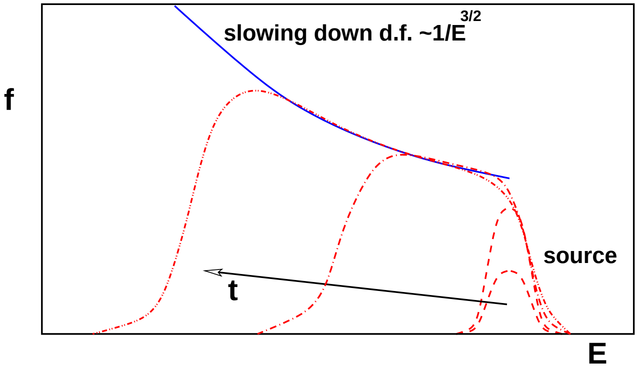
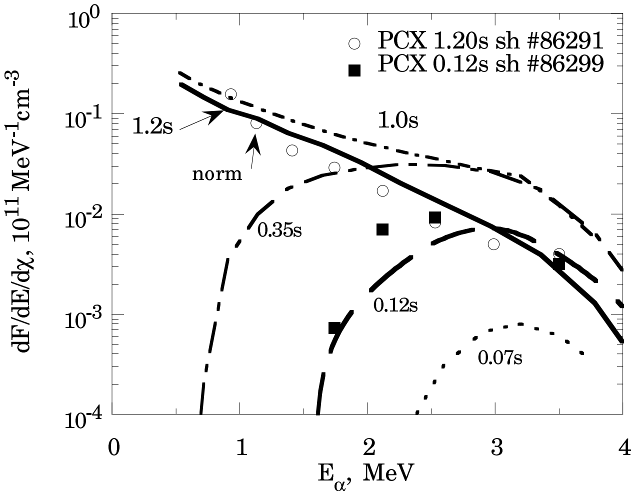
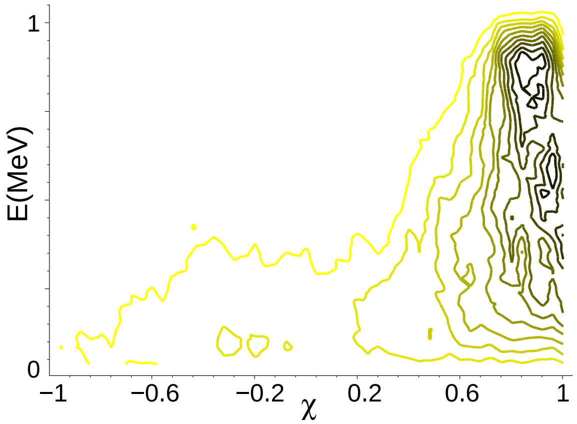




Energetic fusion α's are classically confined in DT plasma

Realistic distribution function reveals complex structures.

Example of beam ions distribution in ITER in ($\chi = v_{\|}//v, E_{\alpha}$) plane.



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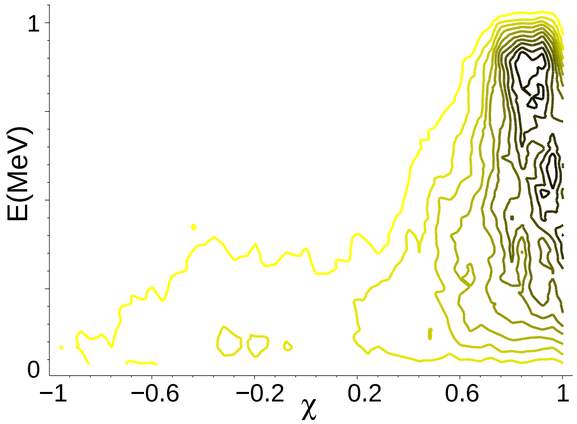

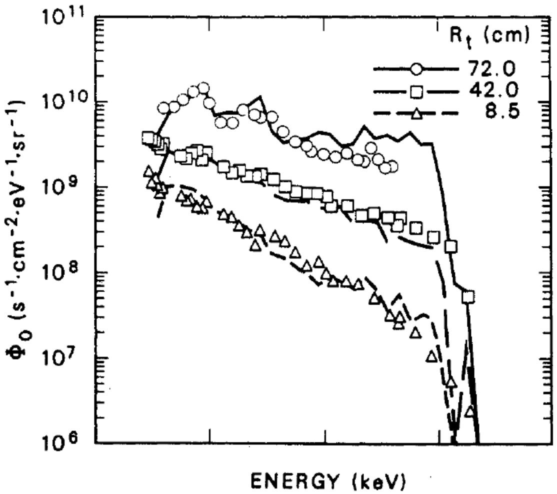

FIG. 21. Comparison of experimental and predicted fast neutral particle spectra during hydrogen beam injection into ISX-B [193]. The analysers scan in the horizontal midplane; R_t *represents the tangency radius. 30 keV co-going neutrals were injected at* $R_{tan} = 74.5$ *cm (*$R_0 = 93$ *cm).*

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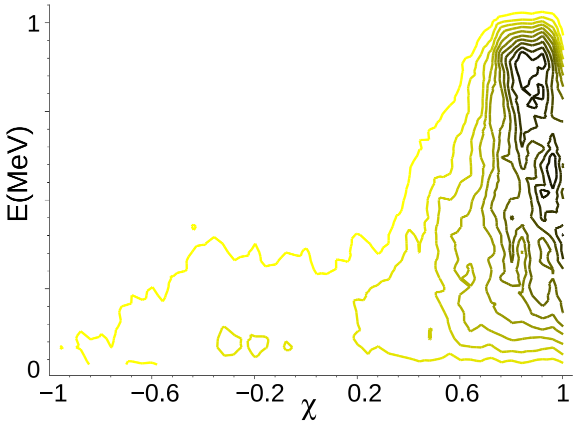

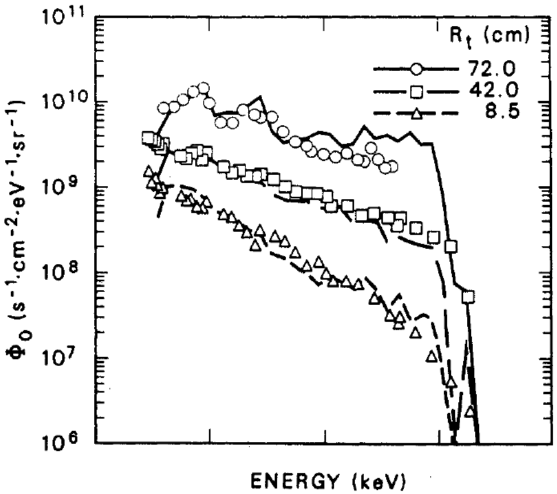

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Energetic particles are classically confined: Coulomb scattering

Another example: effect of losses

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Extra "loss" term in kinetic equation (Cordey, Goldston, Mikkelsen, '81) $\Rightarrow$:

$$\frac{1}{\tau_{se} v^2} \frac{\partial}{\partial v} \left(v^3 + v_*^3 \right) f - \frac{f}{\tau_{loss}} + S = 0$$

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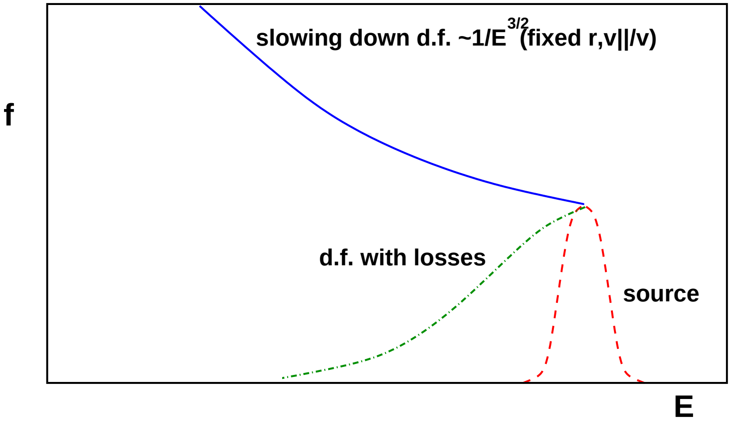

Direct spectrum measurement carries information about particle confinement

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At finite τ_{loss} we obtain

$$f = \frac{Cn_bH}{v^3+v_*^3}\left(\frac{v^3+v_*^3}{v_{b0}^3+v_*^3}\right)^{\tau_{se}/3\tau_{loss}}$$

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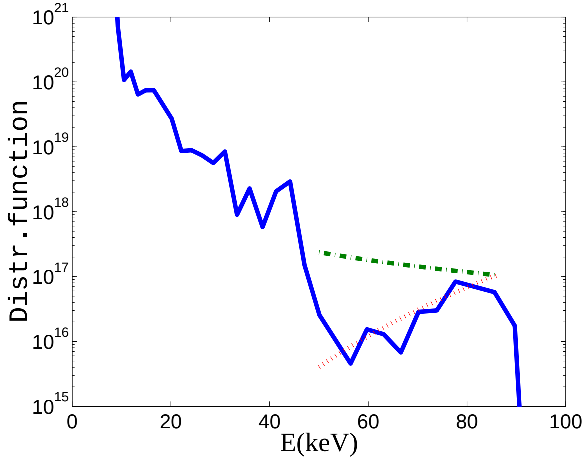

NPA (NSTX, Medley) against theory: what is τ_{loss}?
(don't mind low energy part)

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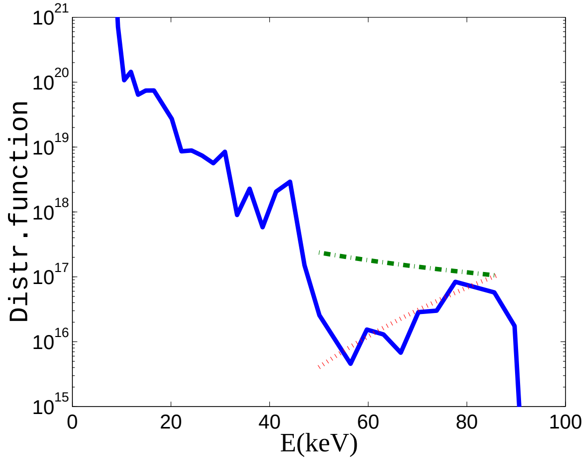

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Implications;

- $\tau_{loss} = \tau_{se}/15$, i.e. $\tau_{loss} = 4msec$
- only high energy part is affected $E_b > 60keV$.

Conclusions

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Classical confinement leads to the characteristic “slowing down” distribution function

Distribution of fusion products is close to “isotropic”

But issues remain on how the distribution of present day EP’s used for heating is described

- beam ion distribution
- ICRH minority particles

Once again distribution of EP effects:

- stability properties of plasma, EP - plasma interactions
- power balance
- confinement
- diagnostic needs